



Physical challenges & numerical issues in controlled fusion plasmas

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Physical challenges & Numerical issues in Controlled fusion plasmas

Y. Sarazin

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Ack.: X. Garbet, Ph. Ghendrih, G. Dif-Pradalier

SMAI 23/06/2021 – mini-symposium "Mathematical modelling of thermonuclear fusion plasmas"

- ▶ Basics on controlled fusion & ITER project
- ▶ Physical challenges:
 - Confinement
 - Heat & particle Exhaust
 - Disruptions
- ▶ Focus on three critical issues:
 - Fast particles: hybrid MHD, turbulence
 - Plasma-wall interaction: the kinetic side
 - Coulomb collisions: towards an efficient (non-linear) operator



Hydrogen isotopes

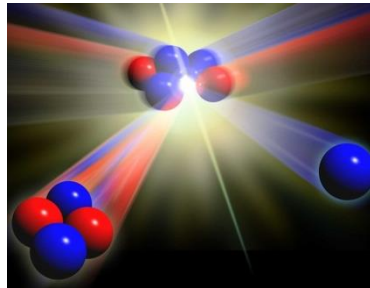
Stable **Unstable**

Helium

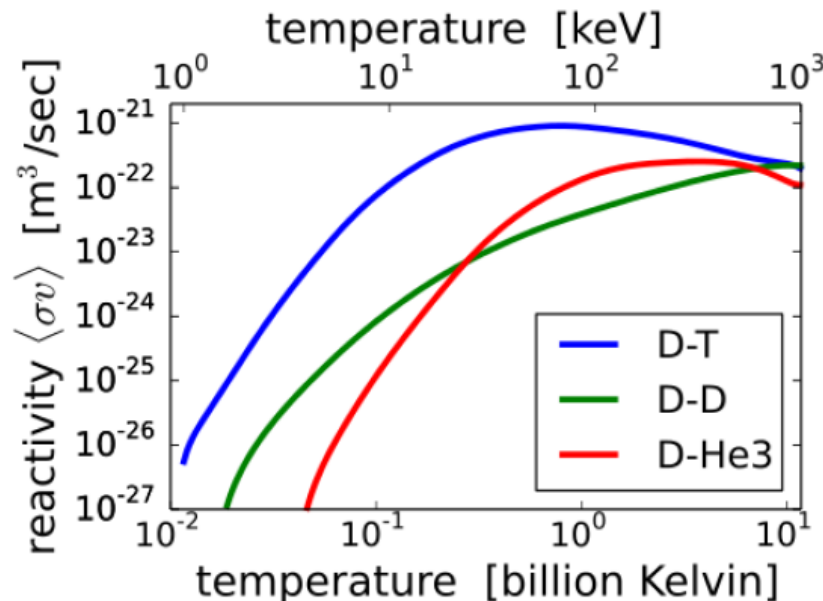
stable φ & χ

Neutron

- **must be produced**
- **not key for ITER**

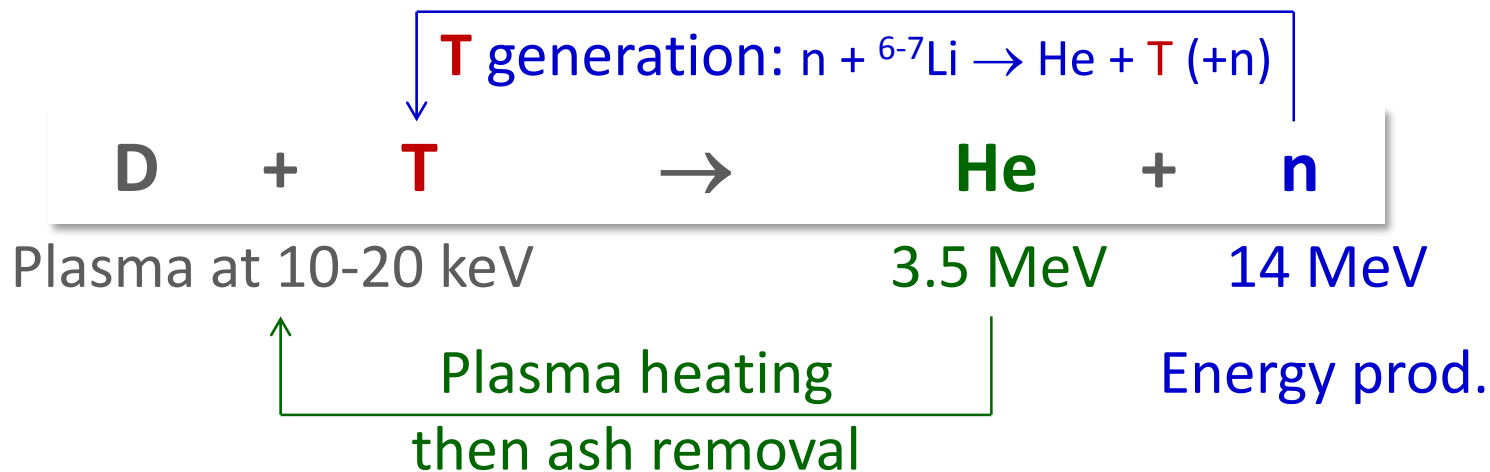


- impact on device
- used to produce T
- **not key for ITER**



Threshold condition for fusion

 $E_{\text{th}} \geq 10 \text{ keV} \rightarrow$ **Plasma state** $\rightarrow$ interaction distance 10^{-13} m



► **Power balance** at equilibrium: sources = losses

$$P_{\alpha} + P_{add} = \frac{W_{internal}}{\tau_E}$$

Internal energy $\sim nT$

P_{α} — α -heating

$\sim n^2 \langle \sigma v \rangle_{DT}(T)$ — additional heating

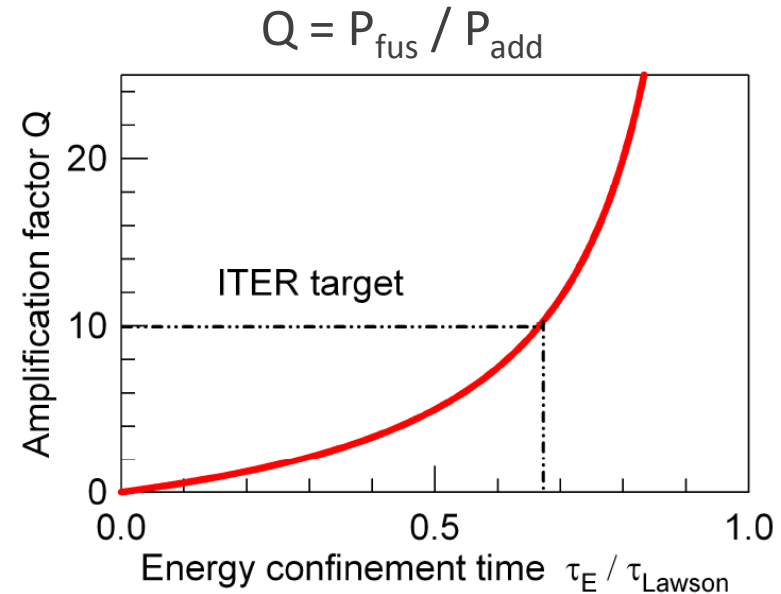
τ_E — Energy confinement time

► Self-heated Fusion (ignition)

$$nT\tau_E > 6 \cdot 10^{21} \text{ m}^{-3} \cdot \text{keV} \cdot \text{s}$$

MHD stability
Turbulent transport

$$\tau_E = \frac{\text{energy content}}{\text{loss power}} \Rightarrow \text{governs fusion performance}$$



