Key role of phase dynamics & diamagnetic drive on Reynolds stress in fusion plasma turbulence

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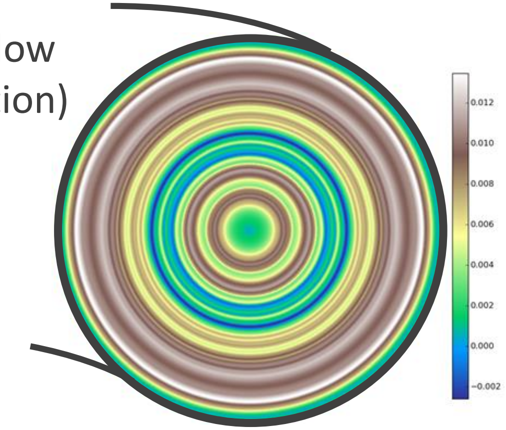
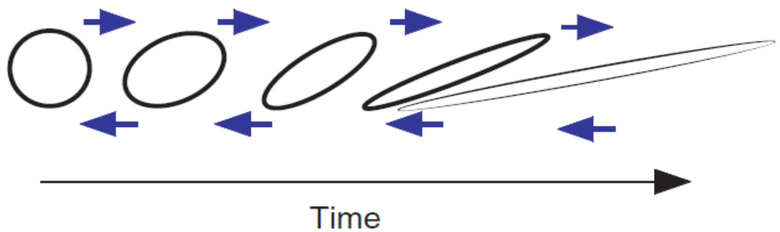
- Turbulence rotates poloidally: mean flow velocity $V_E \sim -E_r$

Mean poloidal flow (GYSELA simulation)



- Radial shear of V_E (i.e. E_r') $\Rightarrow$ turbulent vortex decorrelation
  $\rightarrow$ Dominant mechanism for turbulence control
  $\rightarrow$ Transport barriers, optimized confinement



[Biglari-Diamond-Terry 1990; Itoh-Itoh 1996; Terry 2000]

- **Turbulence** generates zonal & mean flows via Reynolds stress

- **Issue** addressed in this talk:
  What dominant mechanisms govern Reynolds stress dynamics ?

Structure of the poloidal Reynolds' stress

- Start from GK equation, focusing on E×B advection:

$$\partial_t \bar{f} + \underbrace{\{J.\phi, \bar{f}\}}_{} = RHS$$

with gyro-average operator $J \approx 1 + \frac{\mu}{2}\nabla_\perp^2$

Poisson brackets (account for E×B advection: $\mathbf{v}_E.\boldsymbol{\nabla}\bar{f}$)

- Account for quasi-neutrality (adiabatic electrons):

$$n_0\Omega = \langle J.\bar{f}\rangle_v \approx \langle \bar{f}\rangle_v + \frac{1}{2}\nabla_\perp^2 p_\perp$$

Potential vorticity $\Omega = \phi - \langle\phi\rangle - \nabla_\perp^2\phi$

- Where it comes: transport of potential vorticity

$$\partial_t\Omega - \nabla_{\perp,i}\{\phi + p_\perp, \nabla_{\perp,i}\phi\} = \langle J.RHS\rangle_v$$

[Parra-Catto PPCF 2009; Abiteboul PoP 2011; Ajay JPP 2020]

- Integrate & flux-surface average → equation on $\langle u_{E\theta}\rangle = \partial_r\langle\phi\rangle$

$$\boxed{\frac{\partial\langle u_{E\theta}\rangle}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2(\pi + \pi^\star)\right] = rhs}$$

[Diamond-Kim PoP 1991; Smolyakov PoP 2000; McDevitt PRL 2010]

Reynolds' stress

$$\pi = \langle\tilde{u}_{Er}\,\tilde{u}_{E\theta}\rangle$$

Diamagnetic Reynolds' stress

$$\pi^\star = \langle\tilde{u}_r^\star\,\tilde{u}_{E\theta}\rangle$$

with $\begin{cases} \tilde{u}_{Er} = -\frac{1}{r}\partial_\theta\tilde{\phi} \\ \tilde{u}_r^\star = -\frac{1}{r}\partial_\theta\tilde{p}_\perp \end{cases}$

[Sarazin, Dif-Pradalier, Garbet et al., PPCF 63 (2021) 064007]

- Comparative weight & properties of
  Reynolds stress $\pi = \langle \tilde{u}_{Er}\, \tilde{u}_{E\theta} \rangle$ *vs.*
  diamagnetic Reynolds stress $\pi^\star = \langle \tilde{u}_r^\star\, \tilde{u}_{E\theta} \rangle$

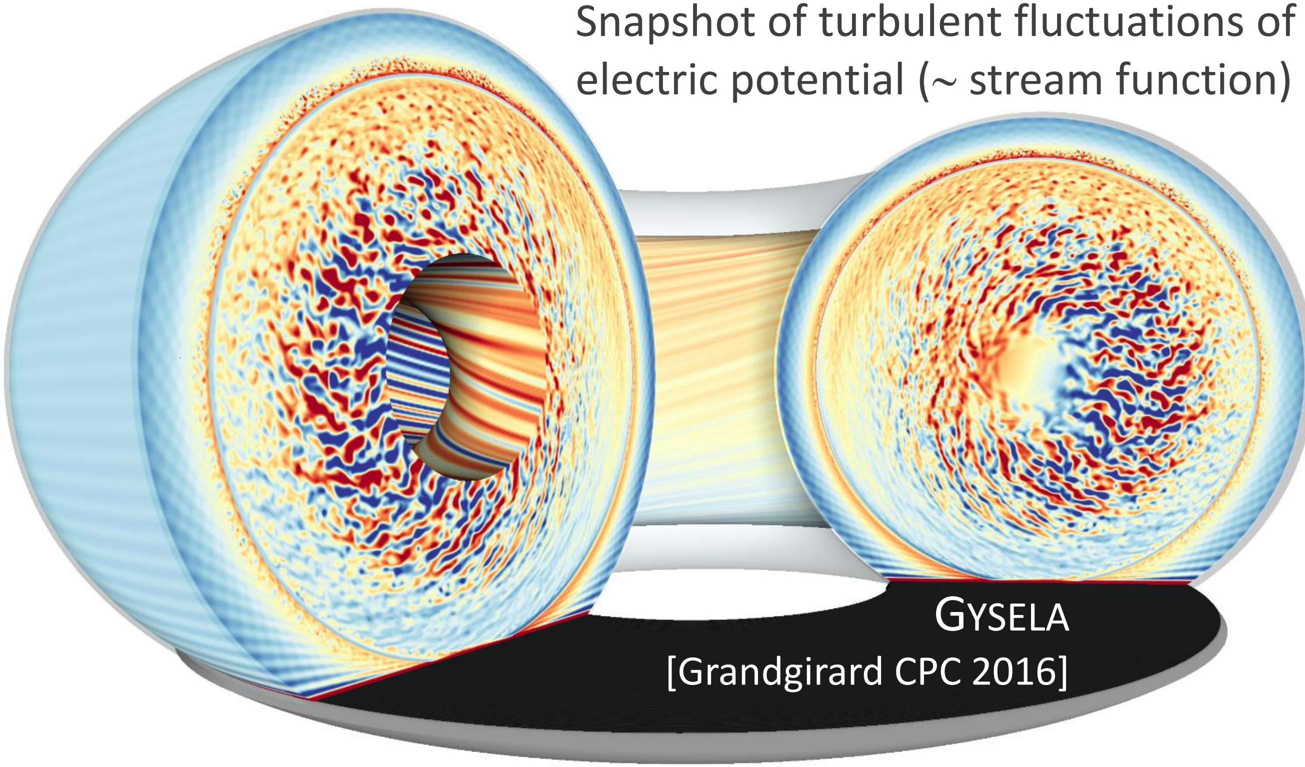
- Dynamics of the Reynolds stress: specific role of the phase

