Design and optimization of a self-resonant impedance matched coil for Wireless Power Transfer

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***Abstract*— In this paper, we report the design and the optimization of high-quality-factor self-resonant coils for Wireless Power Transfer (WPT) relying on resonant inductive coupling. The main benefit of self-resonant coils is their potential to increase the quality factor of WPT systems, enabling to reach higher efficiencies at large distances. An analytical modeling of self-resonant coils is proposed and validated by finite element simulations (ADS Keysight). Self-resonant coils prototypes are then introduced and characterized, and a new topology for impedance-matched self-resonant coils enabling to further increase the performances of WPT systems is finally reported.**

Keywords—Wireless Power Transfer, self-resonant coil, impedance matching

I. INTRODUCTION

Wireless Power Transmission (WPT) is an old idea brought in the 1890s by Nikola Tesla. This idea of transmitting data or energy by magnetic fields led to the emergence of electromagnetic induction, enabling to transmit data or energy using two coils. Nowadays, this technology has evolved to be used in various products such as RFID [1], medical implants [2], domestic appliance [3]... However, this technology has its drawbacks, especially in terms of transmission distance for power. To increase the power transfer distances, the idea of making the coils resonate was introduced.

Resonant inductive WPT differs from standard inductive coupling by employing high-quality-factor resonant coils operating at their resonance. This method allows the transfer of significant power over distances (i.e. several coil diameters) thanks to the high quality factor compensating the low mutual inductance between the coils [4]. This work aims to implement this technology for applications such as drone charging, in which the power receiver is distant from the transmitter.

A resonant inductive coupling WPT system is composed of five main elements [5] (Fig. 1): an ac or dc power source supplies the power inverter generating an ac power at the resonance frequency of the resonant coils composing the coupler. The coupler transmits the energy from the primary to the secondary coil through air via electromagnetic coupling. Finally, the ac-to-dc rectifier delivers a useful dc power to a load (e.g. charging a battery or powering electronics).

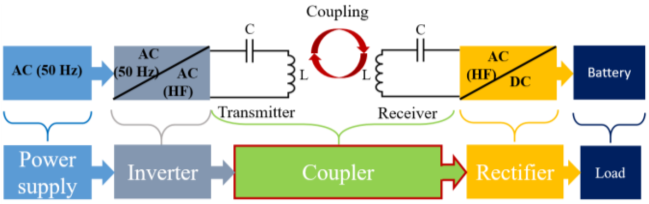


Fig. 1. General schematic of a resonant wireless power transfer (WPT) system.

This paper focuses on the design and the optimization of self-resonant electromagnetic couplers on PCB. A simple resonant coupler is composed of two coupled resonant coils, i.e. a pair of inductance and capacitance. These components can be discrete or distributed in a circuit. Resonant couplers using discrete components facilitate the manufacturing of WPT products and are the most commonly-used solution in state-of-the-art WPT systems (. However, they have a limited quality factor at high powers. Resonant couplers with distributed components are more challenging to design but allow more design flexibility and better performances. The resonance frequency of the coupler is chosen in the MHz range (13.56 MHz) in order to decrease the overall size of the electronics and increase the performances over distance. Some of the coils are also designed with a 50 Ω input impedance to match typical RF power sources impedances.

In this paper, the essential characteristics of a resonant coupler are described and discussed in Section 2. Then, self-resonant coils with distributed capacitance are optimized and compared with resonant coils with discrete components in Section 3. Finally, a new topology of a self-resonant and impedance matched coil is proposed in Section 4 to further increase the performances of WPT systems.

II. RESONANT WPT PERFORMANCES

A. Resonant coil parameters

In resonant WPT, each resonant coil (Fig. 1) is composed of a main inductor and a capacitor in order to obtain a highly resonant system [4]. Other reactive elements can be added to adapt the impedance seen by the source or the load to improve the performances. Capacitive elements used with the main coupled coils are either discrete components or distributed in a circuit.

The resulting assembly presents a resonance and/or an anti-resonance and may be modeled by an equivalent circuit with appropriate capacitance arrangement. For example, if the capacitance is in series with the coil, the impedance reaches a minimum at the peak of resonance, resulting in a maximum of current. For a parallel capacitance, the impedance reaches a maximum at the anti-resonance peak.

By assuming low losses in the system, a LC-circuit (anti)-resonates approximatively at the pulsation ω_0:

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{1}$$

with L the equivalent inductance and C the equivalent capacitance of the coil.

