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A unique probe for nuclear structure in a future European radioactive ion –electron collider

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Project to be developed in collaboration with:
- Nuclear experiment groups: mainly physicists at GANIL, IJCLab and LPC Caen, SCRIT team at RIKEN and Tohoku University, physicist groups at TUD Darmstadt and GSI.
- Nuclear theory groups, mainly theorists from CEA DAM CEA DRF DPhN, TUD Darmstadt.

Abstract
Electron scattering on radioactive nuclei could provide nuclear observables with an unprecedented radial sensitivity for exotic systems. In these nuclei, unique quantum phenomena occur but basic properties such as charge densities are still completely unknown. Precise densities extracted from the scattering data could be confronted to modern structure calculations. Such studies would provide better insight both on the accuracy of the many-body treatment of the new techniques and on the validity of the microscopic characteristics of realistic nuclear interactions used in state-of-the-art nuclear models.
Our goal is to build an electron beam accelerator implanted at a facility providing a variety of radioactive ion beams (RIBs), such as GANIL, to perform electron-radioactive ion (e-RI) collisions and measure cross sections and excitation spectra*. The measurements of electron scattering can be renewed taking advantage of the combined progress done in the fields of theory, accelerator techniques and experiments. Developments in the accelerator design and electron-ion collision techniques would offer the increased luminosities required for the project from 10^{26} up to 10^{28-29} $cm^{-2}s^{-1}$ for the main physics cases of the first step, expanding to 10^{29-31} $cm^{-2}s^{-1}$ in the long-term range for the inelastic form factors.*

Background. *We recall in our document the main scientific motivations of our contribution given in March to the international committee "Nuclear structure from electron-ion collisions" [**ERIBd20**] in the framework of the GANIL prospectives. It follows and expands proposals raised in the NuPECC Long Range Plan 2017 Perspectives in Nuclear Physics [**LRP2017**]. Then we focus on the conditions for the feasibility of the potential machine, describing the main technical constraints, questions and challenges of the project that the community has to handle to realize this project in the years 2030.*

Context*. We mention that two complementary machines could be conceived and built in Europe to operate electron-ion collisions, one built at GANIL using the techniques presented in this document and alternative one at GSI/FAIR (with the techniques presented in a separate contribution to this LRP). Our common goals of new nuclear observables deserve two projects at the European level, as was the case in the years 70-80 when electron accelerator facilities were spreading all over the world to make electron-nucleus scattering Experiments. Timescale of the projects and of the experimental exploitation would not be the same and would offer complementary approaches to reach the goals. The discussions of the communities involved in these two contributions will naturally take place within the NuPECC framework, taking the opportunity of the future discussions around the final report for the LRP. The possibilities for common R&D, task sharing and future works (detection systems, running of experiments) on both facilities will be also examined once both projects are entering in a conceptual design phase, hopefully after the process of the LRP 2022-2024.*

An electronic microscope on the nuclear structure

I. Objectives – Extension of our knowledge on nuclear densities

I. 1. Past milestones on nuclear densities

Since the pioneering work of R. Hofstadter distinguished by the Nobel prize in 1961 [***Hof53***], decades of experimentation [***FroP87, Ae91***] have demonstrated that electron scattering is one of the best probes to study the structure of hadronic systems such as nuclei and nucleons. The spatial resolution offered by electrons of several hundreds of MeV allows to extract a variety of spatially-dependent distributions (radial charge density, charge transition density, magnetic current distributions) very constraining for nuclear models and going beyond integrated quantities (mean square radii, electromagnetic transition probabilities).