Means: 1st principle gyrokinetic simulations (GYSELA)
reduced nonlinear model

Probing the Reynolds stress with GYSELA

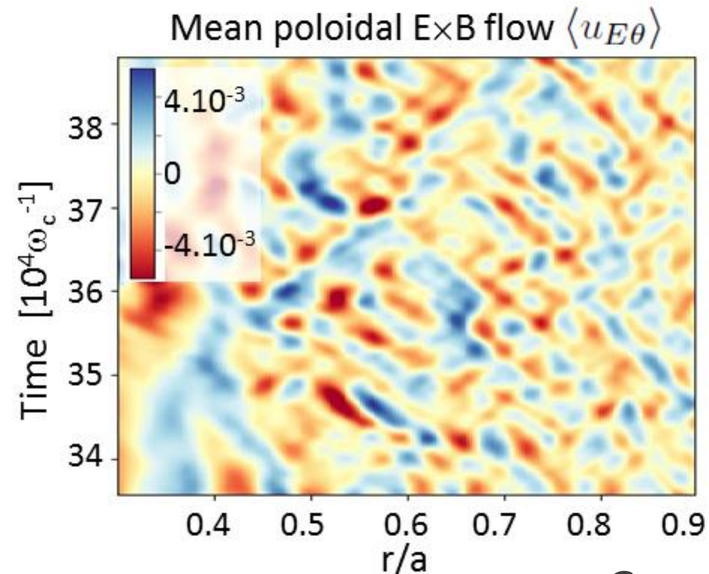
- Flux-driven **5D GyroKinetics**
  - Kinetic description → gyro-centers distribution function (Boltzmann eq.)
  - Self-consistency = Quasi-Neutrality (Poisson eq.)
  - Adiabatic electrons in cases under study

Snapshot of turbulent fluctuations of the electric potential (~ stream function)

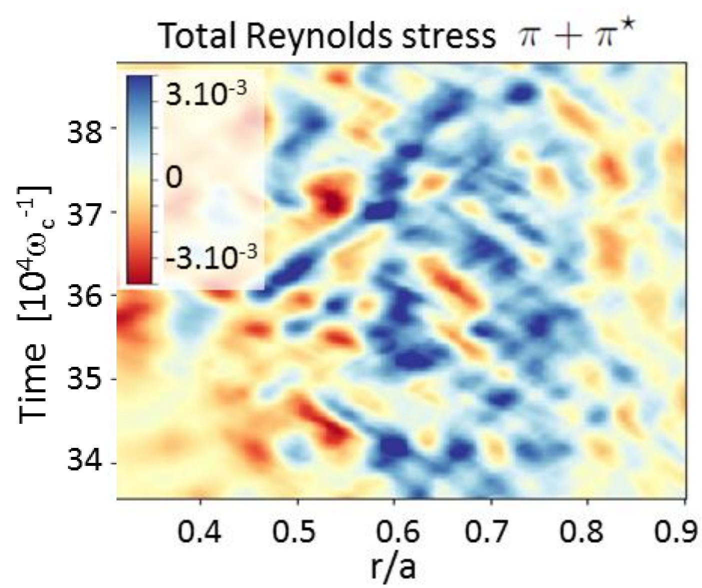


Reynolds' stress: avalanche-like patterns

- Analysis of a statistically saturated turbulent regime
- E×B turbulent flux exhibits bursts and avalanches
- Imprint of avalanche-like patterns on E×B mean flow
- Similar pattern on the total Reynolds' stress $\pi + \pi^\star$



GYSELA



Diamagnetic contribution is larger (factor ~2)

- Diamagnetic Reynolds stress about 2 times larger than E×B contribution

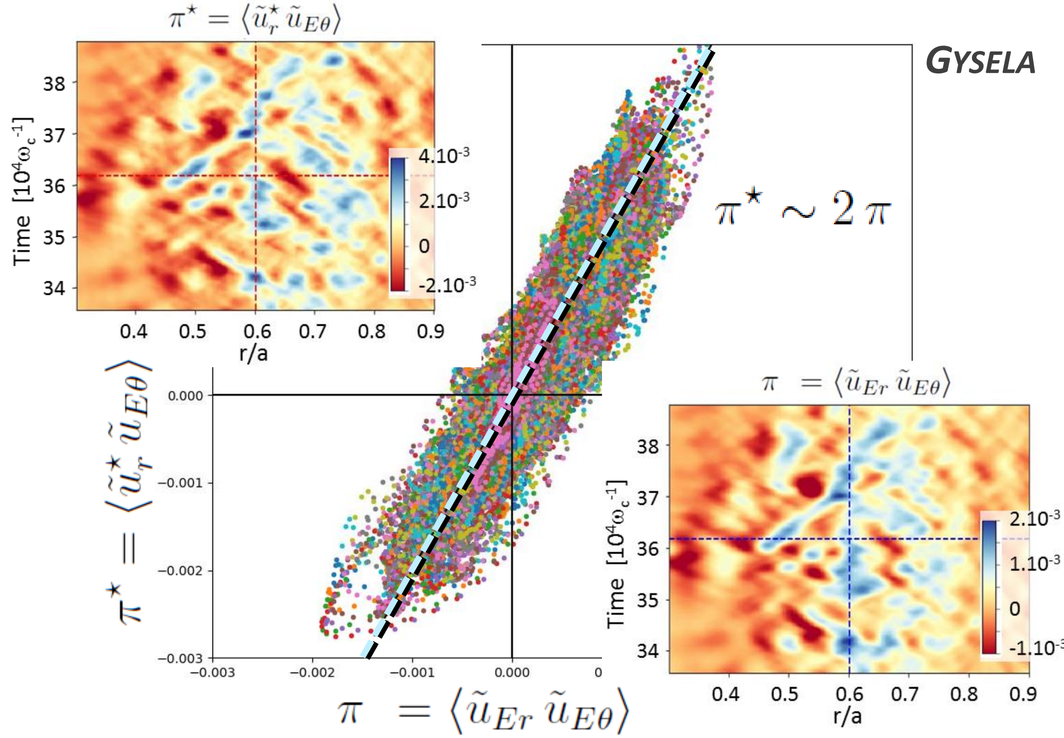
- Diamagnetic $\pi^\star = \langle \tilde{u}_r^\star \, \tilde{u}_{E\theta} \rangle$ and E×B $\pi = \langle \tilde{u}_{Er} \, \tilde{u}_{E\theta} \rangle$ contributions are **in phase**