► Magnetic confinement

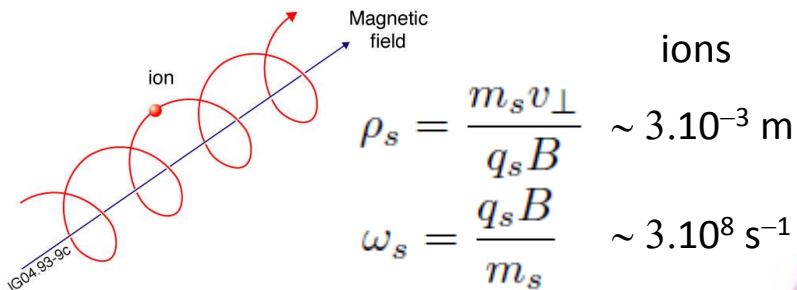
$T \approx 20 \text{ keV}$ (threshold reaction)

$n \approx 10^{20} \text{ m}^{-3}$ ($\beta = P_{\text{kin}} / P_{\text{magn}} < 1$)

$\Rightarrow$ Pressure $\sim$ few atm
($P_{\text{Sun}} \sim 10^{11} \text{ atm}$)

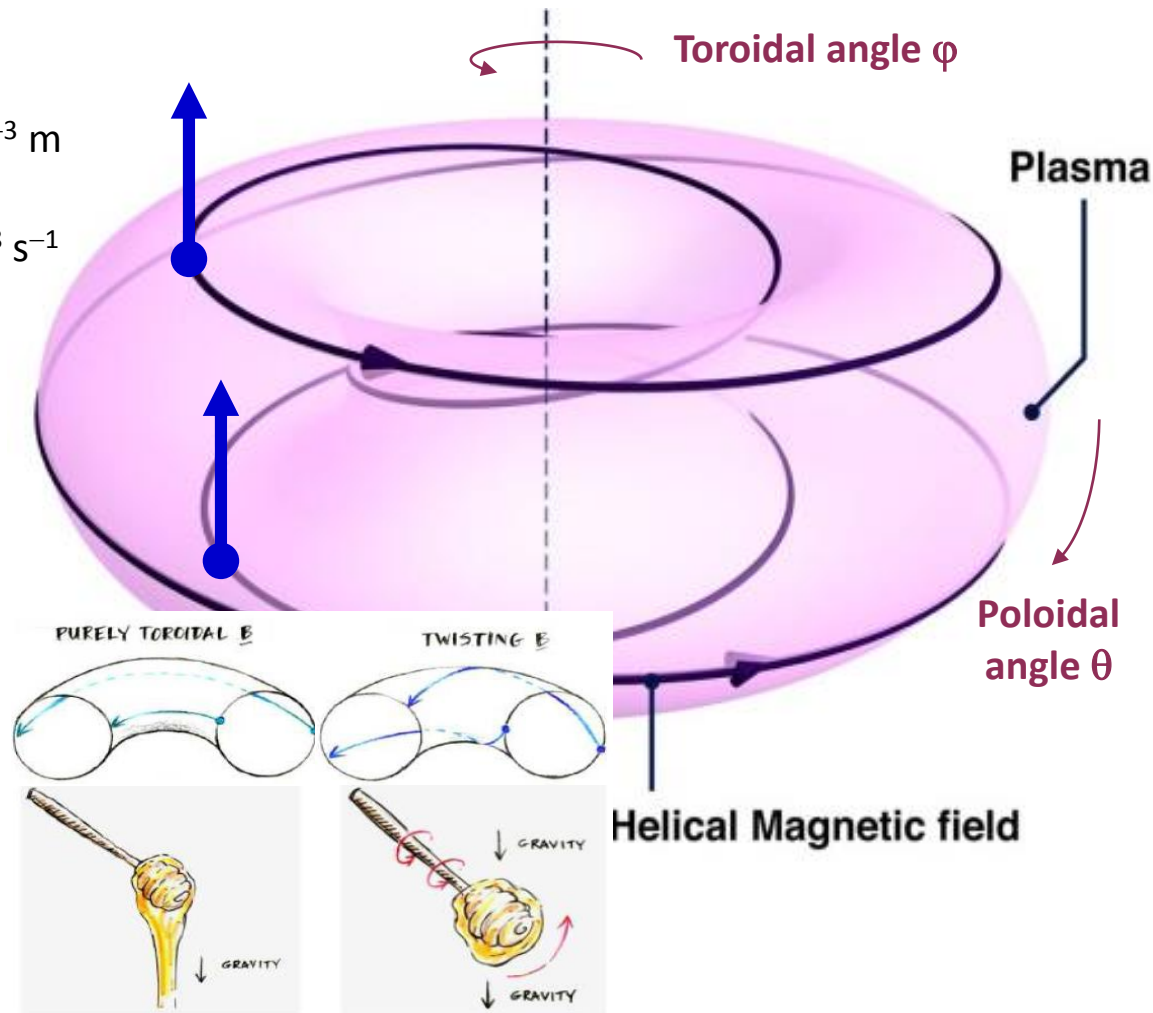
$\Rightarrow$ Requires $\tau_E \approx 3 \text{ s}$

- Particles confined by **strong magnetic field B**



- Closed field lines
 - ⇒ **particle vertical drift**
 - ⇒ helicity to compensate

