Compact End-Fire Arrays: from Theory to Directivity and Gain Maximization

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***Abstract*— End-fire arrays are among the best candidate for high directivity and compact dimensions. Several solutions for the synthesis of superdirective arrays have been proposed in the literature. However, what is called a "superdirective" array suffers inevitably high losses and very poor radiation efficiency. More appealing is the joint optimization of directivity and efficiency, or the intrinsic gain, of the antennas. This paper set side by side the directivity and gain optimization methods based on the Spherical Wave Expansion (SWE) theory. The synthesis of superdirective and supergain end-fire arrays is proposed when Huygens sources, which attain the highest level of directivity, or simple bent dipoles are selected as elements of the array. On behalf of the spherical modal expansions, the results obtained for different optimizations of the two arrays are examined.**

***Index Terms*—antennas, end-fire arrays, gain, optimization.**

I. Introduction

A radiating source is considered superdirective when its directivity is higher than the normal defined by Harrington [1] as N^2+2N, where N is the degree of the SWE of the field. The value of N is normally linked to the size of the radiansphere enclosing the antenna as $N=kR$, with k the wave number and R the radius of the sphere enclosing the source. Having a focused beam in a desired direction of the space, rather than an omnidirectional pattern, would improve the efficiency and security of the wireless link, mitigate inferences and reduce electromagnetic exposure. On the other hand, the compact antenna size for their integration into objects constitute a fundamental limit on antenna bandwidth and gain [2]. However, it has been demonstrated, firstly by Uzkov in 1946 [3] and recently in [4], that the end-fire directivity of a uniformly distributed linear array increases when the inter-element spacing tends to zero, and the excitation coefficients associated with each radiating element are optimized accordingly. The synthesis of the optimal array excitation has been investigated with analytical approaches based on Array Factor theory [5], Characteristic Modes [6], and SWE theory [7]; and with numerical approaches [8]. Nevertheless, the issue of poor radiation efficiency and high losses, characterize arrays with element spacing significantly reduced with respect to the wavelength ($< 0.2\lambda$). When the gain is intended to be maximized, in the practical case of lossy antenna, few works deal with the optimization of compact end-fire arrays. The first example of supergain is attributed to Yaghjian [5], who obtained a gain of 7 dBi with a two-element parasitic Yagi-like antenna array. Other recent works dealing with the supergain end-fire array synthesis for more than two elements and different element spacing are based on the array factor theory [9], a convex-optimization method [10], and the SWE theory [11].

This work has the objective of emphasizing the difference between the directivity and the gain optimizations in their mathematical formulation and the effects of the solutions. To provide a meaningful example, dipole-based and Huygens source-based end-fire arrays are simulated and optimized. The choice of these two types of elements is not casual, as they are both directive antennas, but with different radiation efficiencies. The paper is organized as follows: in Section II the synthesis of superdirective and supergain end-fire arrays are described, and in Section III the different antennas and array designs are illustrated. Following in Section IV the results from the two optimizations are presented and compared, and finally a discussion concludes the paper.

II. Linear End-Fire Array: Synthesis Procedure

The synthesis of superdirective linear end-fire arrays based on the SWE theory has been already presented in the literature [7]. Considering an array of P equally spaced elements, this method calculates the set of complex excitation coefficients a_p, for maximum directivity, in a chosen direction (θ_0,ϕ_0). Recently, a modification that takes into account losses and optimizes for the maximum gain has been proposed in the literature [11]. The optimizations consist in the following steps

1) First step: the far-field radiation pattern for each of the $p=1,\ldots,P$ elements is exactred from full-wave simulations, together with radiation efficiency $\eta_{rad,p}$.

2) Second step: the etracted far-fields are transformed in SWE and P vectors form the matrix of elements $Q_{smn,p}$ that represents the initial radiation pattern of each element.