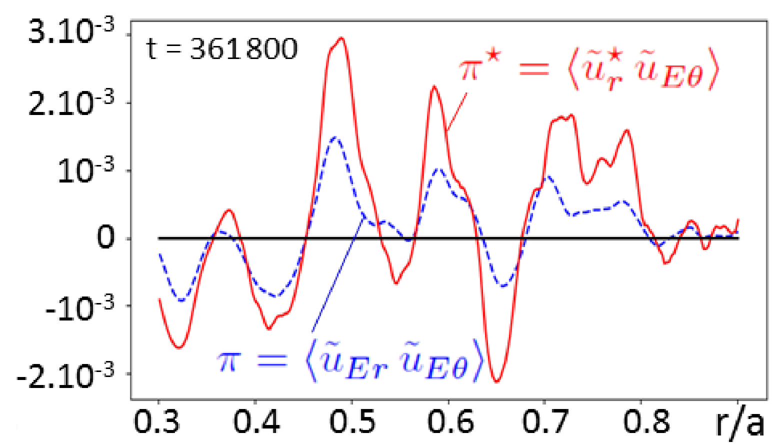
  (minima & maxima coincide)

- Consequence:

  Pressure is NOT simply advected by E×B flow ≠ passive scalar

  $\Rightarrow$ possible impact on analysis of exp. data   Cf. e.g. [Vermare PoP 2018]





Pressure $p_\perp$ is NOT simply advected by v_E

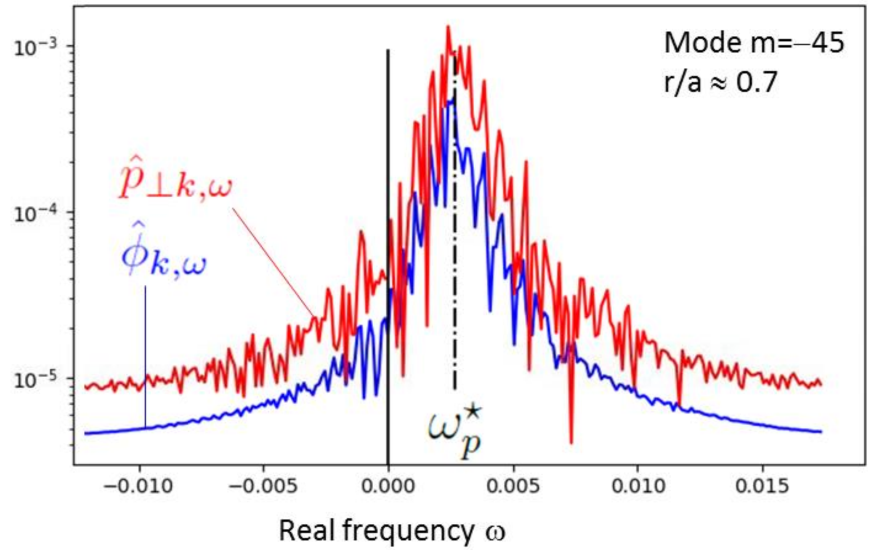
- Let's assume $\partial_t p_\perp + \mathbf{u}_E.\nabla p_\perp = 0$

  Then $\hat{p}_{\perp k,\omega} = -\frac{\omega_p^\star}{\omega}\hat{\phi}_{k,\omega}$

  ($\omega_p^\star = k_\theta \partial_r \langle p_\perp \rangle$ ion diamagn. freq.)

  So that $\pi^\star = -\sum_{\omega,k} k_r k_\theta \frac{\omega_p^\star}{\omega} |\hat{\phi}_{k,\omega}|^2$