The strength of this resolving power is the ability to go from a one-dimensional excitation function of a nucleus to the full nuclear response surfaces of this system (also called dynamic structure functions) using electron elastic and inelastic scattering. These longitudinal and transverse dynamic structure functions **S_L(q, w)** and **S_T(q, w)** are functions of **q**, the transferred momentum in the electron scattering off target, and of **w** the total energy change in the reaction (**w=0** for the elastic scattering). **These observables contain all information on the distribution of the nuclear electromagnetic current density.**

The richness of the above-mentioned resolving power combined with the precise knowledge of the electromagnetic scattering process (via QED) motivated worldwide research programs of electron scattering on stable nuclei from the 50s until the end of the 90s whose results are at the heart of our current understanding of nuclei. To reach these achievements, intensive cross-section measurements were done varying the electron beam energy to extend the range of transfer momenta and took place in the different facilities developed for this purpose during this period. The worldwide panorama for these studies included: in the United States, the MIT Bates Laboratory, the SLAC followed by the Jefferson Laboratory (starting, end of 90s, the research on the quasi-elastic electron scattering on nuclei and on the nucleon structure) and, for instance, in Europe, the Saclay Laboratory in the 80-90s. These labs provided most of the nuclear data we have up to now, compiled in the tables [***ANDT87***] and used as references to discuss the ground state radii and densities of stable nuclei, and also the transition densities to excited states via (e,e') and (e,e'p) spectroscopy.

The most illustrative and textbook results of these worldwide efforts are shown in [***FroP87***] (*Figure 1 on the right side*): the elastic scattering cross sections of electron on ^{208}Pb were measured worldwide in various energy ranges, giving a complete data set on about 12 orders of magnitude for momentum transfers from ~0.5 up to 3.9 fm^{-1}.

From the model-independent analysis of these data, the density could be extracted precisely down to very small radius, and compared to the state-of-the-art theory calculations available in this period. Another typical example illustrating the insight on the microscopic configurations is the charge distribution of a 3s proton in ^{206}Pb which was extracted from the difference of its measured charge density with ^{205}Tl in [***Fro83***].

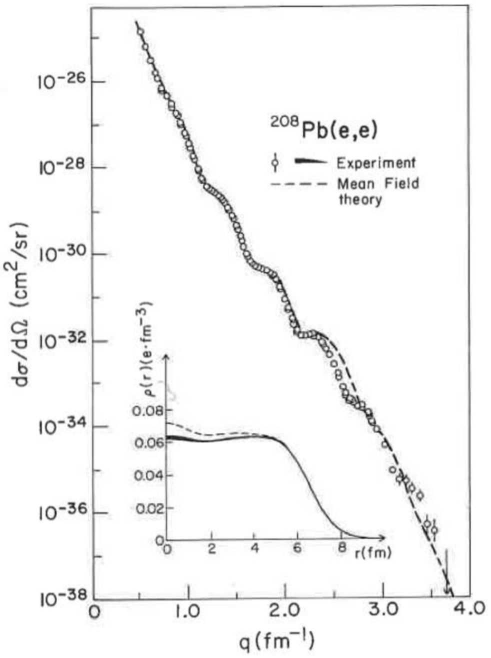

*Figure 1. The density distributions could be extracted for ^{208}Pb from a synthesis of the (e,e) cross section data collected worldwide [**FroP87**].*

The point-like nature of the electrons (excellent spatial resolutions) and the fact that the electromagnetic interaction is weak (low re-scattering rates and theoretically well-constrained tools, e.g. perturbation theory) make the reaction mechanism under control. The history of this research field demonstrates that electrons constitute optimal probes for a clean extraction of several properties of atomic nuclei. Details on the variety of observables accessible using these processes are provided below.

I.2 Next-generation nuclear structure studies with electron-RI collisions

At the end of the 80s, the area of radioactive ion beam facilities started. Since it was not possible to make targets of the short-lived isotopes, nuclear structure investigations have been pursued mainly using hadronic probes. Integrated information was obtained for root mean square (rms) radii of charge densities (r_{ch}) via laser/muonic spectroscopies of unstable radioactive ion nuclei, and for nuclear matter radii from RI collisions with hadronic probe (proton, heavy ion beams). The present knowledge we have on structure density observables for radioactive nuclei are a combination of measurements obtained from these probes [***Diffpp***]. Despite the discussion on the accuracy of the models used to extract the information from the measurements, if we focus on the benchmark microscopic models used to obtain the structure characteristics (rms radii, moments), the precision on these values are due to the involved physical process itself, resulting in charge rms at the level of ~± 0.005 -0.01 fm ; matter rms r_m at ± 0.1 fm, and neutron skin up to r_n-r_p ~ ± 0.1 fm.