- Magnetic field lines twist around **tori called magnetic surfaces**

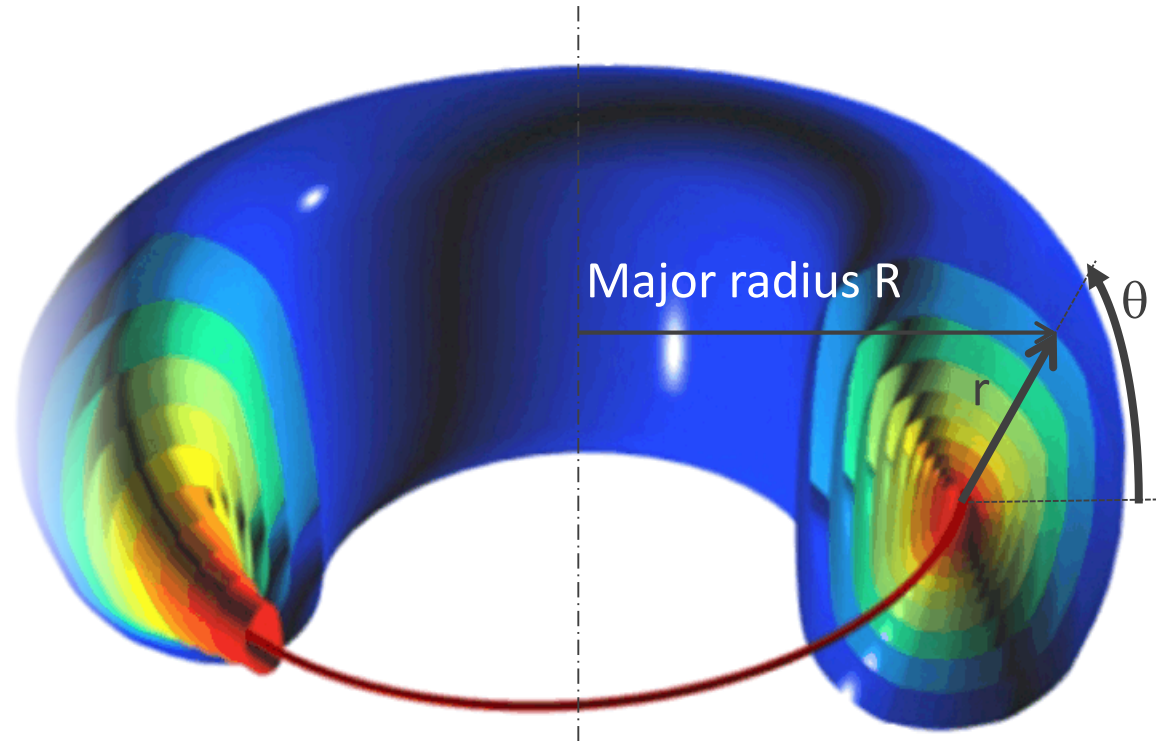


- **Strong anisotropy** of the transport: $\parallel$ to $\mathbf{B} \gg \perp$ to $\mathbf{B}$
 $\Rightarrow$ Fast homogenization of magnetic flux surfaces

- Magnetic surfaces are (at leading order) isobar, iso-temperature, iso-current

- **Lorentz force** balances **pressure gradient**

$$\mathbf{j} \times \mathbf{B} = \nabla p$$



Nested magnetic surfaces

- **Strong anisotropy** of the transport
⇒ Fast homogenization of magnetic surfaces

- Magnetic surfaces are
(at leading order) isobar,
iso-temperature, iso-current

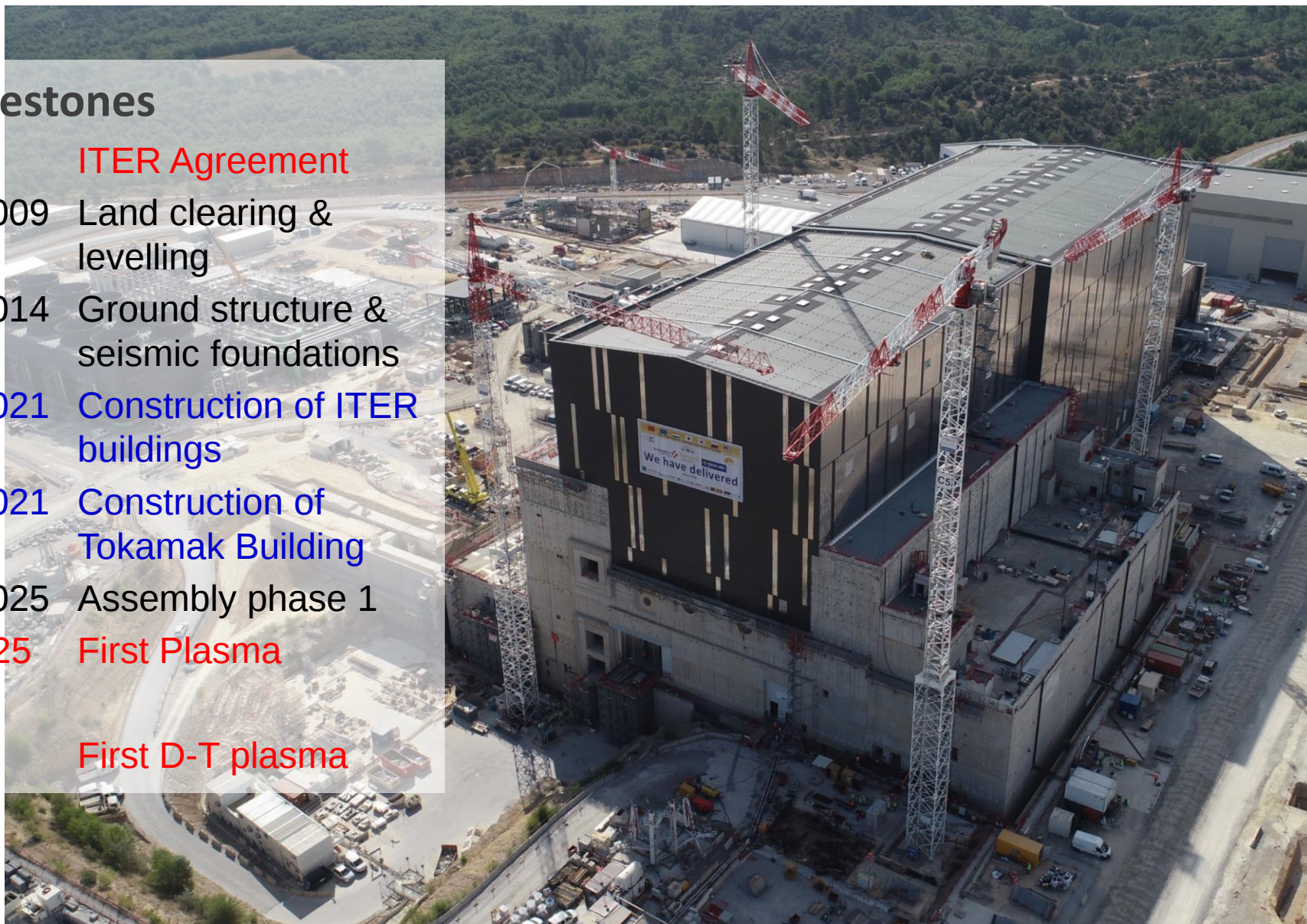
- **Lorentz force** balances
pressure gradient

$$\mathbf{j} \times \mathbf{B} = \nabla p$$



► Milestones

- 2006 ITER Agreement
- 2007-2009 Land clearing & levelling
- 2010-2014 Ground structure & seismic foundations
- 2010-2021 Construction of ITER buildings
- 2014-2021 Construction of Tokamak Building
- 2018-2025 Assembly phase 1
- Dec 2025 First Plasma
- ...
- 203X First D-T plasma