  Opposite to $\pi = \sum_{\omega,k} k_r k_\theta |\hat{\phi}_{k,\omega}|^2$



- **GYSELA simulations** (ITG turbulence):
  - $\omega \approx \omega_p^\star$

Pressure $p_\perp$ is NOT simply advected by v_E

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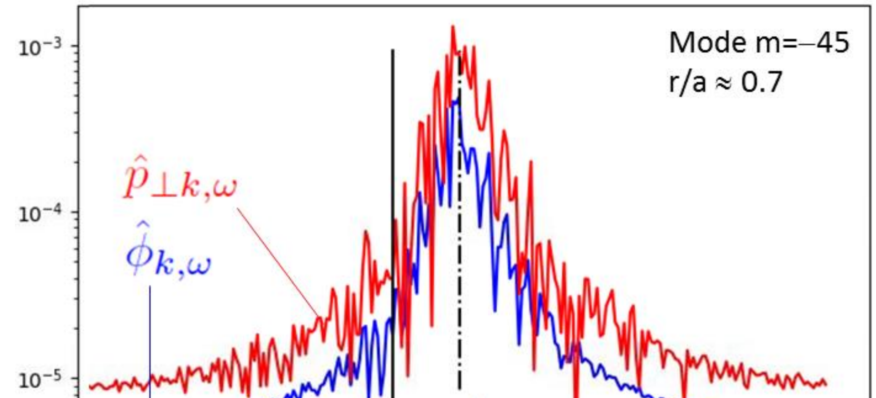
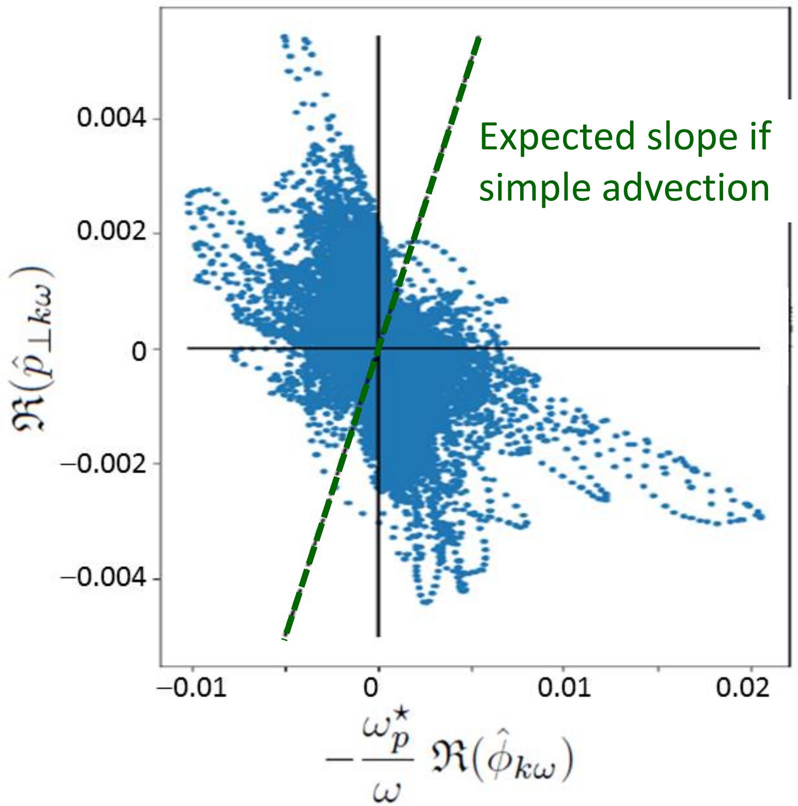
  Opposite to $\pi = \sum_{\omega,k} k_r k_\theta |\hat{\phi}_{k,\omega}|^2$

- **GYSELA simulations** (ITG turbulence):
  - $\omega \approx \omega_p^\star$
  - No clear relationship between $\hat{p}_{\perp k,\omega}$ and $\hat{\phi}_{k,\omega}$

- **Linear GK relationship**: resonances

$$\hat{p}_{\perp k,\omega} = -\left\langle \frac{\omega^\star - \omega_d - k_\parallel v_\parallel}{\omega - \omega_d - k_\parallel v_\parallel} \mu B F_M \right\rangle_v$$





[Sarazin, Dif-Pradalier, Garbet et al., PPCF 63 (2021) 064007]

- Comparative weight & properties of
  Reynolds stress $\pi = \langle \tilde{u}_{Er}\, \tilde{u}_{E\theta} \rangle$ *vs.*
  diamagnetic Reynolds stress $\pi^\star = \langle \tilde{u}_r^\star\, \tilde{u}_{E\theta} \rangle$

- Dynamics of the Reynolds stress: specific role of the phase

Means: 1st principle gyrokinetic simulations (GYSELA)
reduced nonlinear model

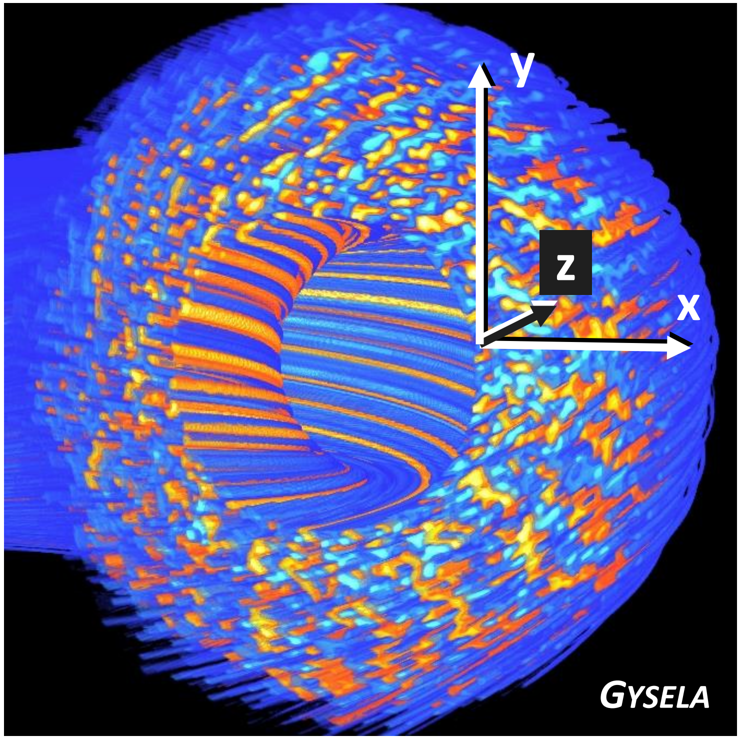
Reduced model for interchange turbulence (I)

- Magnetized plasma $\mathbf{B} \neq$ Cst
  Isotherm, cold ions, only resonant modes $k_{//}=0$

$$\begin{cases} \partial_t n + \mathbf{v}_E . \nabla n = S & \text{Density} \\ \partial_t \Omega + \mathbf{v}_E . \nabla \Omega + g \partial_y n = 0 & \text{Vorticity} \end{cases}$$

$$\mathbf{v}_E = \mathbf{z} \times \nabla \phi$$

$\sim 2(\rho_s/R)$ effective gravity
(only retained compressibility term
$\rightarrow$ governs interchange instability)



- **Scale separation**: mean $\langle ... \rangle(x,t)$
  & fluctuations $\widetilde{...}(x,y,t)$
  Mean poloidal flow $V_E = \partial_x \langle \phi \rangle$

$$\begin{aligned} n(x,y,t) &= N(x,t) + \tilde{n}(x,y,t) \\ \Omega(x,y,t) &= V_E'(x,t) + \nabla_\perp^2 \tilde{\phi}(x,y,t) \end{aligned}$$

- Retaining **single wave vector** k_y

$$\begin{aligned} \tilde{n}(x,y,t) &= n_k \, \mathrm{e}^{ik_y y} + cc \\ \tilde{\phi}(x,y,t) &= \phi_k \, \mathrm{e}^{ik_y y} + cc \end{aligned}$$