At the resonance, the WPT performances depend on the losses in the system represented by a quality factor Q. In coils, the losses come mainly from conductive losses and the skin effect at high frequencies, and the coil quality factor is given by:

$$Q_{coil} = \frac{\omega L}{R_L} \tag{2}$$

with R_L the equivalent series resistance. Typical coil quality factors are in the range of few 100's at 13.56 MHz.

In capacitors, losses come mainly from dielectric losses, and the capacitor quality factor is given by:

$$Q_{capacitor} = \frac{1}{\omega C R_C} \tag{3}$$

with R_C the equivalent series resistance. In the case of discrete capacitors, the quality factor can be large but are limited at high voltages. In the case of distributed capacitance, the quality factor is limited by surrounding media, mainly by the substrate material:

$$R_C = \frac{\tan\delta}{2\pi f C} \tag{4}$$

with $\tan\delta$ the dielectric loss constant.

For an RLC circuit, the overall quality factor (Q_{system}) is a combination of all the component's quality factors and is mainly limited by the lower one (${Q_{system}}^{-1} = \sum {Q_i}^{-1}$).

B. Coupler parameters and WPT performances

The coupling between the transmitter coil Tx and the receiver coil Rx is described by the coupling coefficient k:

$$k = \frac{M}{\sqrt{L_{Tx} L_{Rx}}} \tag{5}$$

with L_{Tx} and L_{Rx} the inductances of the Tx and Rx coils respectively, and M the mutual inductance between them. When the coils are very close to each other, they are highly coupled, and k tends to 1. When the coils are far apart, k decreases rapidly with the distance. In this case however, a high quality factor can counterbalance the effect of the lower coupling coefficient on the performances.

Indeed, as shown by [6], the kQ product is of prime importance to describe the system performances, especially the maximum obtainable efficiency given by:

$$\eta_{max} = max\left(\frac{P_{out}}{P_{in}}\right) = \frac{k^2Q^2}{1+k^2Q^2} \tag{6}$$

with P_{in} and P_{out} the input and output powers of the coupler respectively.

Three cases of coupling can be defined (the system is assumed to be symmetrical with identical quality factors):

- Weak coupling when $kQ < 1$: the two coils Tx and Rx have a weak influence on each other. A single resonance peak is observed and $\eta_{max} < 50$ %.
- Critical coupling when k$Q = 1$: this coupling allows for maximum energy transfer between the two coils with $\eta_{max} = 50$ %.
- Strong coupling when $kQ > 1$: the two coils Tx and Rx have a strong influence on each other and $\eta_{max} > 50$ %. The phenomenon of frequency splitting is observed (i.e. the system presents two resonance frequencies different from ω_0 requiring a frequency tuning if the distance between coils, and then k, changes).

In a drone charging application, working at the critical coupling ($kQ = 1$) has the advantage of a rapid charging with 50 % efficiency over a distance that is even greater that the quality factor is higher, while avoiding the frequency splitting.

In the next section, the quality factors and the performances of coils designed to resonate at 13.56 MHz with discrete and distributed components on various PCB substrates are compared.

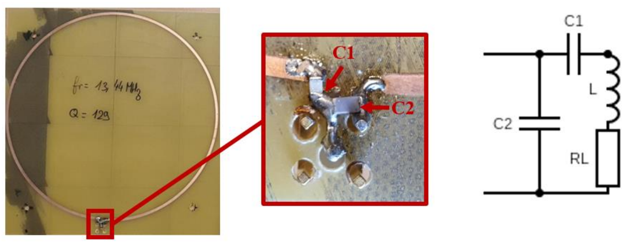
III. Self-Resonant Coils

The resonant coils are designed on PCB for ease of fabrication. A first resonant coil is designed with discrete capacitors to set a reference. Then two self-resonant coils are designed on FR4 and ROGERS 4003C with distributed capacitances to resonate at 13.56 MHz. The coil is designed on PCB for ease of fabrication and volume optimization while having good electrical characteristics for high power. These coils are more delicate to design because their characteristics depend on the geometry and the substrate material parameters such as their dielectric constant and dielectric losses.

A. Reference resonant coil with discrete capacitors

The simple reference coil is a single 15 cm diameter 2 mm wide turn loop on FR4 (Fig. 2a). In this case, the PCB does not significantly interfere with the performances since distributed capacitance is small. Its inductance is estimated to 490 nH with the formula of a planar circular coil [7].