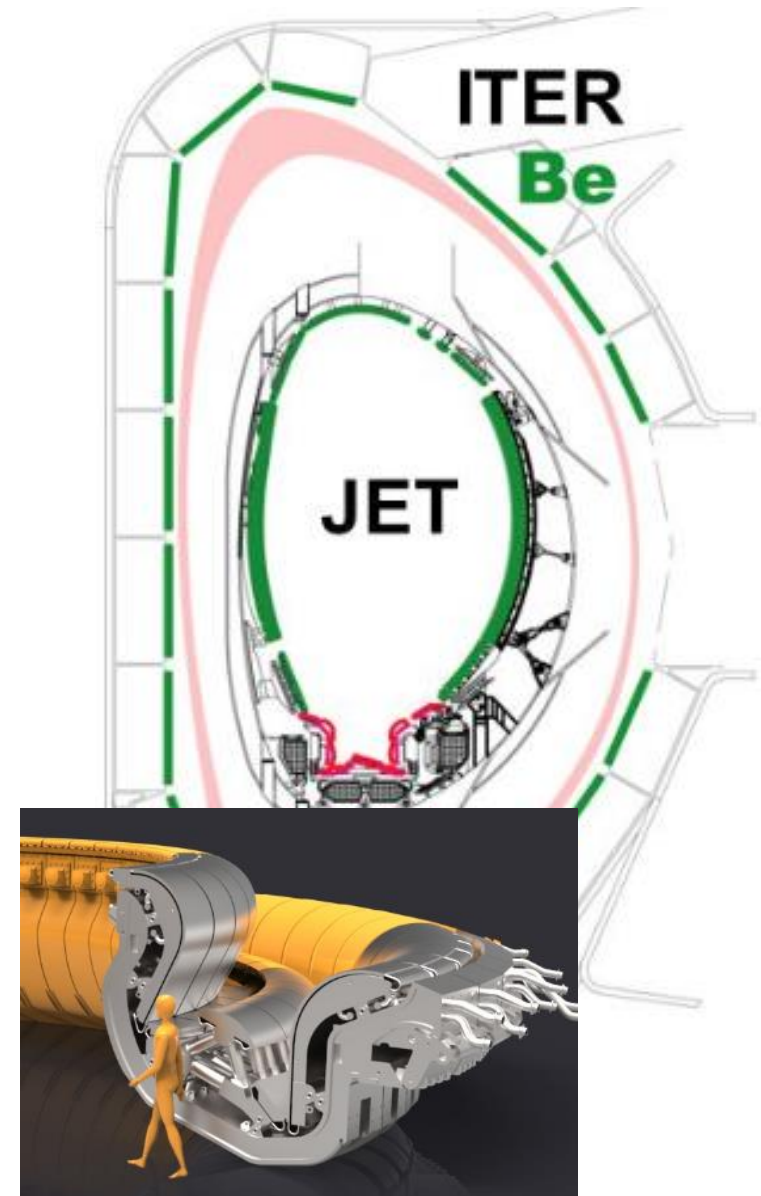
ITER work-site (December 2020)

CEA, IRFM



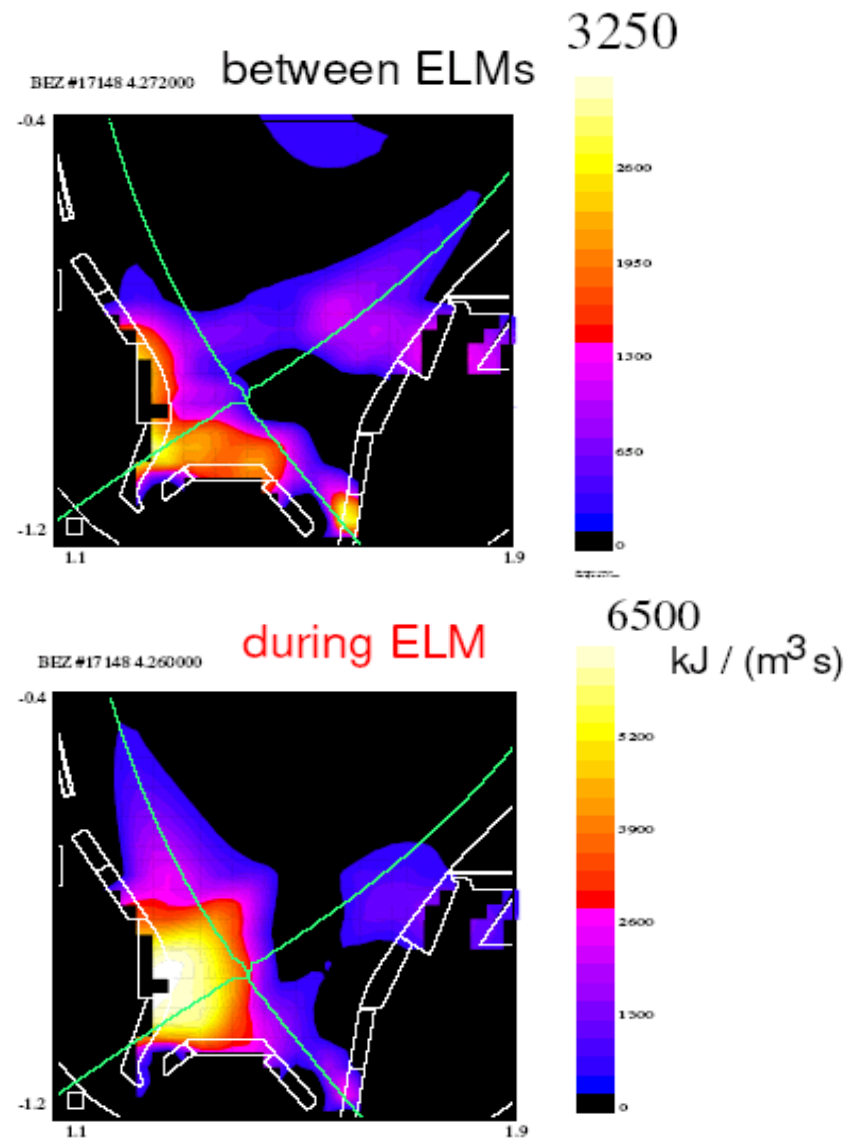
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- ▶ Interface with plasma facing components: in general, hot & diluted plasma
 - ⇒ emission of **impurities**
 - ⇒ inefficient **pumping**
- ▶ Preferable solution: **dense, cold & radiating plasma**
 - appropriate device = **divertor**
 - Heat exhaust ($\sim 10 \text{ MW.m}^{-2}$)
 - Pumping ashes (He)
 - Controlling impurity influx
 - Fueling the plasma with D-T
 - Resilience to transient heat loads