Via future e-RI experiments, we propose first to investigate directly the charge density distributions of these exotic systems in which unique quantum phenomena emerge.

We plan to start an extensive program to measure (e,e) elastic scattering cross sections to extract directly the charge density distributions through a model-independent analysis (*e.g. using analytical functions such as Fourier-Bessel)* and to compare them to theoretical predictions. Detailed densities are much more demanding than integrated quantities (such as rms radii) and encapsulate different correlation effects. As such, they offer an unprecedented test bench for state-of-the-art nuclear structure models. Their availability over a wide range of unstable isotopes would thus systematically provide model-independent constraints very complementary to information from other probes like (p,p) scattering.

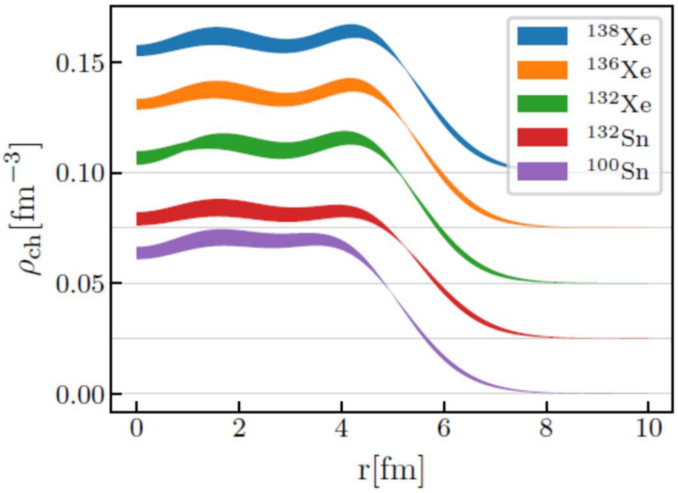

*Figure 2. Ab initio computation of charge densities for Sn and Xe isotopes. Figure from Fig.4 of [**Th132Xe**] ("the charge density is shifted upward by 0.025 fm^{-3} between each two nuclei, and the colored bands indicate the theoretical error associated with model space convergence.") These calculations were found to provide quantitative reproduction of the charge densities for the ^{132}Xe: the densities are consistent with the experimental ones (analytical functions 2pF) extracted from the analysis of the ^{132}Xe(e,e) measurements done with the SCRIT device [**Scrit17**].*

Correspondingly, new theoretical tools will be put in place for electron-nucleus collisions taking advantage of the renewal of the calculations for the e-RI scattering observables. On the one hand, some of the knowledge acquired in the past on electron collisions with stable nuclei has to be recovered and revised in light of recent advances. For instance, for the e-RI elastic scattering observables of differential cross sections and electric charge form factors, we can apply the techniques elaborated in [***TheRI08***], using a relativistic eikonal approximation of the Dirac equation. On the other hand, newly developed approaches need to be adapted and generalized to the description of new processes and observables. In particular, the interpretation of the quasifree (e,e'p) scattering processes would benefit from a revision of the reaction mechanism to investigate final state interactions, including consistently nuclear short- and long-range-correlation effects [***TheoLRC09***].

For the (e,e) scattering at 500 MeV with the ^{132}Sn and ^{136}Xe ions, at luminosities 10^{29} cm^{-2} s^{-1}, we should be able to extract from the cross sections the charge densities which could be directly compared to the structure model calculations, like the *ab initio* presented in [***Th132Xe***] (***Fig. 2***).

Furthermore, as pointed out in recent works [***rmsN20***], insights onto the mean square radii of the neutron densities could also be obtained from the analysis of the 4th moment of the nuclear charge density extracted from the electron elastic scattering. It would offer new data used as a benchmark for nuclear theories.