[Sarazin PoP 2000;
Bian PoP 2003;
McMillan PoP 2009]

Reduced model for interchange turbulence (II)

[Sarazin PoP 2000;
Bian PoP 2003;
McMillan PoP 2009]

1D nonlinear system (+ Dissipation)

- **Mean fields** ($\in \mathbb{R}$)

$$\begin{cases} \partial_t N &= -\partial_x \Gamma + S \\ \partial_t V_E &= -\partial_x \Pi \end{cases}$$

Turbulent Flux $\Gamma = \langle \tilde{v}_{Ex} \tilde{n} \rangle = -\Im(2k_y\, n_k \phi_k^*)$

Reynolds stress $\boxed{\Pi = \langle \tilde{v}_{Ex} \tilde{v}_{Ey} \rangle = -\Im(2k_y\, \phi_k^*\, \partial_x \phi_k)}$

- **Fluctuations** ($\in \mathbb{C}$)

$$\begin{cases} \partial_t n_k &= -ik_y\,(V_E\, n_k + v_\star\, \phi_k) \\ \partial_t \Omega_k &= -ik_y\,(V_E\, \Omega_k + g\, n_k - V_E''\, \phi_k) \end{cases}$$

Potential vorticity

$$\Omega_k = \left(\partial_x^2 - k_y^2\right) \phi_k$$

Diamagnetic drift

$$v_\star = -\partial_x N$$

Linear instability & stabilizing role of V_E

- **Linear instability** = "interchange" ~ Rayleigh-Bénard

$$\begin{aligned} \partial_t n_k &= -ik_y \left(V_E \, n_k + v_\star \, \phi_k \right) \\ \partial_t \Omega_k &= -ik_y \left(V_E \, \Omega_k + g \, n_k - V_E'' \, \phi_k \right) \end{aligned}$$

Diamagnetic drift $v_\star = -\partial_x N$

Potential vorticity $\Omega_k = \left(\partial_x^2 - k_y^2 \right) \phi_k$

- **Dispersion relation**

$$\omega_\pm - k_y V_E = \frac{k_y \, V_E''}{2k_\perp^2} \pm \frac{k_y}{k_\perp} \left(\frac{V_E''^2}{4k_\perp^2} - g v_\star \right)^{1/2}$$

Doppler shift

Stabilization
$\Rightarrow$ Instability Threshold

Instability drive = curvature
$\times$ mean gradient

$\rightarrow$ Linear growth rate γ

$$V_E = Cst \quad \Rightarrow \quad \gamma = \frac{k_y}{k_\perp} \sqrt{g v_\star}$$

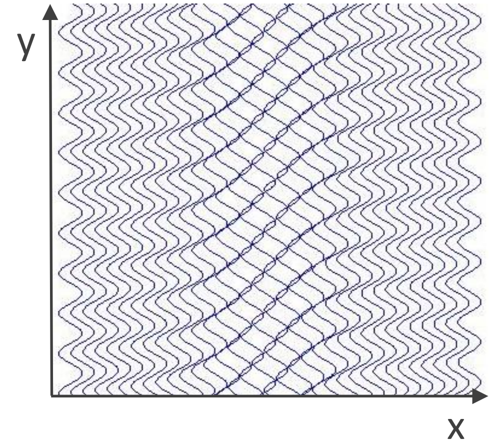
Reynolds stress, phase & amplitude

- Discriminate **phase** and **amplitude** of fluctuations

$$\begin{cases} n_k = |n_k| \; e^{i\varphi_n} \\ \phi_k = |\phi_k| \; e^{i\varphi_\phi} \end{cases} \Rightarrow \begin{cases} \Gamma = -2k_y \; |n_k||\phi_k| \; \sin\Delta\varphi_{n\phi} \\ \boxed{\Pi = -2k_y \; |\phi_k|^2 \, \partial_x\varphi_\phi} \end{cases}$$

Phase gradient $\partial_x\varphi_\phi$

$\equiv$ Phase shift of k_y waves



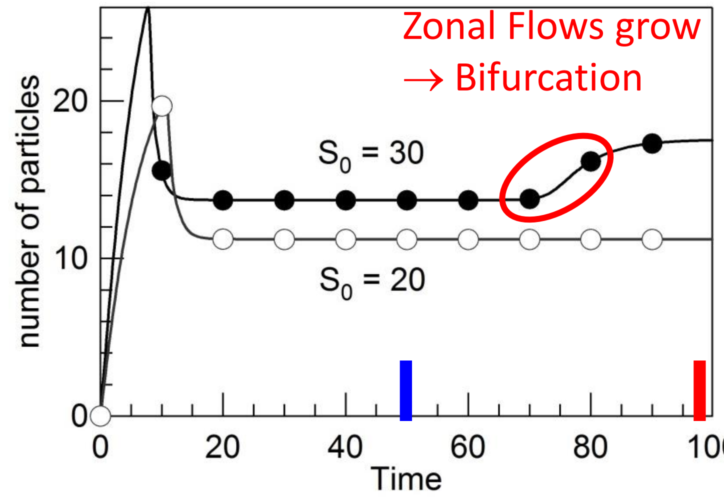
- **Reynolds' stress** Π can grow due to
  - Amplitude $|\phi_k|^2$
  - Phase gradient $\partial_x\varphi_\phi \sim k_x$

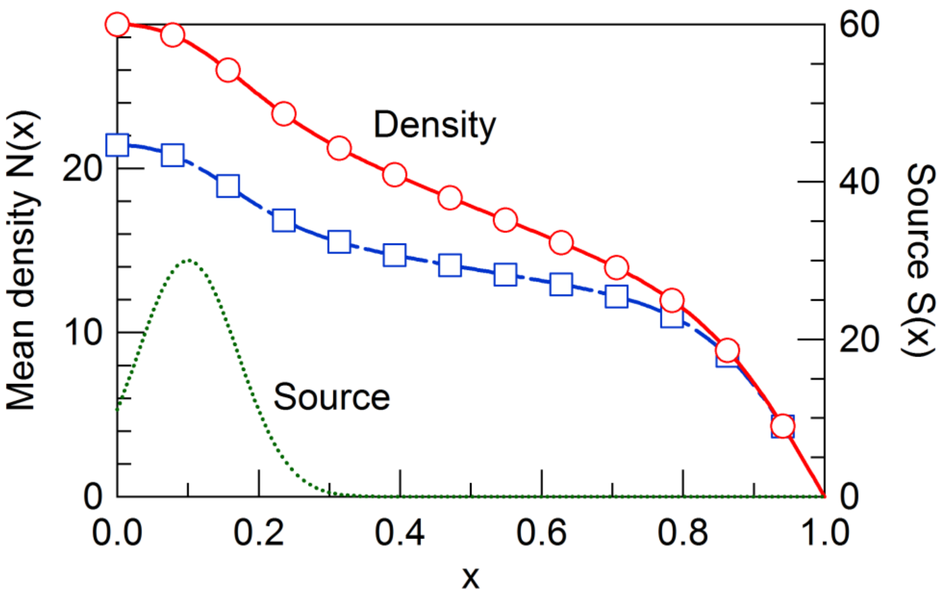
Note: phase curvature already identified as a possible key player
"phase patterning can generate ZF w/o turbulence inhomogeneity": $\partial_x\Pi \sim |\phi_k|^2 \; \partial_x^2\varphi_\phi$