- ▶ Interface with plasma facing components: in general, hot & diluted plasma
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Asdex-Upgrade



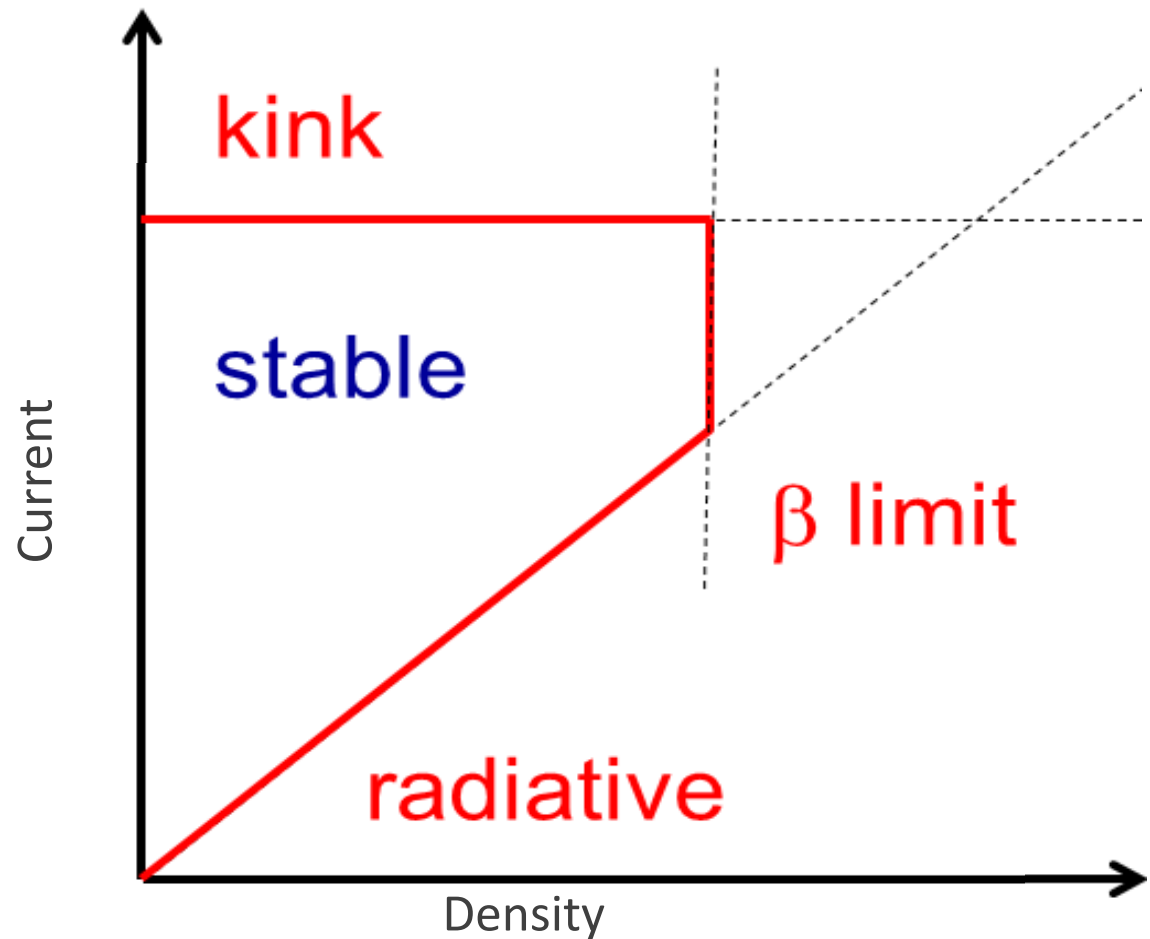
► **Magneto-hydrodynamic (MHD)**

instabilities driven by

- Pressure
- Current
- Radiative losses

► **Dynamics of MHD**
instabilities

- Steady-state
- Oscillations
- Disruption



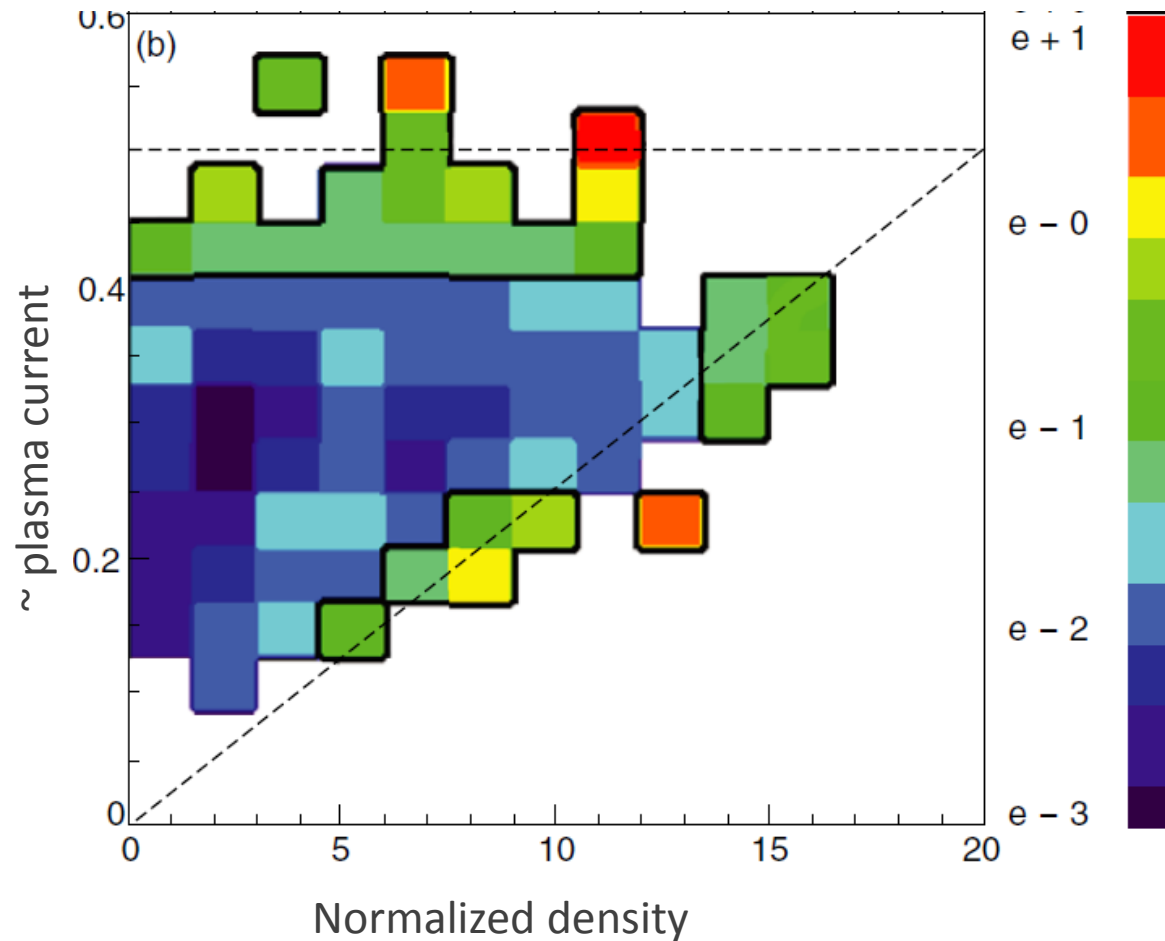