I.3 Observables and luminosity

For a simple case of a spin-0 nucleus, the form factor F(q) (with q the momentum transfer) is a Fourier Transform of the nuclear charge densities (ρ_{ch}). In the general case for the scattering of electrons on a spin-J finite-size nucleus, the cross sections are function of the square of the longitudinal F_L and transverse F_T form factors [***Don75***]. For a spin-0 nucleus, the cross section can be expressed as a product of the Mott cross section (point charge cross section) and of the form factor $F_L(q) = F(q)$. Neutron densities could also be investigated via the magnetic form factors extracted from cross sections of the (e,e) measurements. F_T can be expanded in sum of the square of the magnetic (odd) multipole M_i (I = 1, 3, .. up to Λ = 2Jo). The extraction of the F_T form factors gives access to the entire shape of the valence nucleons, if the cross sections are measured in a wide range of momentum transfer q, this was the case for typically electron energies up to 500-700 MeV.

We discuss here the required luminosities, looking at the main effects of the Z^2 and $1/q^4$ dependence of the cross sections. Extracting charge densities require high enough momentum transfers, typically up to 3-4 fm^{-1} for medium-heavy nuclei (Z > 10) which means working at electron energies at least up to 500 MeV. In this case, the extreme measurement of the cross sections should go down to 10^{-38} cm^{-2} sr^{-1}.

We discuss the orders of magnitude of the luminosity L for the desired observables presented in ***Tab.Obs*** [*for light ($Z^2 \leq 100$); medium ($100 < Z^2 \leq 32^2 = 1024$) or heavy ($Z^2 > 1024$) nuclei]*. For comparison, we consider the figures of L in the case of the electron scattering on stable targets. For an electron beam of intensity I_e (A), impinging a target containing N_T nuclei (of mass and element numbers A and Z) over an interacting area of S cm^2, the instantaneous L (geometrical) is defined as (with Q_e the electron charge): $L = (I_e/Q_e) * (N_T/S)$ cm^{-2} s^{-1}. In the case of the most complete studies done in the past on the structure of stable nuclei, at the high momentum transfers (3-4 fm^{-1}), the luminosities were ranging from 12 to 1.2 10^{36} cm^{-2} s^{-1} for *A* ~ 20 to 200.

Observables **deduced quantities**	**Reactions** *(q: momentum transfer)*	**Type of nucleus**	**Required luminosity L**
rms charge radii	**(e,e)** *elastic at small q*	Light ($Z^2 \leq 100$)	L: 10^{24} $cm^{-2}s^{-1}$
Charge density distribution with 2 parameter Fermi function (2pF) ρ_{ch}	**(e,e)** *First min. in elastic form factor*	Light Medium Heavy	**L: 10^{28} 10^{26} $cm^{-2}s^{-1}$ 10^{24}**
Charge density distribution with 3pF ρ_{ch}	**(e,e)** *2nd min. in elastic form factor*	Medium Heavy	**L: 10^{29} $cm^{-2}s^{-1}$ 10^{26}**
F_L, F_T Magnetic form factors→ Proton, neutron transition densities *Direct access to neutron-skin*	**(e,e)** *2nd min. in elastic form factor*	Odd-even Medium Heavy	**L: $10^{30}$$cm^{-2}s^{-1}$ 10^{29}**
Energy spectra, width, strength, decays, collective excitations	**(e,e')**	Medium-Heavy	L: 10^{28-29} $cm^{-2}s^{-1}$
Extraction of the density distribution *using functionals (series of Fourier-Bessel functions …)*	**(e,e)** **(e,e')**	Light Medium-Heavy	**(e,e) (e,e') L : 10^{30-31}** **(e,e) (e,e') L ~10^{29-30}**
Spectral functions, correlations	**(e,e'p)**		**10^{30-31} $cm^{-2}s^{-1}$**

Table Obs. *New structure observables from e-RI collisions and required luminosity.*

However, to reach the charge density parameter in simple form like the 2 or 3pF function, the measurement can be done at smaller momentum transfers, around 0.5-2.5 fm^{-1} where the cross sections are a factor 10^{6-10} higher, thus requesting much lower luminosities, typically 10^{26-30} cm^{-2} s^{-1}, depending on the type of ions (since the cross sections evolve with a Z^2 factor). The insight one can get into density distributions depends on the accuracy of the measured form factor and the range of momentum transfer covered.