3) Third step: the optimal coefficients a_p are calculated by the matrix inversion product

$$a_p = Q^{\dagger}_{smn,p} \cdot Q^{\max}_{smn} \tag{1}$$

where the objective function $Q^{\max}_{smn}$ is defined for maximum directivity (2) and maximum gain (3) separately as

$$Q_{smn}^{\max,DIR} = \left(\vec{K}_{smn}\left(\theta_0,\phi_0\right)\right)^* \qquad (2)$$

$$Q_{smn}^{\max,GAIN} = \left(\frac{\vec{K}_{smn}\left(\theta_0,\phi_0\right)}{\sqrt{1+\delta_n\left(kR\right)}}\right)^* \qquad (3)$$

The $\vec{K}_{smn}\left(\theta_0,\phi_0\right)$ are the spherical vector functions as defined in [12]. In (3) the term $\delta_n\left(kR\right)$ is the dissipation factor, calculated for each spherical function, and *kR* is the radiansphere circumscribing the p-th element. The dissipation factor quantifies the attenuation of each spherical mode, depending on the order *n*, the size of the radiator *kR*, and its loss resistance r_{loss}, calculated from the efficiency $\eta_{rad,p}$ as $r_{loss,p} = \left(1-\eta_{rad,p}\right)/\eta_{rad,p}$.

III. HUYGENS SOURCES AND ELECTRICAL DIPOLES

Considering an electric (magnetic) current distribution in space, it generates an electric (magnetic) dipole. The length of the current defines the resonance frequency of the dipole. Assuming an infinitesimal length (l_{dipole} = λ), its radiated field presents a theoretical directivity of 1.5 (1.77 dBi). When an electric and a magnetic dipole are properly combined in space, the total radiation generates a field with a directivity equal to 3 (4.77 dBi), and if the two currents are phase-shifted by 90°, a cardioid shape radiation pattern is presented. This is the most directive radiating source and is called the Huygens source. Although it is considered in theory a single radiating entity, this source is composed of two independent electric and magnetic components. The dipole and the Huygens source are two examples of superdirective radiators. Moreover, dipole-type elements are often employed in the design of end-fire arrays, to achieve enhanced directivity within a very low profile. The modal expansion of these sources and their use in arrays is discussed in the following paragraphs.

A. The unitary elements

An electrical dipole and a Huygens source have been designed and simulated using CST MW Studio software. The working frequency chosen is 850 MHz, and the elements consist of microstrip lines of copper ($\sigma \approx 5{,}8\cdot 10^7$) of thickness 0.07 mm, on a Roger 5880T substrate ($\varepsilon_r \approx 2.33$) with a thickness of 0.8 mm. To be electrically small (kR < 1) the dipole is bent 3 times, with an arm length of 0.1λ, 0.1λ, and 0.03λ and a width of 1.3 mm, 5 mm, and 8 mm respectively. The geometry is shown in Fig. 1 with its SWE of the simulated far-field. The spherical TM modes with $m=\pm 1$, typical of the electrical dipole oriented along +y direction, are shown. The gain is 1.6 dBi, slightly lower than the theoretical 1.77 dBi because of the miniaturization. For the Huygens source, to the rear part of the substrate a Capacitively Loaded Loop (CLL) is added, with a radius of

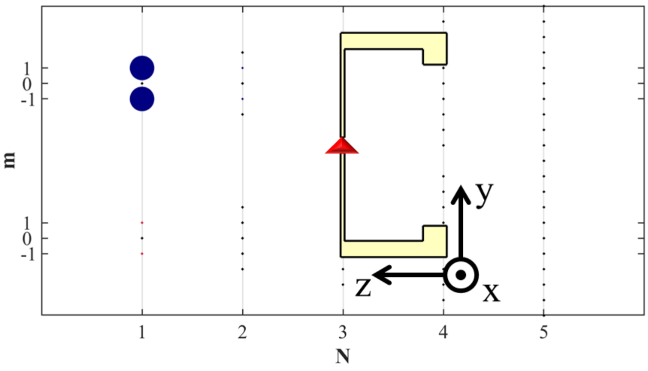

Fig. 1 Geometry of the designed Bent dipole (left) and the SWE of the radiated field calculated from full-wave simulation (right).

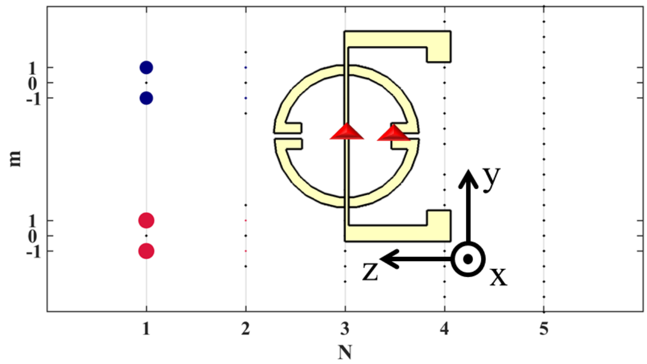

Fig. 2 Geometry of the designed Huygens source (left) and the SWE of the radiated field calculated from full-wave simulation (right).