To obtain a 13.56 MHz resonance and 50 Ω input impedance, a capacitor bridge (C_1-C_2) as shown in Fig. 2b is used.



(a) (b)

Fig. 2. (a) Reference resonant coil, impedance matched with discrete capacitors. (b) Equivalent electrical circuit of the reference resonant coil.

This coil was realized with C_1 = 330 pF and C_2 = 2700 pF and its characteristic impedance obtained with the vector network analyzer (VNA) E5061B from Keysight is shown in Fig. 3.

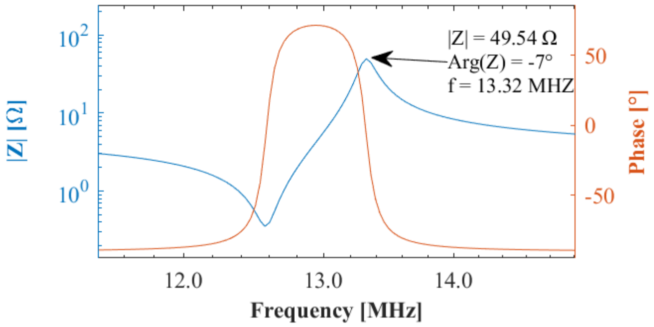
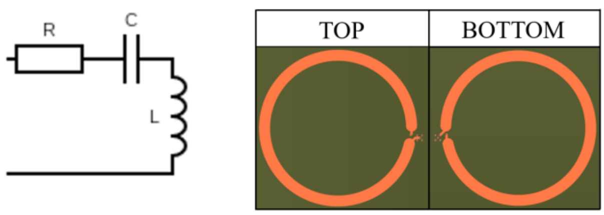


Fig. 3. Impedance of the simple resonant matched coil with discrete capacitors measured with VNA.

Its Q-factor is estimated to 129 with a mutual coupling measurement with a VNA [8]. This Q-factor is good for low level of voltages. However, discrete capacitors needed to withstand high voltages for high power transfer would decrease this quality factor and the overall performances.

B. Self-resonant coil with distributed capacitance

Self-resonant coils using distributed capacitance on PCB are described in the literature [9,10,11]. The self-resonant coils designed in this paper are inspired by [9] and are made of two multi-turns loops printed on two sides of a PCB (Fig. 4a). This particular topology has never been presented and differ from the state of the art by higher Q. The equivalent circuit of such a coil is an inductor in series with a resistor and a capacitor (Fig. 4b). The two turns are not connected to each other by a conductor, but are each connected to a terminal of the generator. The two conductive surfaces facing each other form the distributed capacitance in series. The current will pass through a portion of the first loop, then will pass on the second side through the substrate by capacitive effect, and finally flow through the rest of the second loop.



(a) (b)

Fig. 4. (a) Equivalent electrical circuit of the self-resonant coil. (b) Layout of a self-resonant coil made of two single turn loops printed on two sides of a PCB substrate.

The inductance is given by equation from [7]:

$$L = 0.002\pi(D-w)\left(\ln\left(\frac{4(D-w)}{w}\right)-0.5\right)10^{-4} \quad (7)$$

with D the outer diameter and w the loop width. The capacitance is obtained by the plane capacitance formula, when fringing fields are neglected:

$$C = \varepsilon_0\varepsilon_r\frac{A}{e} = \varepsilon_0\varepsilon_r\frac{\pi(4wD-4w^2)}{4e} \quad (8)$$

with e the thickness of the substrate, A the area of the two facing conductors and ε the dielectric constant of the material used.

The equivalent series resistance R is given in [9]:

$$R = \frac{\rho_{copper}\pi(D-w)}{w}\left(\frac{2}{S_d\left(1-e^{-\frac{t}{S_d}}\right)}-\frac{4}{3t}\right)+\frac{\tan\delta}{2\pi f C} \quad (9)$$

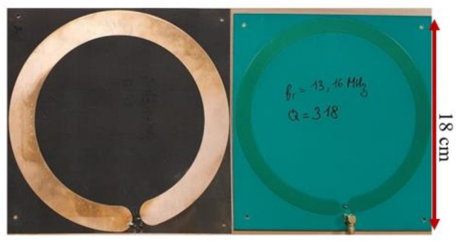
with ρ_{copper} the copper's resistivity, S_d the skin depth, t the track thickness, and $\tan\delta$ the dielectric loss constant. The equivalent resistance depends on conductive losses of the inductance (in red) and on dielectric losses of the capacitance (in blue). The topology presented in Fig. 4a with wide tracks tends to decrease the conductive contribution. In order to further increase the quality factor, we can change the substrate material: for example, among materials considered hereafter, ROGERS 4003C ($0.0021 < \tan\delta < 0.0027$) is 5 times less lossy than FR4 ($0.015 < \tan\delta < 0.02$) and is a better choice for our application.