This translates into luminosity constraints to access different structure observables. It is worth mentioning that the main ideas discussed below on the required observables were outlined in the NuPECC long-range plan (LRP) in 2016-2017 [***LRP2017***]. In this context, it was pointed out that electron beams of 400-800 MeV provide the ideal spatial resolution scale of about 0.5 fm to study charge distributions. The recommendations of the NuPECC

community were written as follows: « *Ion-electron colliders represent a crucial innovative perspective in nuclear physics to be pushed forward in the coming decade. They would require the development of intense electron machines to be installed at facilities where a large variety of radioactive ions can be produced* ».
As an example, for elastic scattering:

- Global indicators are accessible first, like rms radii and diffuseness, to model densities by simple analytical functions (such as 2pF), as done extensively for stable nuclei reported in nuclear data tables [***ANDT87***]. This can be achieved with luminosity starting from **10^{24} for heavy nuclei to 10^{28} cm^{-2}.s^{-1} for lighter ones** (lower Z).
- When form factors are measured precisely over an extended momentum transfer, the charge density can be extracted via a model independent analysis [***Sick74***]. It corresponds to differential cross sections measured up to the second minimum and translates into luminosities in the range **10^{26-29} cm^{-2}.s^{-1}** (again depending on Z).

From luminosities around 10^{29} cm^{-2}.s^{-1} and higher, the study of other processes mentioned previously can be reached offering unprecedented possibilities as stated in [***LRP2017***] : « *As a long-term goal, such facilities would allow (e,e') inelastic scattering with selectivity to the transferred angular momentum, (e,e'f) electro-fission with detection of fragments, and (e,e'p) quasi free-scattering studies with radioactive ions* ».
An objective of the luminosity at **10^{29} cm^{-2}.s^{-1}** is thus considered to reach an entirely new range of research and opening the way for further upgrades. **The long-term goal of a European eRI collider would be to extend progressively the applicability of those methods to a broader range of nuclei allowing systematic model-independent studies leading to the build-up of nuclear data tables away from stability in the years 2040s.**
For the first steps of this project, it is clear from the above table that the potential test cases will correspond to isotopes produced at intensities higher than 10^7 part/s. There will be the feasible nuclei (10^{7-8} and higher) for which the detailed structure information could be reached, and, depending on the Z number of the nucleus, some intermediate test cases with lower requirements on the luminosity and on the intensities (around 10^6).
Whatever the technique, since the electron-ion collisions require nuclei with lifetime above (a few) 100 ms at least, there are physics cases which will not be feasible, corresponding to all the nuclei with too short lifetimes.
For the choice of the beam energy, we can consider that reaching 500 MeV gives us most of the essential physical cases that we want to investigate. The final choice is a compromise between luminosity, cross sections, access to high transfer momenta. The 400-500 MeV is certainly a better choice than the 700-800 MeV regime where the cross sections are lower, and better than the 200-300 MeV range, for which the electron beam properties are degraded as a function of the decreasing energy.
In summary, the main physics cases will correspond to the (e,e) and (e,e') experiments done at luminosities 10^{26-29} (for nuclei with Z^2 >100, medium and heavy isotopes), leading to the extraction of the form factor at large q values (4 fm^{-1} with the electron energy at 500 MeV) which will give access to the charge and transition densities for the radioactive nuclei with lifetimes >100 ms and produced at rates higher than 10^7/s.