0.06λ and the width of the microstrip 1.5 mm. The geometry of the antenna and the SWE of the simulated far-field are depicted in Fig. 2. The energy is almost equally distributed into TE and TM modes with $m=\pm 1$, as the superimposition of an electric and magnetic dipole oriented along *+y* and *–x* axis, respectively. The directivity simulated is 4.9 dBi and the gain is 4.5 dBi, with a cardioid-shaped radiation pattern.

B. Three-elements end-fire array

The solutions of the directivity and gain optimization problems are demonstrated to be the same when the array elements are sufficiently spaced, or equivalently when lossless antennas are considered. An end-fire array of *P=2* elements is a special case where, for an efficient radiatior choice ($\eta_{rad} \approx 1$) directivity and gain optimization produce the same results as far as the spacing is greater than 0.01λ. The increase of the array elements number toughens the mutual coupling and intensifies losses, and the spacing must increase accordingly. Increasing the number of elements *P*, good trade-off gain/size can be found for a spacing of 0.15λ and 0.2λ, in the case of 3- and 4-elements end-fire arrays, as reported in [11]. Considering the Huygens sources, the elements count must be doubled, and for these reasons *P* is set to 3 in the following analysis. Then, the end-fire arrays designed and simulated consist of the elements discussed in Paragraph III.A, displaced along the +z direction, spaced with a center-to-center distance of 0.15λ.

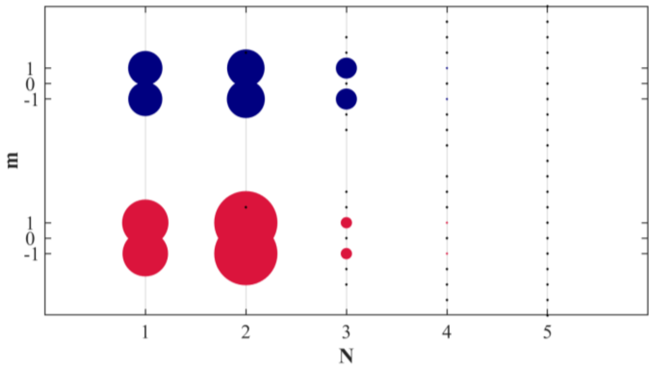

Fig. 3 Spherical modes obtained with the SWE of the field for maximum directivity of 3 bent dipoles-based end-fire array.

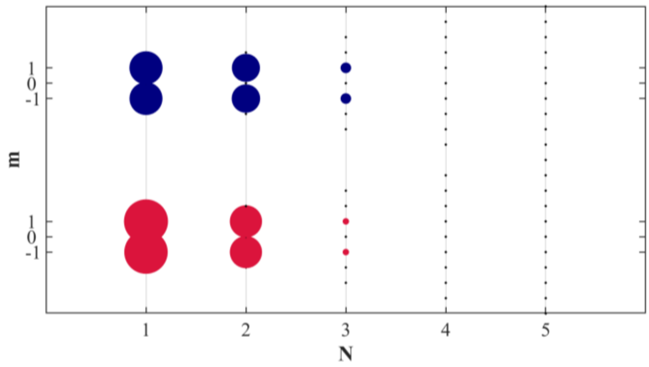

Fig. 4 Spherical modes obtained with the SWE of the field for maximum gain of 3 bent dipoles-based end-fire array.

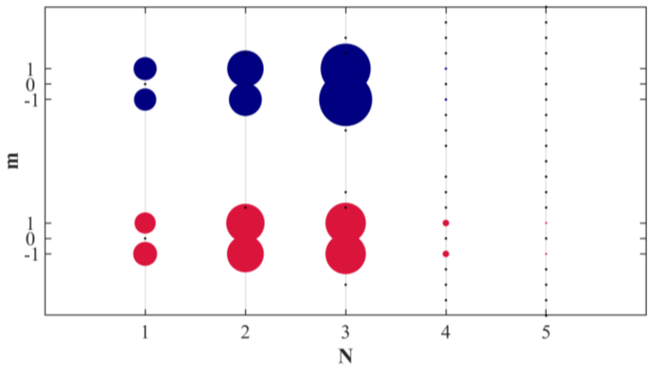

Fig. 5 Spherical modes obtained with the SWE of the field for maximum directivity of 3 Huygens source-based end-fire array.