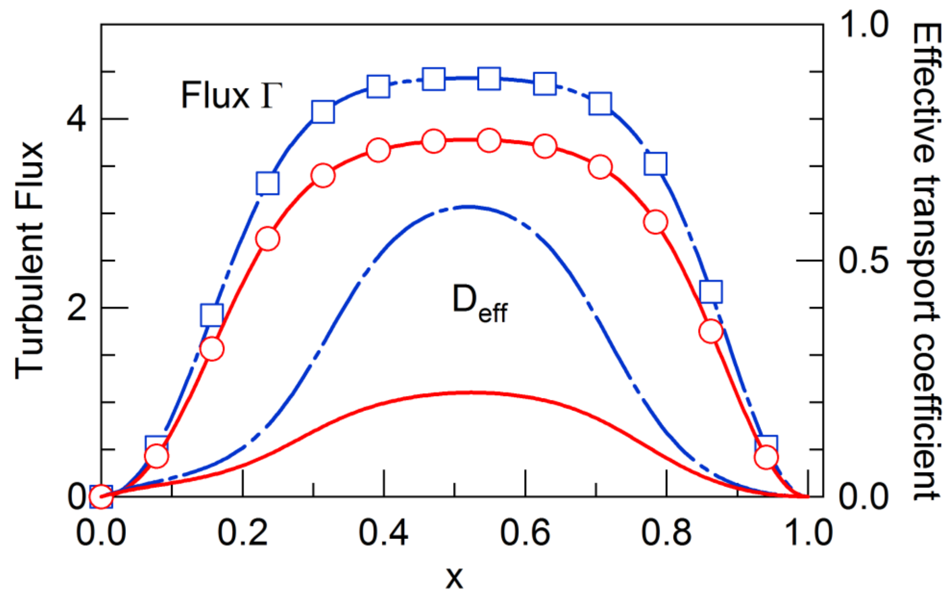
[Z. Guo, P. Diamond PRL 2016; Xu NF 2020]

Reynolds stress governed by phase instability

- **Zonal Flows** are excited above a threshold in source magnitude S_0
- Turbulence magnitude & effective transport are reduced at their onset







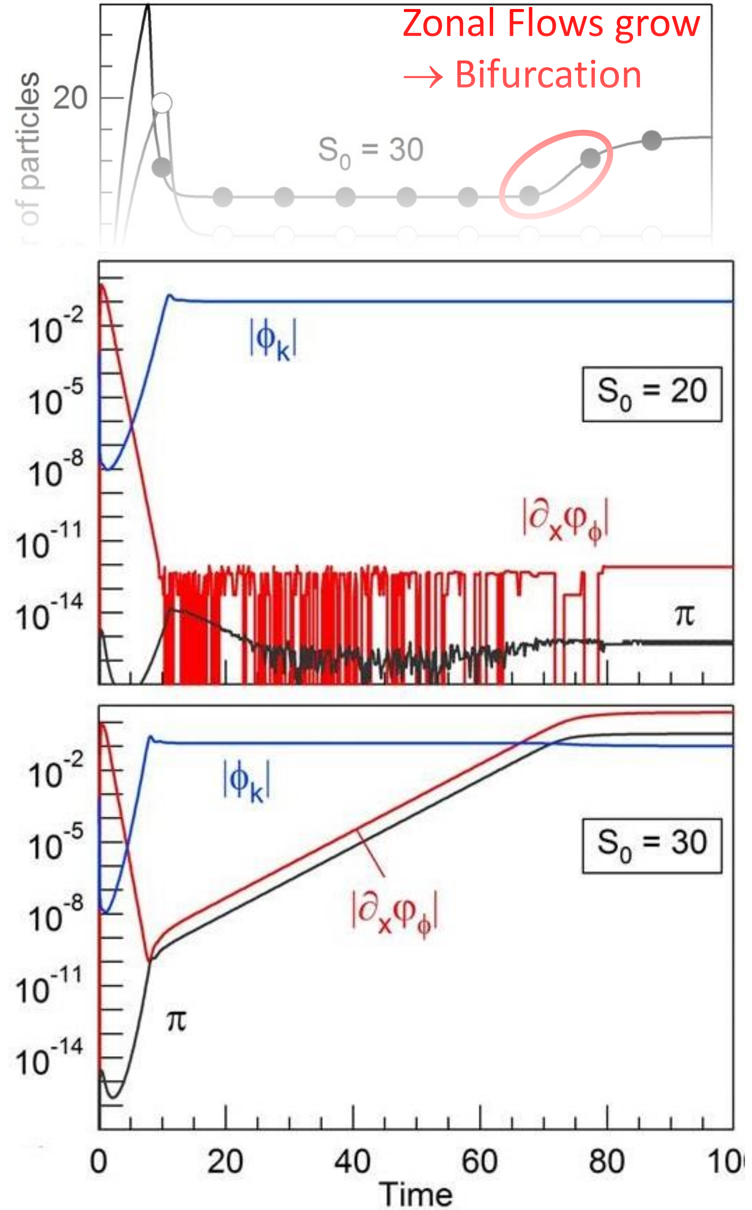
Reynolds stress governed by phase instability

