

Digital in-line holography for the characterization of two phase flows in astigmatic systems

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Abstract: This study details the characterization of two phase flows using digital holography in astigmatic systems through a generalized convolution method derived from the Collins formula.

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1. Introduction

Digital Holography (DH) is well-known for allowing, in a single shot, the complete 3D reconstruction of any particulate flow. As highlighted by Lamadie et al. [1], DH involves a very simple measurement setup, that is not sensitive to vibrations and to most typical environmental factors, and is therefore particularly suitable for the characterization of two phase flows. The authors demonstrated the feasibility of the measurements for a liquid/liquid flow, considering a cuboid flow cell. However, most standard devices designed for chemical engineering (for instance pulsed columns, centrifugal extractor, etc.) exhibit a cylindrical geometry inducing astigmatism, thus rendering the classical hologram simulation and focusing methods, inappropriate.

Two main approaches are possible to tackle astigmatism. The first one, introduced by Collins et al. in 1970 [2] is a general expression of the Rayleigh-Sommerfeld integral, including optical system in paraxial approximation thanks to an ABCD transfer matrix. It gives the relationship between the output and input of the complex amplitude distributions of the light field passing through an ABCD optical system. In this context, Rayleigh-Sommerfeld diffraction integral could be analytically expressed, as proved by Verrier et al. [3] or used through a generalized Fresnel Transform (GFT) propagator as described by Palma et al. [4]. The second approach, introduced by Ozaktas et al. [5], relies on the Fractional Fourier Transform (FrFT) properties. According to Ozaktas, all quadratic phase systems (QPS) can be interpreted as a magnified FrFT with a phase curvature.

In this contribution, the characterization of a two-phase flow in a cylindrical cell using DH is described. The first section is devoted to hologram simulation in astigmatic optical system through the GFT propagator and a supplementary Generalized Impulse Response (GIR) propagator. The second section introduces a Best Fitting Approach (BFA) based on GFT and GIR propagators to retrieve the 3D position and size of spherical particles. To improve the positioning of the particles along the optical axis z , a focusing method based on the maximal intensity position of the photonic jet is developed.

2. Hologram simulation in astigmatic geometry

Astigmatic optical systems exhibit different properties along the two axes (x and y) perpendicular to the direction of propagation z . In digital holography, focusing methods usually involve a numerical model for the simulation of the holograms. While in free space this point is easily achieved using a Fresnel Transfer (FT) function or an Impulse Response (IR) propagator [6], this approach is no longer applicable to astigmatic optical systems. Indeed, for such system it is mandatory to differentiate the propagation in x and y directions, and accordingly the kernel functions of the propagators must be asymmetrical.

As mentioned in the introduction, in the paraxial approximation all optical systems can be described by an ABCD matrix [7]. To deal with astigmatic optical systems, this approach is particularly relevant as it allows to use two different matrices $ABCD_x$ and $ABCD_y$ to describe the system the along x and y axis respectively.

In free space, the Rayleigh-Sommerfeld integral describes the diffracted amplitude A_z at distance z for a monochromatic wave. In astigmatic optical systems, the diffracted amplitude can be expressed through the Collins formula [2, 4]. This formula is a generalization of the free space diffraction integral for an optical system represented by an ABCD matrix in the paraxial approximation. Following this formula, it reads for the wave amplitude A_z :

$$A_z(x, y) = \frac{\exp(ikz)}{i\lambda\sqrt{B_x B_y}} \iint A(u, v) \exp\left(i\pi \frac{A_x u^2 - 2xu + D_x x^2}{\lambda B_x}\right) \exp\left(i\pi \frac{A_y v^2 - 2yv + D_y y^2}{\lambda B_y}\right) dudv \quad (1)$$

with $A(u, v)$ the complex amplitude distribution of the light field before propagation, λ the wavelength, z the distance of propagation along the optical axis and $k = 2\pi/\lambda$ the wave number. The coefficients A_x, B_x, C_x, D_x and A_y, B_y, C_y, D_y are the elements of the wave matrix in x and y direction respectively.

For each particle, the simulation of the recorded intensity is performed by three basic operations, *i*) propagation of the incident wave from the source to the particle plane using Eq. (1) and $ABCD_x = ABCD_{1x}$, $ABCD_y = ABCD_{1y}$ matrices, *ii*) multiplication of the calculated field, at the z -particle position, by the particle transmission function T , and *iii*) propagation of the resulting field from the particle plane to the sensor position using Eq. (1) and $ABCD_x = ABCD_{2x}$, $ABCD_y = ABCD_{2y}$ matrices.

As for free space, the main difficulty is to evaluate numerically the Collins formula. In literature several approaches are presented to address this issue. Among them, Ozaktas expresses Eq. (1) as a magnified FrFT with a phase curvature [5]. Another approach is to derive an analytical expression of the resulting amplitude after the two propagations. Indeed, as demonstrated, by Verrier *et al.*, the amplitude A_z can be calculated thanks to an analytical solution by considering an incident Gaussian beam profile [8].

Finally, as for free space, the Collins integral can be rewritten as a convolution product as follow:

$$A_z(x, y) = \frac{\exp(ikz)}{i\lambda A_x A_y \sqrt{B_x B_y}} \exp\left(i\pi \frac{C_x x^2}{\lambda A_x}\right) \exp\left(i\pi \frac{C_y y^2}{\lambda A_y}\right) \left[A\left(\frac{u}{A_x}, \frac{v}{A_y}\right) * h(u, v) \right](x, y) \quad (2)$$

with $h(u, v) = \exp(i\pi u^2/\lambda A_x B_x) \exp(i\pi v^2/\lambda A_y B_y)$ the Generalized Impulse Response (GIR). A Generalized Fresnel Transfer (GFT) function H can be expressed by calculating the Fourier transform of the GIR:

$$H(u, v) = \exp(-i\pi A_x B_x \vartheta_x^2) \exp(-i\pi A_y B_y \vartheta_y^2) \quad (3)$$

with $\vartheta_x = x/(\lambda A_x B_x)$, $\vartheta_y = y/(\lambda A_y B_y)$ the spatial frequencies along x and y respectively.