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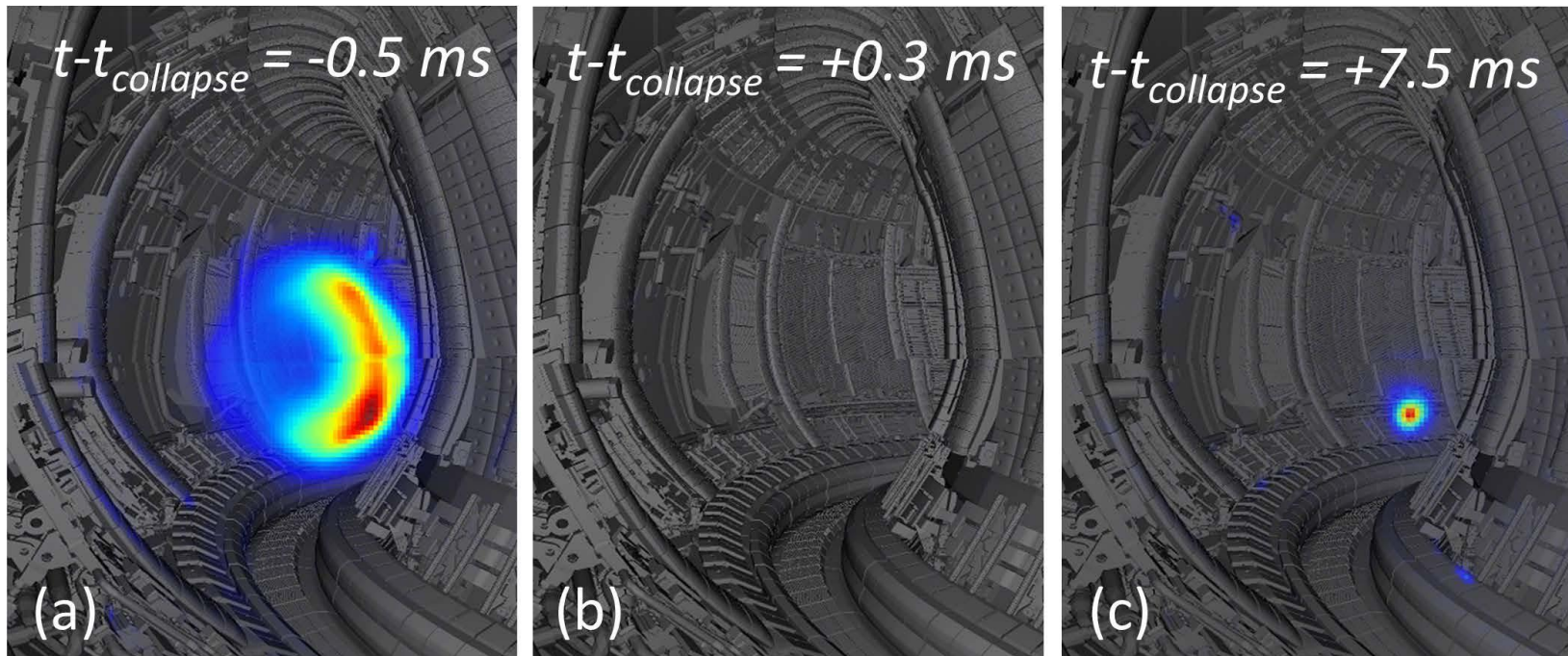
- Steady-state
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Disruptivity at JET
De Vries et al. (2009)



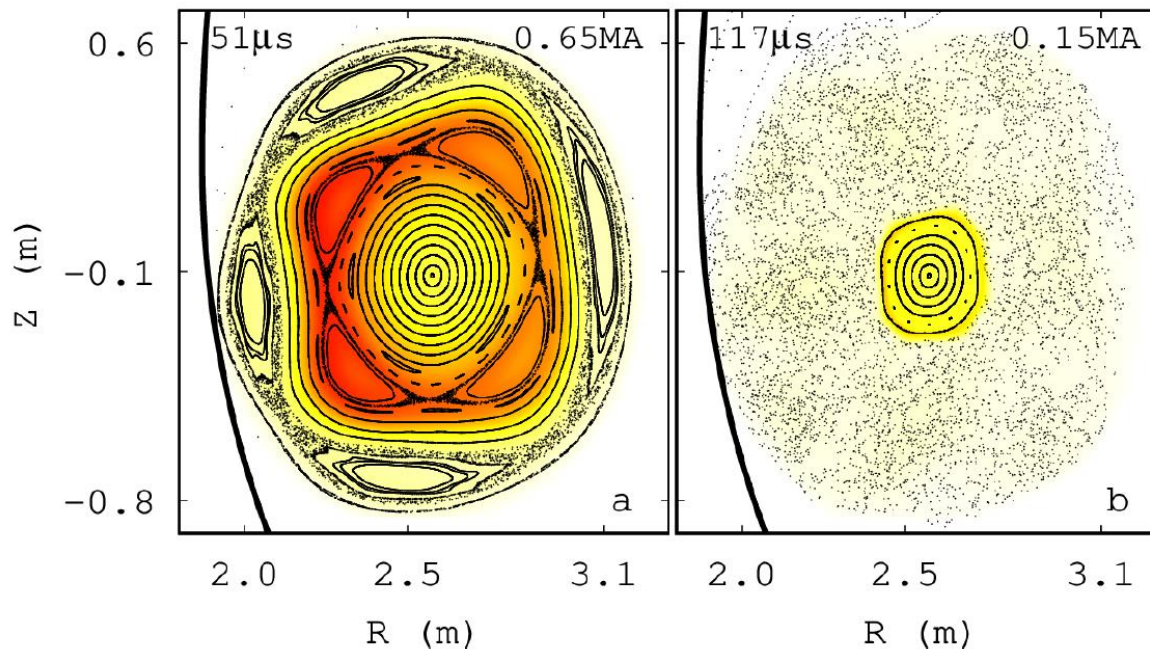
- ▶ **Disruptions** = sudden (few ms) loss of confinement
 - Huge forces on plasma vessel
 - Runaway Electron beam $v_e \sim c \rightarrow$ up to 100 MJ in ITER
- ▶ **Strategies**: detection, **avoidance** and/or **mitigation**