- **Zonal Flows** are excited above a threshold in source magnitude S_0
- Turbulence magnitude & effective transport are reduced at their onset
- ZF are driven by an instability of the phase gradient (reminiscent of the tilting instability) [Drake PFB 1992; Guzdar PFB 1993; Rosenbluth PoP 1994; Fedorczak PPCF 2013]
- Estimate of phase instability growth rate

$$\begin{cases} \dfrac{\partial(\partial_x\varphi_\phi)}{\partial t} = -k_y \dfrac{\partial V_E}{\partial x} & \leftrightarrow \quad \dfrac{\mathrm{d}k_r}{\mathrm{d}t} = -k_\theta\, V'_E \\ \dfrac{\partial V_E}{\partial t} = 2k_y \dfrac{\partial}{\partial x}\left(|\phi_k|^2\,\partial_x\varphi_\phi\right) & \end{cases}$$

$$\Rightarrow \frac{\partial^2(\partial_x\varphi_\phi)}{\partial t^2} \approx \boxed{-2k_y^2 \frac{\partial^2\,|\phi_k|^2}{\partial x^2}}\, \partial_x\varphi_\phi$$

~ (phase instability growth rate)^2



Reynolds' stress governed by phase instability

- **Zonal Flows** are excited above a threshold in source magnitude S_0
- Turbulence magnitude & effective transport are reduced at their onset
- ZF are driven by an instability of the phase gradient (reminiscent of the ti... instability) [Drake PFB 1992 ... Rosenbluth PoP 1994 ...]

⇒ What about in **tokamak plasma turbulence**?

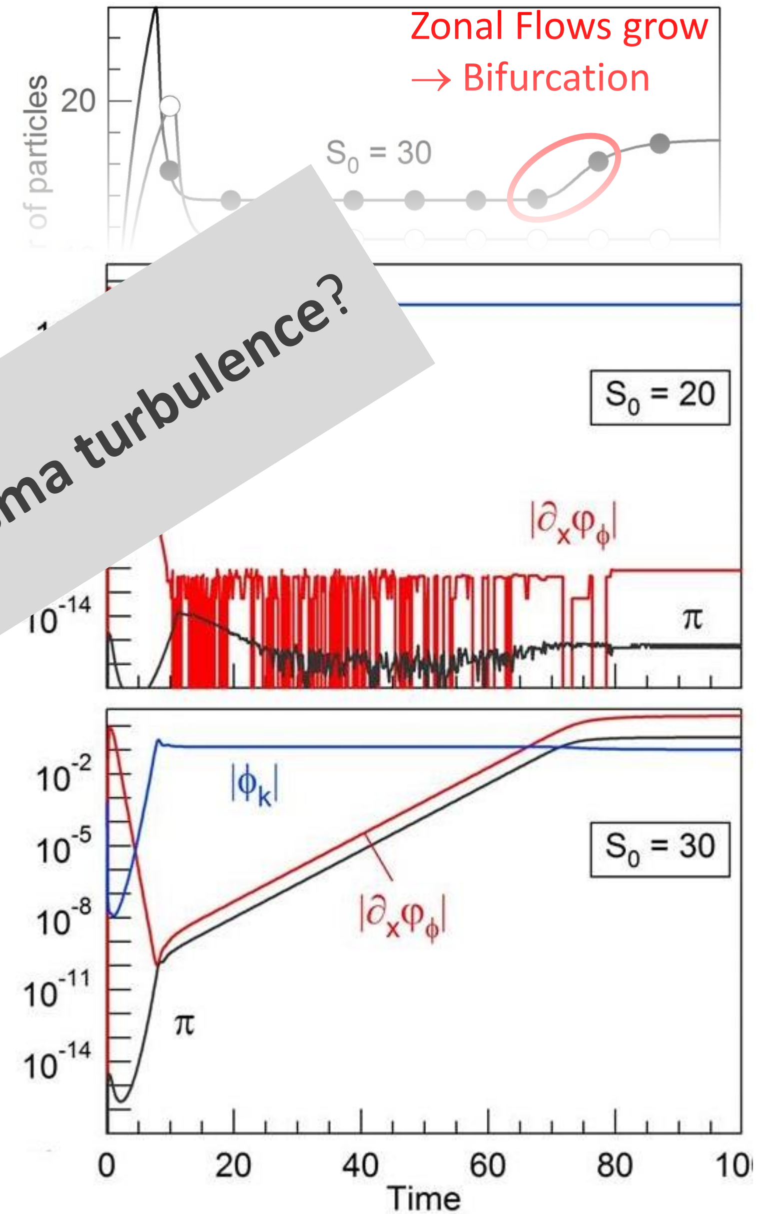
- Estimate of phase in... ...rate

$$\frac{\partial(\partial_x\varphi_\phi)}{\partial t} = - \ldots \quad = -k_\theta\, V_E'$$

$$\frac{\partial V_E}{\partial t} = 2k_y \frac{\partial}{\partial x} \ldots \varphi_\phi)$$

$$\Rightarrow \frac{\partial^2(\partial_x\varphi_\phi)}{\partial t^2} \approx \boxed{-2k_y^2\frac{\partial^2\,|\phi_k|^2}{\partial x^2}}\,\partial_x\varphi_\phi$$

~ (phase instability growth rate)²



Reynolds' stress governed by phase instability

- Similar decomposition with **3D output GYSELA data** (ion turbulence)

$$\Pi = -\sum_{m>0,n} \frac{2m}{r} |\hat{\phi}_{mn}|^2 \, \partial_r \varphi^{\phi}_{mn} = \sum_{m>0,n} \Pi_{mn}$$

$$= \mathcal{I}(r,t)\, \boldsymbol{\varphi}'(r,t)\, \Delta(r,t)$$

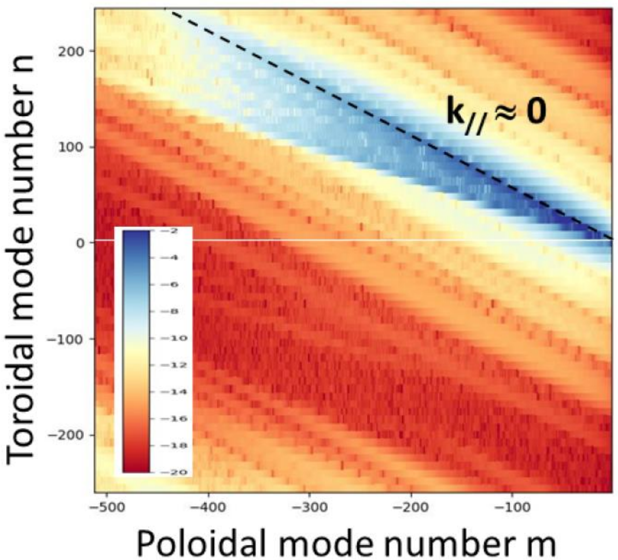
2D Fourier decomposition
(*m*,*n*) = (poloidal, toroidal) mode numbers