Then the evaluation of Eq. 1 is achieved through two successive Fourier transforms:

$$A * h = FT^{-1}(FT(A).H) \quad (4)$$

Required sampling conditions on the supplementary quadratic phase $\exp(i\pi C_x x^2/\lambda A_x) \exp(i\pi C_y y^2/\lambda A_y)$ regarding Shannon theorem, bring sampling issues for the propagation (*iii*) that can be addressed by considering a general system merging the two initial ABCD systems. This generalized convolution method allows to describe the particle transmission function T without limitations regarding shape and relative refractive index of the particles. A comparison between Verrier approach, GFT approach and experimental acquisition is presented in Figure 1 for the astigmatic experimental setup described in [1]. A good agreement is observed between all holograms.

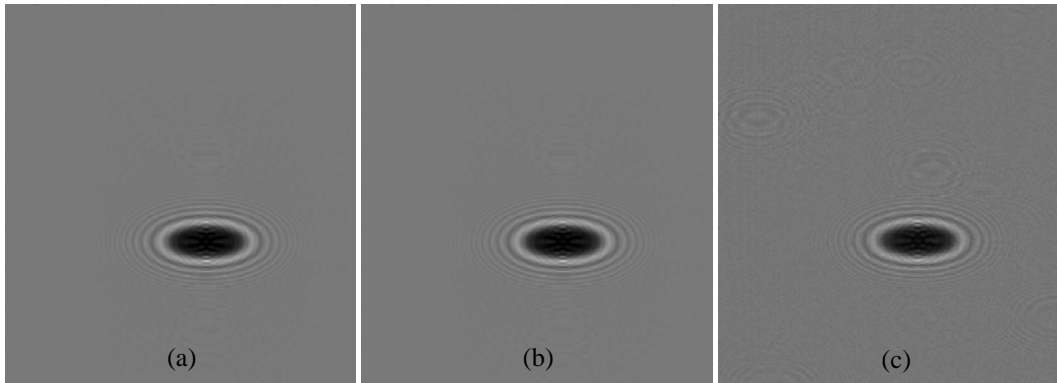


Figure 1 : Intensity recorded for a 1mm diameter oil droplet rising in water through an astigmatic experimental [1] - (a) Analytical solution, (b) GFT, (c) experimental acquisition.

3. Particles parameters retrieval

The particles characterization from a recorded hologram is realized through two main steps.

In the first step, a Best Fitting Approach (BFA) is applied to retrieve the (x, y) position and the diameter D of the particles. This latter consists in a simple least square minimization between the experimental acquisition and the numerical holograms simulated with the approach detailed in the previous section.

The second step consists in finding the best focusing plane along the z -axis. To achieve this, the field is reconstructed in successive planes along the optical axis by convoluting the recorded hologram with GIR or GFT. Then, using the photonic jet generated by a particle when interacting with a coherent light, a maximum intensity position z_{\max} is detected thanks to the Tenengrad variance [9]. This maximum corresponds to the “focal length” of an equivalent ball shaped lens. Following these assumptions, the z -position of the center of the particle can be easily expressed as:

$$z \approx z_{\max} - \frac{nD}{4(n-1)} \quad (5)$$

With n the relative refractive index of the particle.

An example of the reconstruction of a polydisperse population from an experimental hologram acquired in an astigmatic experimental setup is given in Figure 2, in the case of rising oil droplets

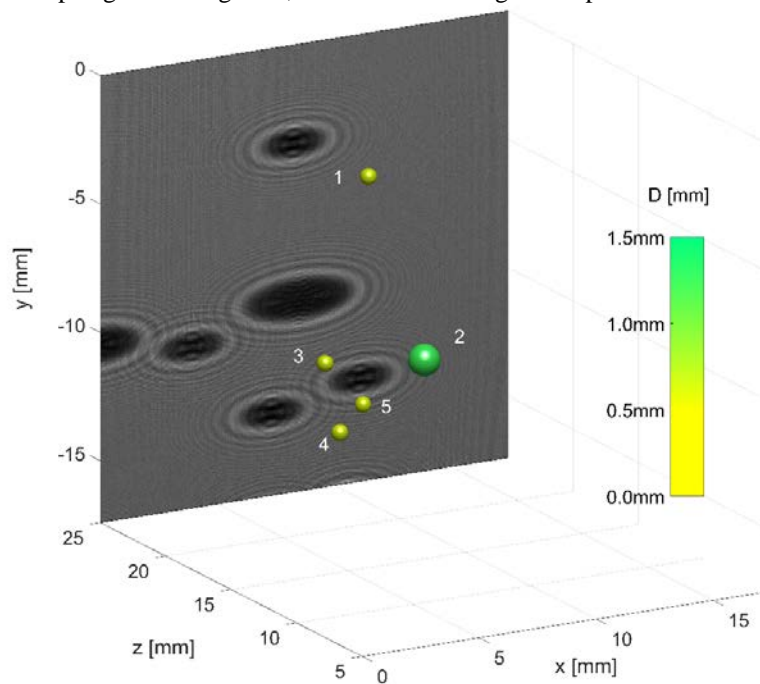


Figure 2: Reconstruction of five rising oil droplets in an astigmatic experimental setup.

4. References

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