For each material, the coil geometry is optimized in order to obtain coils resonating at 13.56 MHz with the best quality factor. The optimization is performed with Matlab (fmincon) with set constraints on the resonance frequency and on the geometry to maximize Q. The dielectric constants of the two considered materials are different ($\varepsilon_r = 4.6$ for FR4, $\varepsilon_r = 3.38$ for ROGERS 4003C), which affects the capacitance as shown in (8) and results in different optimized geometries.

For a 0.8-mm-thick FR4 substrate, a theoretical maximum Q-factor of 50 is obtained for a 18.3 cm diameter and 21 mm wide coil (L=298 nH, C=543 pF, R=0.44 Ω). As expected, dielectric losses dominate and represent 92 % of the losses.

For a 0.406-mm-thick ROGERS 4003C substrate, a theoretical maximum Q-factor of 231 is obtained for a 16.3 cm diameter and 11.5 mm wide coil (L=330 nH, C=403 pF, R=0.12 Ω). In this case, the dielectric losses are on part with the conductive losses, and the Q-factor increases by a factor of 4.6 between the two materials.

After fabrication and characterization with a VNA (Fig. 5), these coils were found to have a resonance frequency of 13.17 MHz (FR4) and 13.16 MHz (ROGERS 4003C), and quality factors of 42 and 318 respectively, which represents an experimental gain of 7.6 in Q-factor. The differences in frequency and quality factor can be mainly explained by the uncertainties on the thickness of the dielectric. Indeed, it is shown in equation (9) that the resistance is inversely proportional to the capacity. However, the two PCBs have thinner thicknesses than desired, so the capacity is greater (8) and the resistance lower. This leads to a quality factor that will be greatly impacted because the capacitive losses are dominant compared to losses by conduction and skin effect.



(a) (b)

Fig. 5. Self-resonant coils on FR4 (a) and on ROGERS 4003C (b).

C. Comparison of resonant coils performances

The maximum efficiencies of these three resonant coils are shown in Fig. 6 as a function of the inter-coil distance and their coupling coefficient k.

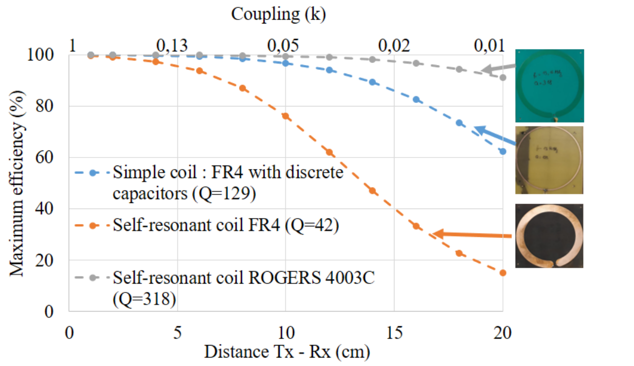


Fig. 6. Maximum efficiency for different Tx-Rx distances and different resonant coils.

The FR4 being a lossy dielectric, the corresponding coil has a lower Q-factor than a resonant coil with discrete capacitors. The maximum power transfer efficiency of such coils decreases faster with the distance ($\eta_{max} = 50\,\%$ at 13.5 cm instead of 22 cm). However, ROGER 4003C being a low-loss subtrate, the corresponding coil overperforms the resonant coil with discrete capacitor performances ($\eta_{max} = 50\,\%$ at 31 cm instead of 22 cm). We do not expect the quality factor of the coil with discrete capacitance to increase by changing materials because this only affects the capacitive losses, and these are fixed and dependent on the components.

However, the main drawback of the two self-resonance coils is their very low impedance at their resonance (0.58 Ω and 0.08 Ω respectively for FR4 and ROGERS 4003C coils). In the next section, we propose a new topology of self-resonant coil to realize both self-resonnance and impedance matching without discrete capacitors.