II. Techniques for the electron-ion collider: ion trap and electron accelerator

Since the 2000s, the physics of electron-ion collisions [***eRIB17***] has been part of the main international projects (ELISE at FAIR, DERICA at Dubna, MUSES at RIKEN-RIBF) aiming to probe the nuclear structure of exotic nuclei. However, the technical difficulties for colliding an intense electron beam with a RIB between 10^6 and 10^9/s were not solved. Due to the complexity and cost of the electron-ion collider machine design, an alternative was explored and found with the SCRIT (Self Confined Radioactive Ion Target) [***Scrit05***], project launched in the years 2004-2005 at RIKEN in Japan. At that time, it represented the world's first attempt of an electron-scattering facility for exotic nuclei [***Scrit04***]. It consists of a dedicated electron storage ring device with circulating electron bunches colliding with trapped ions. **The "ion trapping"phenomenon forms a local target** which makes electron scattering off short-lived radioactive nuclei possible in principle. In the years 2015-2016, SCRIT became operative at RIKEN for physics runs, results were obtained at E_e= 151, 201 and 301 MeV with the stable isotopes ^{132}Xe [**Scrit17**]. A few **10^{27}cm^{-2}.s^{-1}** represents up to now the current limit of the device. The luminosity was reached with a mean electron beam intensity of **200 mA** and around **10^8 target ions**. The electron beam size was "2 mmH x 1mmV (σ) at the center of SCRIT". The quality of the data for ^{132}Xe(e,e) were sufficient to be compared to recent *ab initio* calculations [***Th132Xe***]. The numbers of the running conditions for the SCRIT facility represent some benchmarks to explore the techniques of the ion trap self-confinement to be used for the electron –RI collisions. As explained above, these studies require luminosities ranging from 10^{26} to $10^{29,}$ up to 10^{30-31} for (e,e'p). Taking into account i) the physical constraints, ii) the state of the art of electron machines, and their potential upgrade in the near future and iii) realistic conditions of the ion-trap techniques, we obtained the required parameters: an accelerator of electron beams at 100 mA intensity –with potential upgrade at 200 mA; energies between 500 and 700 MeV; an ion trap device able to confine up to 10^{7-8} ions/s, which represents the true challenge of the project.

Technical solutions, issues; synthesis of the timeline of the project

If we now compare possible technological solutions for the electron accelerator, we have to examine the machine characteristics together with the potential solutions for the project. Compared to an Electron Recovery Linac (ERL)-type design limited to few 10 mA intensity, the synchrotron, with possible intensity at 100 mA, matches the physics constraints. Moreover, the European institutes working at the RIB facilities have the expertise of this category of machine, which would optimize the engineer developing time for the implantation of the new electron accelerator. It has well-known cost estimate. Furthermore, the operation of a synchrotron is much less demanding than the other machines. It constitutes the most advantageous technical solution. **Exploring the running conditions, identifying the technical constraints and the beam limitations for the operation of the electron-ion collisions would be the purpose of the full design study in a 5-year period.** With the know-how of the electron synchrotron technology, the planned schedule for the design and building of the electron accelerator would be straightforward. The challenge of the project lies in the testing of the ion trap capabilities to reach the targeted luminosities of around 10^{29} – that are hardly insured, because of (amongst other effects), ion heating effect (trap issues) or intra beam collisions (electron accelerator issues related to the maximum intensity around 100 mA in the interaction point). Several questions about the ion trap technique require detailed studies, including both simulations and benchmark tests with a demonstrator for a quantitative investigation of the various physical processes and of the main difficulties to overcome: ion heating, ion beam capture thanks to charge breeding, ion charge state reprocessing in a RF trap filled by buffer gas, intra beam scattering effects at the interaction point. The ion trapping performances such as maximum space charge capacity, overlap between ions and electrons, as well operational parameters like the possible duty cycle including the recirculation of the multi-charged ion cloud are to be carefully determined.

Conclusions. *The program accessible with e-RI collisions at luminosities* $\mathbf{10^{26-29}}$ $cm^{-2}s^{-1}$ *would open entirely new perspectives to study nuclear structure away from stability, firstly with direct extraction of nuclear proton densities. In this respect, we consider that it is timely to launch R&D for a future facility in Europe. For heavier nuclei with high enough (e,e) cross sections, magnetic form factors will also be accessible experimentally, giving access to information on neutron densities, too. Systematic measurements of (e,e) and (e,e') cross sections will enable the extraction of experimental nuclear form factors for nuclear densities directly comparable to the state-of-the art calculations. Physics cases can be enlarged gradually, following the evolution of the RI beam developments at the facility where the electron accelerator-ion trap machine will be implanted. It would thus offers to the nuclear physicists bright scientific perspectives for several decades.*

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