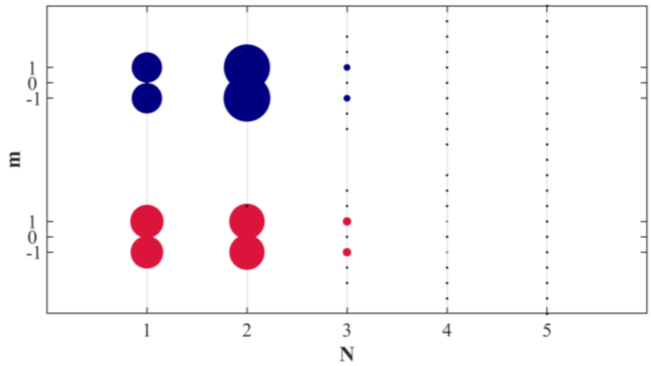

Fig. 6 Spherical modes obtained with the SWE of the field for maximum gain of 3 Huygens source-based end-fire array.

IV. Results Of The Optimizations

Once the arrays are designed (Section III.B) the synthesis procedure for directivity and gain maximization described in Section II can be operated. Through full-wave simulations the radiated far-field and radiation efficiency are calculated and results are extracted, for each element of the array $p=1,\ldots,P$. Then, the optimization toolbox developed in Matlab is executed, and the SWE of the fields imported are calculated and compared with the optimal modal distribution for maximum directivity (2) and maximum gain (3) respectively. In the gain optimization, an additional step calculates the dissipation factor for the radiated spherical modes of each element, accounting for the size and the efficiency, known from the simulation. Finally, the set of optimal excitations for each element is calculated.

A. *Three-elements bent dipoles end-fire array*

The bent dipole behaves essentially as a straight dipole (see Fig. 1), and by bending its arms the vertical length can be reduced by a factor of 2. However, the bent arms create a second dimension compromising with the usually mono-dimensional profile of the half-wave dipole. Hence, the performance expected from a 3-element array is the same as in the case of non-miniature dipoles. The maximum directivity obtained is 10.2 dBi, in line with the limits presented in [4], with a corresponding gain of 8.1 dBi. The SWE of the optimized far-field is illustrated in Fig. 3. The TM modes for n =1,2 and 3 are distinguishable and are the natural multipole modes of the electric dipole. The translation of the second and third elements from the origin of the coordinate systems causes the emergence of TE modes [12], naturally not present in the SWE of the electrical dipole in the origin of the coordinated system (Fig. 1). Simultaneously, this effect is further increased by the coupling in the array. When the excitation coefficients are calculated taking into account losses, the directivity decreases to 9.9 dBi, with an increased gain of 8.5 dBi. Moreover, a different spherical modal distribution is obtained as depicted in Fig. 4. The same modes excited by the array are present, but with different weights. According to the SWE higher order modes are the most directive but also the less efficient, and present high dissipative effects. Then, what emerges from the expansion of the field is a distribution of the energy not on the most directive higher order modes, but on the more efficient modes that however contribute to increasing the directivity. The different generated radiation patterns in the case of maximum directivity and maximum gain are depicted in Fig. 7.

B. *Three-elements Huygens source end-fire array*

The second case of optimization is performed on the Huygens source-based end-fire array. The geometry of the array is the same as the bent dipole case, where the difference is the presence on the rear part of the substrate of a CLL for each bent dipole. Each Huygens source is realized with two feeding ports, one for the CLL and the other for the bent dipole. The results of the optimizations are then 3×2=6 complex excitation coefficients. When the excitations are calculated using (2), the maximum directivity simulated is 12 dBi, slightly above the theoretical limits reported for Huygens source-based end-fire array calculated in [4]. The corresponding gain is 0.7 dBi, which is clear evidence of the severe losses that follow directivity optimization when the space between elements is limited, and/or the number of elements increases (6 in this case). The corresponding SWE of the far-field obtained for maximum directivity is reported in Fig. 5. It is flagrant how the excitation for maximum directivity distributes power toward higher-order modes, which are notoriously more directive and less efficient. Oppositely happens when the coefficients are calculated using (3), where the dissipation factor is introduced in the optimum problem. The maximum gain obtained in this case is 8.75 dBi, slightly higher than the case of bent dipoles, and the directivity is reduced to 10 dBi. A substantial efficiency improvement is observed when the optimal problem is pre-conditioned with the information of losses. The impact of using (3) instead of (2) can be observed in the SWE of the radiated far-field obtained for maximum gain, depicted in Fig. 6. The TE and TM modes for n=1,2 are fully excited and most of the energy is used by the antennas to radiate. The remaining energy is then distributed over the n=3 modes.