Turbulence intensity

$$\sum_{m>0,n} \frac{2m}{r} |\hat{\phi}_{mn}|^2$$

Phase gradient: $\boldsymbol{\varphi}'(r,t) = -\sum_{m>0,n} W_{mn}\, \partial_r \varphi^{\phi}_{mn}$

Mode weight $\log |W_{mn}|$ $(r/a \approx 0.72)$



$$W_{mn}(r) = \frac{\langle |\Pi_{mn}| \rangle_t}{\langle \Pi \rangle_t}$$

Reynolds' stress governed by phase instability

- Similar decomposition with **3D output GYSELA data** (ion turbulence)

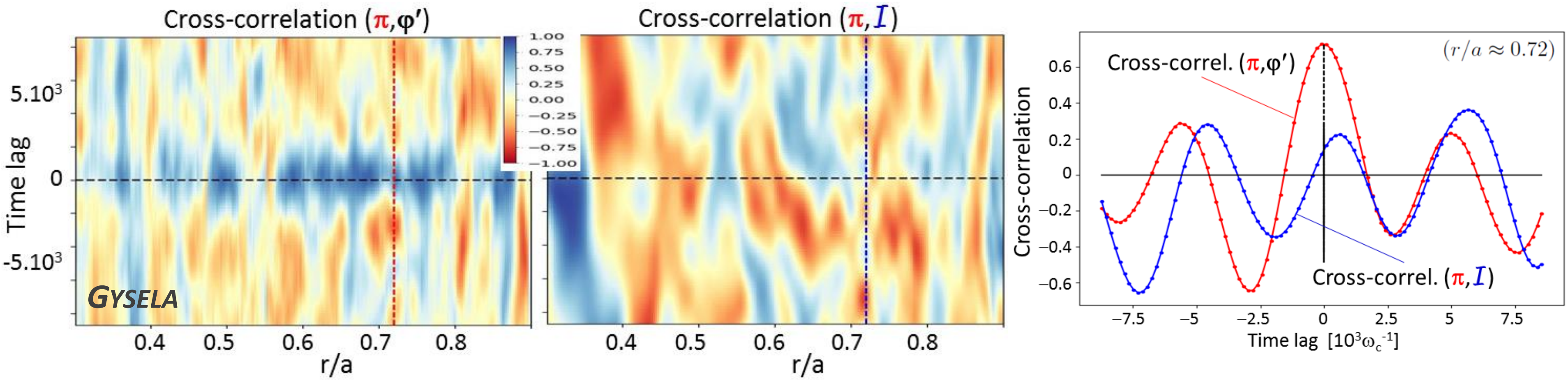
$$\Pi = -\sum_{m>0,n} \frac{2m}{r} |\hat{\phi}_{mn}|^2 \, \partial_r \varphi^{\phi}_{mn} = \sum_{m>0,n} \Pi_{mn}$$
$$= \mathcal{I}(r,t)\, \varphi'(r,t)\, \Delta(r,t)$$

2D Fourier decomposition
(m,n) = (poloidal, toroidal) mode numbers

Turbulence intensity

Phase gradient: $\varphi'(r,t) = -\sum_{m>0,n} W_{mn}\, \partial_r \varphi^{\phi}_{mn}$

- Reynolds stress Π well correlated with φ', not with $\mathcal{I}$
  $\Rightarrow$ **phase dynamics is critical**



Conclusion & Discussion

- **Zonal & mean flows excited by turbulence**
  - Reynolds stress $\pi = \langle \tilde{u}_{Er}\, \tilde{u}_{E\theta} \rangle \approx -\frac{1}{B^2\, r} \left\langle \partial_\theta \tilde{\phi}\, \partial_r \tilde{\phi} \right\rangle$
  - Diamagnetic Reynolds stress $\pi^\star = \langle \tilde{u}_r^\star\, \tilde{u}_{E\theta} \rangle \approx -\frac{1}{enB^2\, r} \left\langle \partial_\theta \tilde{p}_\perp\, \partial_r \tilde{\phi} \right\rangle$

- **Diamagnetic Reynolds stress in phase with & larger than E×B RS**
  - Pressure does not behave as a passive scalar
  - Numerical evidence of its critical role in the build up of large scale flows at the plasma boundary → cf. **talk Dif-Pradalier** (this conf.)
  - Experimental evidence? Role in triggering transport barriers?

- **Reynolds stress dynamics governed by the phase gradient**
  - Growing evidence that phase is a key ingredient to RS drive
  - Robustness w.r.t. turbulence properties (e.g. electron turbulence)?
  - Role of phase instab. in triggering staircases / transport barriers?
  - What mechanism governs the transport of phase, if any ?

Back up slides

Diamagnetic Reynolds stress – fluid approach

- Drift velocities

$$\mathbf{u}_\perp^{(1)} = \mathbf{u}_E + \mathbf{u}_i^* = \frac{\mathbf{B}\times\nabla\phi}{B^2} + \frac{\mathbf{B}\times\nabla p_\perp}{enB^2}$$

$$\mathbf{u}_\perp^{(2)} = \mathbf{u}_{pol} \approx -\frac{m_i}{eB^2}\left[\partial_t + (\mathbf{u}_E + \mathbf{u}_i^*).\nabla\right]\nabla_\perp\phi$$

- Charge conservation $\nabla.\mathbf{j} = 0$

$$\nabla.\mathbf{j}_{pol} \approx -\frac{nm_i}{B^2}\left(\partial_t + \mathbf{u}_E.\nabla\right)\nabla_\perp^2\phi - \frac{m_i}{eB^3}\nabla.\left[\left(\frac{\mathbf{B}\times\nabla p_\perp}{B}.\nabla\right)\nabla_\perp\phi\right]$$

$$\approx -\frac{nm_i}{B^2}\left(\partial_t\nabla_\perp^2\phi + \frac{1}{B}\{\phi,\nabla_\perp^2\phi\}\right)$$

$$-\frac{m_i}{eB^3}\left(\{\nabla_{\perp,i}p_\perp,\nabla_{\perp,i}\phi\} + \{p_\perp,\nabla_\perp^2\phi\}\right)$$

~ divergence of $\pi^\star = \langle\tilde{u}_r^\star\,\tilde{u}_{E\theta}\rangle$

~ divergence of $\pi = \langle\tilde{u}_{Er}\,\tilde{u}_{E\theta}\rangle$