Mitigation of RE at JET with shattered D2 pellets [Reux et al., PRL 2021]



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Mitigation of RE at JET with shattered D2 pellets [Reux et al., PRL 2021]



Tearing MHD mode
↓
Stochastization
of field lines
↓
RE no longer confined in
100 μs

► Several transport channels

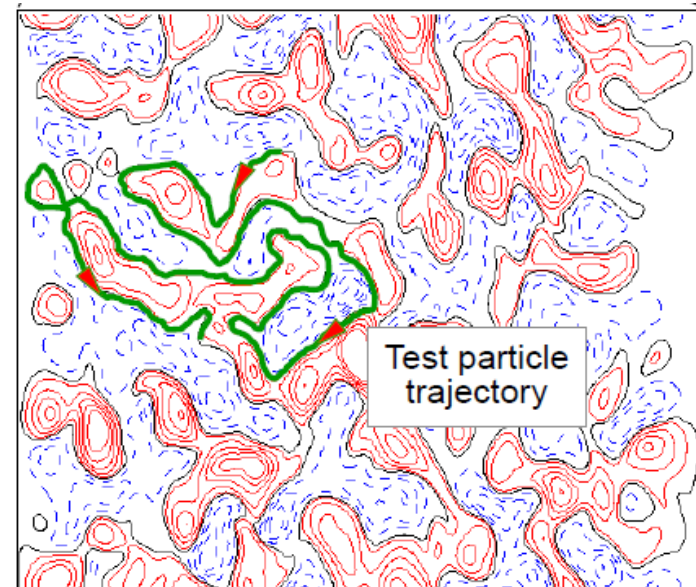
- Heat (τ_E) $\rightarrow$ electrostatic turbulence
- α particles, runaway electrons $\rightarrow$ electromagnetic modes
- Impurities $\rightarrow$ turbulence & collisions

► Key physics:

Resonant and/or non-resonant coupling
between particles and EM fields

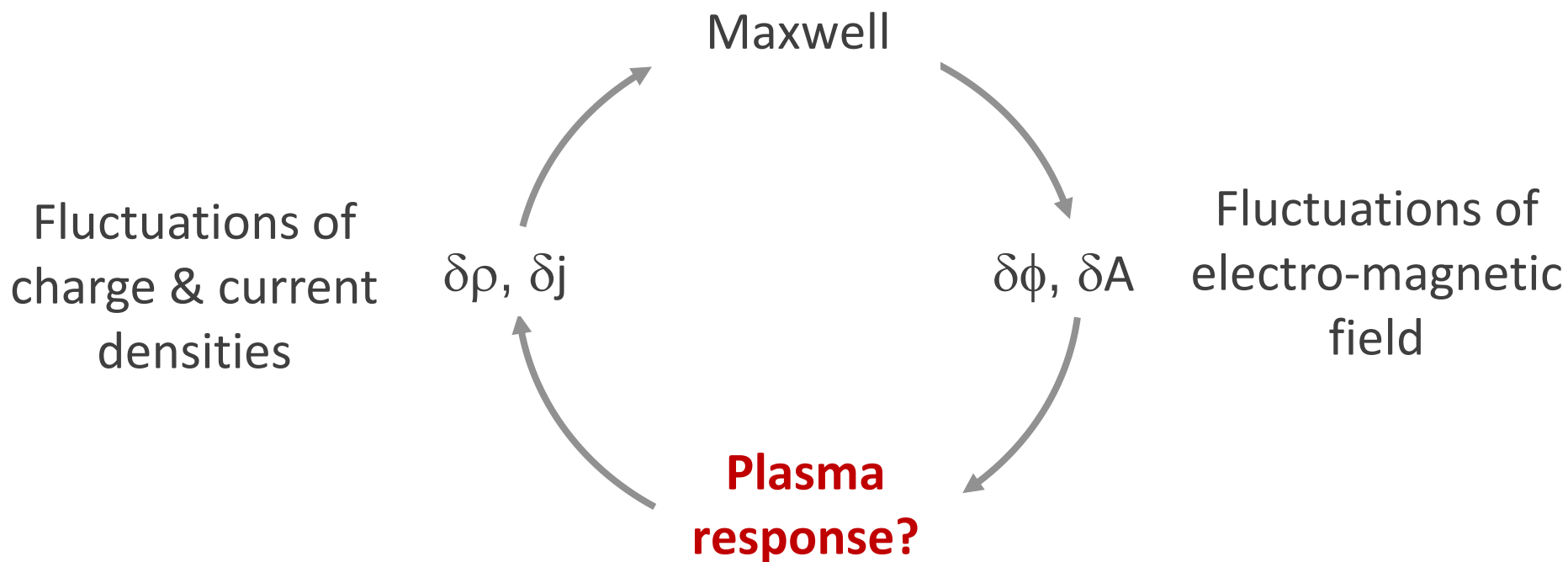
► Numerical issues:

- Model description: fluid moments and/or adiabatic vs. (gyro)kinetic $\rightarrow$ coupling issues
- Mix of slow & fast particles: thermal particles vs. α or runaway electrons, ions vs. electrons $\rightarrow$ resolution issues



Quasi-neutrality: $\rho = 0$ (OK for $k\lambda_D \ll 1$)

Maxwell-Ampère: $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$



3 main descriptions: | MHD / Multi-Fluid
(Gyro-) Kinetic

► **(Gyro)Kinetic:**

- Vlasov-like equation
- Sources & sinks
- Collisions

$$d_t F_s = S + C(F_s)$$

- **(5)6D + time:** $F_s(\mathbf{x}, \mathbf{v}, t)$
- Integrals on velocity to get charge & current densities