V. DISCUSSION AND CONCLUSIONS

In this work, the two methods to optimize directivity and gain, described in Section II, are compared. The elementary sources considered are an electric dipole and a Huygens source, both miniaturized. In the two cases, arrays of three elements were optimized in directivity and gain. The results reported for maximum directivity find those defined by the limits [4]. As for the gain, the results show different behavior for the two types of elements. For the dipole-based array, it is found that three elements sufficiently spaced attenuate the losses due to coupling effects, and the difference between gain for maximum directivity and maximum gain is 0.4 dBi. In contrast, the effect of losses in the Huygens source-based array is much more impactful, given the doubled number of elements in the same space. In this case, the gain obtained for maximum directivity is just above 0.7 dBi, while it exceeds that of the 3 dipoles when the gain is directly optimized. The difference in gain for the two optimizations is greater than 8 dBi.

This study brings out two fundamental aspects of super-directivity and super-gain. The first is related to the choice of the single radiating source and its use in arrays; it is clear that the much higher gain of the Huygens source on the electric dipole has a much less impact when these sources are used in the form of compact arrays. However, the directivity levels show a marked superiority in the case of Huygens sources. The second aspect is related to the use of gain optimization with respect to directivity, which shows a great advantage in terms of efficiency when this would otherwise be compromised to obtain maximum directivity.

TABLE I. OPTIMAL EXCITATION COEFFICIENTS FOR 3-ELEMENTS BENT DIPOLES END-FIRE ARRAY

Optimization	**Optimal coefficients**					
	Ampl. 1	***Phase 1***	***Ampl. 2***	***Phase 2***	***Ampl. 3***	***Phase 3***
Directivity	0.253	-3.83	0.483	-169.85	0.263	25.89
Gain	0.24	-9.41	0.49	-170.36	0.27	34.04

TABLE II. OPTIMAL EXCITATION COEFFICIENTS FOR 3-ELEMENTS HUYGENS SOURCE END-FIRE ARRAY

Optimization	**Optimal coefficients**					
	Ampl. 1	***Phase 1***	***Ampl. 2***	***Phase 2***	***Ampl. 3***	***Phase 3***
Directivity	0.057	107.15	0.218	-29.3	0.12	-75.24
	Ampl. 4	***Phase 4***	***Ampl. 5***	***Phase 5***	***Ampl. 6***	***Phase 6***
	0.349	141.51	0.11	123.08	0.145	-58.88
Gain	***Ampl. 1***	***Phase 1***	***Ampl. 2***	***Phase 2***	***Ampl. 3***	***Phase 3***
	0.089	3.74	0.15	-43.66	0.248	-153.69
	Ampl. 4	***Phase 4***	***Ampl. 5***	***Phase 5***	***Ampl. 6***	***Phase 6***
	0.224	-153.94	0.105	74.83	0.184	14.75

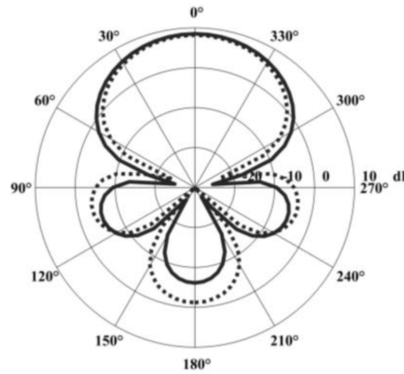

Fig. 7 Horizontal plane radiation pattern (Phi = 90°) of the maximum directivity (bold line) and maximum gain (dotted line) for the case of 3-bent dipoles end-fire array.

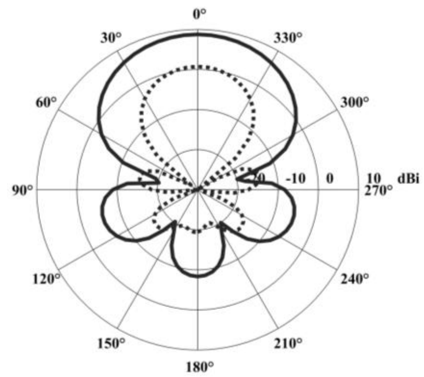

Fig. 8 Horizontal plane radiation pattern (Phi = 90°) of the maximum directivity (bold line) and maximum gain (dotted line) for the case of 3-Huygens sources end-fire array.

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