IV. Impedance Matched Self-Resonant Coils

To maximize the power transmitted by the WPT system, the resonant coil should be impedance matched to the input impedance of the inverter (Fig. 1). To keep a good quality factor and good performances obtained with self-resonant coil on FR4 or ROGERS substrates, we propose a new topology to realize both the impedance matching and the self-resonance only with distributed components.

A. *Proposed LCL topology and optimization*

The circuit presented Fig. 7 contains a main inductance L_p with a capacitor C_p in parallel (two turn loops), and an inductance L_s in series (spiral coil) added for the impedance matching (s stands for series and p for parallel). The two turn loops have a similar geometry to the one seen previously but are now connected together. The diameter of the spiral coil is much smaller than the loops (<1/3) in order to obtain a weak mutual inductance between the two coils which is neglected at first.

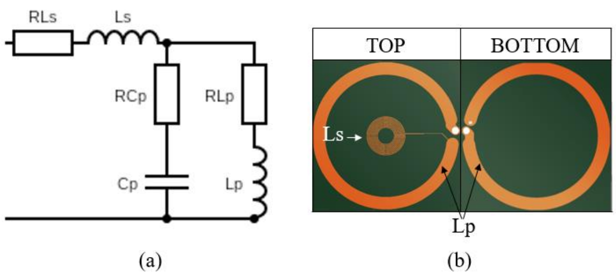


Fig. 7. (a) LCL topology equivalent electrical circuit. (b) LCL coil layout.

The resonance frequency of this circuit is approximated by the following formula in case of strong Q-factor:

$$\omega_0 = \sqrt{\frac{L_S + L_P}{C_P L_S L_P}} \qquad (9)$$

Because the topology is different, (two loops connected in series and not through a capacitance), the inductance L_p is 4 fold higher and the capacitance 4 fold lower than the ones calculated with the formula from [7] and the plane capacitance formula (8). A new formula for L_s is introduced from [12], considering a current sheet approximation:

$$L_s = \mu_0 \frac{N^2 D_{avg}\, c_1}{2} \left(\ln\left(\frac{c_2}{\rho}\right) + c_3 \rho + c_4 \rho^2 \right) \qquad (10)$$

with c_1, c_2, c_3 and c_4 the coefficients depending on the geometry given in [12], ρ the filling factor, N the turns number and D_{avg} the average diameter of the coil.

While the resistance for L_s and C_p are calculated with the same formula (9), the resistance for L_p is multiplied by 2 because of the two turns of the coil.

The geometry is optimized on Matlab to obtain an impedance of 50 Ω at the resonance frequency of 13.56 MHz while maximizing the Q-factor on FR4. The optimized resonant matched coil geometrical parameters are given in Table 1.

TABLE I. Géometrical parameters of the 4 coils

Coil	Geometrical parameter	Description	Value
RLC Coil Discrete capacitance	w	Track width of the coil	2 mm
	D	Outer diameter of the coil	15.2 cm
	N	Turns number of the coil	1
	e	Thickness of the substrate	1.49 mm
RLC Coil FR4	w	Track width of the coil	21 mm
	D	Outer diameter of the coil	18.3 cm
	N	Turns number of the coil	1
	e	Thickness of the substrate	0.76 mm
RLC Coil ROGERS 4003C	w	Track width of the coil	11.5 mm
	D	Outer diameter of the coil	16.3 cm
	N	Turns number of the coil	1
	e	Thickness of the substrate	0.406 mm
LCL Coil	w_i	Track width of the inner coil	1 mm
	s_i	Space between turns of the inner coil	1 mm
	D_i	Outer diameter of the inner coil	6.6 cm
	N_i	Turns number of the inner coil	10
	w_o	Track width of the outer coil	21 mm
	D_o	Outer diameter of the outer coil	25.2 cm
	N_o	Turns number of the outer coil	2
	e	Thickness of the substrate	1.6 mm

According to our analytics equations on MATLAB, this coil resonates at 13.54 MHz with an impedance of 39.3 Ω. The impedance is set deliberately lower because it will be impacted by the weak coupling between the two coils.

B. *Numerical simulation validation with ADS*

To validate the theoretical results, numerical simulations are performed with ADS Keysight on two coils facing each other with a distance on a 1.6 mm thick substrate. The coil impedance is 50.2 Ω at the resonance frequency of 14.08 MHz (Fig. 8), which is in good agreement with the targeted values of 13.56 MHz and 50 Ω (4 % error in frequency and 2 % in impedance).

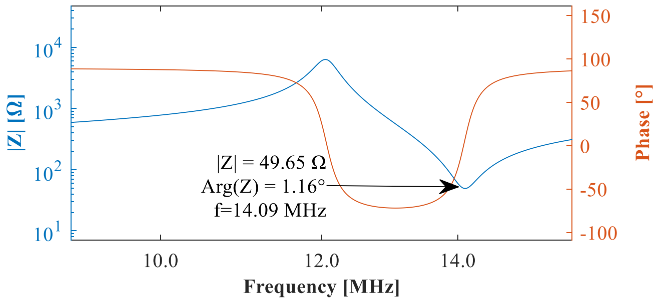


Fig. 8. Simulated impedance of the LCL topology (ADS from Keysight).

The quality factor obtained graphically is about 42 with FR4 substrate, which is consistent with previous results of self-resonant coil on FR4. Therefore, high quality factor should be achieved using ROGERS 4003C. This new topology is therefore of great interest for self-resonant coils as it enables to develop impedance matched self-resonant coils with a good Q-factor by exploiting distributed capacitors only.

C. *Experimental measurement using a VNA*

The coil is made on a 1.6 mm thick FR4 PCB plate (Fig. 9).

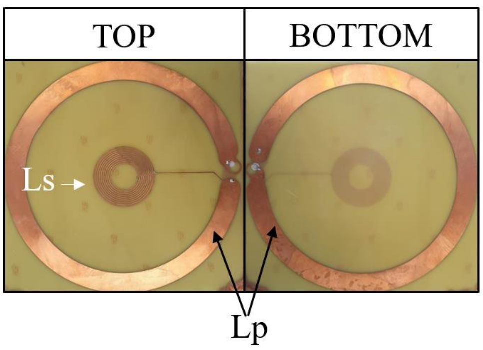


Fig. 9. Self-resonant and impedance matched coil with LCL topology on FR4.

The characteristic impedance of the LCL coil is shown in Fig. 10: at the resonance of 12.67 MHz, the coil impedance is 33.2 Ω. The Q-factor is in the same order of magnitude as the ones of non-matched coils ($Q = 55$).

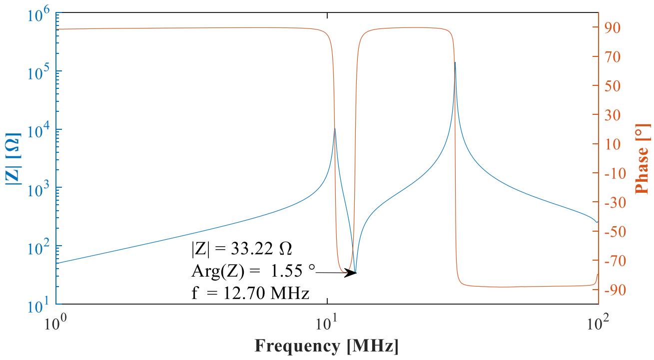


Fig. 1. Fig. 10. Measured impedance of the LCL topology (VNA).

The results obtained give a deviation of 6.4 % and 15.5 % for the frequency and the modulus respectively, between the theoretical and the measured values. This error is mainly due to a deviation in the capacitance (Table 2), mainly explained by an actual thickness of FR4 of 1.46 mm instead of 1.6 mm.

TABLE II. COMPONENTS MEASURED COMPARED WITH ANALYTICAL VALUES

Components	Analytical value	Measured	Error
L_S	5.48 µH	5.18 µH	6 %
L_P	1.90 µH	1.69 µH	11 %
C_P	97.54 pF	128 pF	31 %

This highlights the necessity to characterize the dielectric before running the optimization.

Finally, we note a significant difference between the simulation and the measurements. This difference could be explained by a problem of meshing. COMSOL simulations made afterwards shows great accordance with the analytical and measurements values.

V. Conclusion and Prospects

We have presented the design and the optimization of self-resonant coils both on FR4 and ROGERS 4003C. We have validated theoretically and experimentally the good quality factor of these coils while using only distributed capacitors, which is a great benefit to transmit high power with great range of distance.

A new self-resonant coil topology enabling the impedance matching with the inverter's input impedance has been proposed, simulated and characterized. The next step is to manufacture a coil on ROGERS 4003C substrate and to characterize it in order to decrease the losses and increase the quality factor and performance as demonstrated in part III.

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