► **Fluid & MHD:**

- Conservation equations of particles, momentum, energy

$$\rho d_t \mathbf{V} = -\nabla P + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V}$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}$$

$$d_t = \partial_t + \mathbf{V} \cdot \nabla$$

+ eq on pressure P

- **3D + time:** $\mathbf{V}(\mathbf{x}, t), P(\mathbf{x}, t), \dots$



Both descriptions require **High Performance Computing**

E.g. GK ion turbulence in ITER: $a/\rho_{ci} \sim 10^3$ & $\omega_{\text{turb}} \tau_E \sim 10^5 \rightarrow \sim 3 \cdot 10^{11}$ grid points, ~ 20 Mh.core

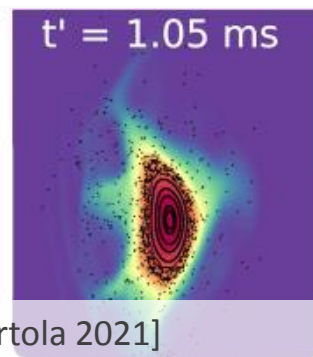
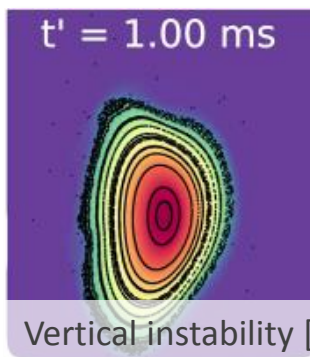
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- Ideal MHD approach valid if $\omega \gg k.v$ & $k_{\perp}\rho \ll 1$

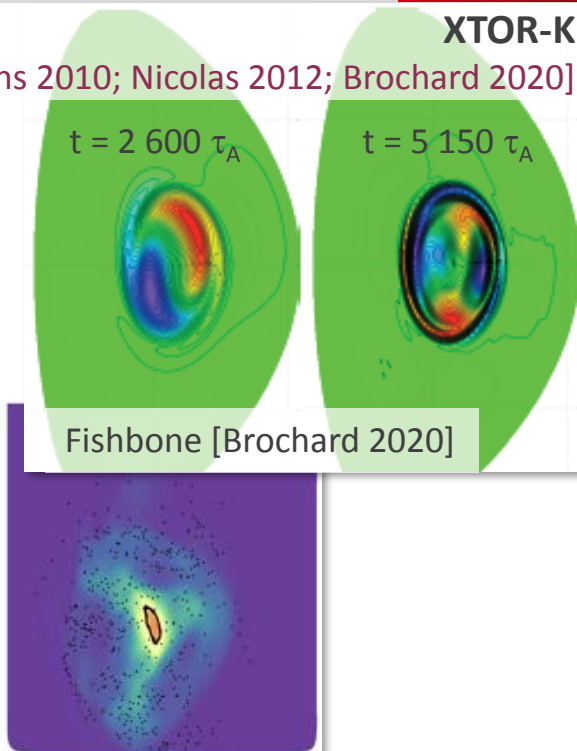
- Works well for ideal global modes ($\omega \tau_A \sim 1$)

$$\tau_A = a/v_A \sim \beta^{1/2} a/v_{th}$$

[Huysmans 2007; Czarny 2008; Hoelzl 2021]



JOREK



Numerical bottlenecks:

- **Implicit schemes** to filter out fast time scales
- Small resistivity ($\eta_{ITER} \sim 10^{-9}$) $\rightarrow$ small scales ($\sim \eta^{1/2}$)
- Nonlinear dynamics is important (e.g. crashes, ...)

Requires many modes



Inversion of (very) **large sparse matrices**

- Accounting for departures from MHD flow is essential for quantitative predictions → Ion flow $\mathbf{V}_i = \mathbf{V} + \mathbf{V}_* + \dots$

MHD flow $\mathbf{V} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + V_{\parallel} \mathbf{b}$

Diamagnetic flow $\mathbf{V}_* = \frac{\mathbf{B} \times \nabla P_i}{enB^2}$

$$mnd_t \mathbf{V}_i = -\nabla P_i + en(\mathbf{E} + \mathbf{V}_i \times \mathbf{B}) - \nabla \cdot \Pi_{gv}$$

- "Gyroviscous cancellation": gyroviscous stress tensor Π_{gv} cancels out with diamagnetic dynamic

[Hinton-Horton '71;
Hazeltine, '85]

$$mn\{\partial_t \mathbf{V}_* + [(\mathbf{V} + \mathbf{V}_*) \cdot \nabla] \mathbf{V}_* + \mathbf{b}(\mathbf{V}_* \cdot \nabla) V_{\parallel}\} = -\nabla \cdot \Pi_{gv} - \nabla \chi$$

Residual pressure $\chi = (P_i/2\omega_c) \mathbf{b} \cdot \nabla \times \mathbf{V}_{\perp i}$ ($\sim \rho_* P_i \ll P_i$) incorporated in P_{total}

- Modified density, momentum, ... eqs

$$d_t \rho + \rho \nabla \cdot \mathbf{V} + \nabla \cdot (\rho \mathbf{V}_*) = 0$$

$$\rho d_t \mathbf{V} + \rho (\mathbf{V}_* \cdot \nabla) \mathbf{V}_{\perp} = -\nabla P + \mathbf{J} \times \mathbf{B}$$



OK in slab geometry
More complex in toroidal

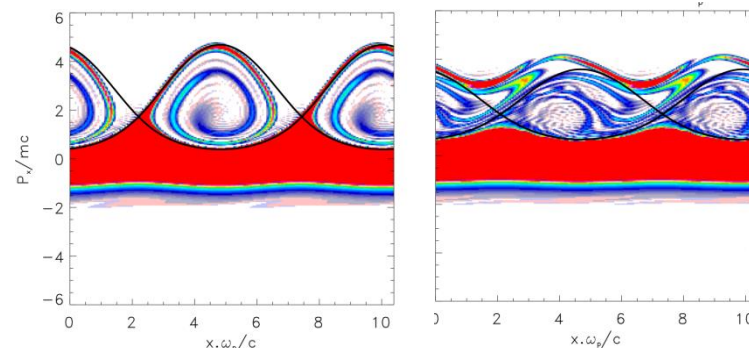
[Ramos, '05]

► Fast particles interact resonantly with $(\mathbf{E}, \mathbf{B})$ fields:

modify MHD modes
drive Energetic Particle driven modes

⇒ Track **wave-particle resonances**

⇒ **kinetic** description

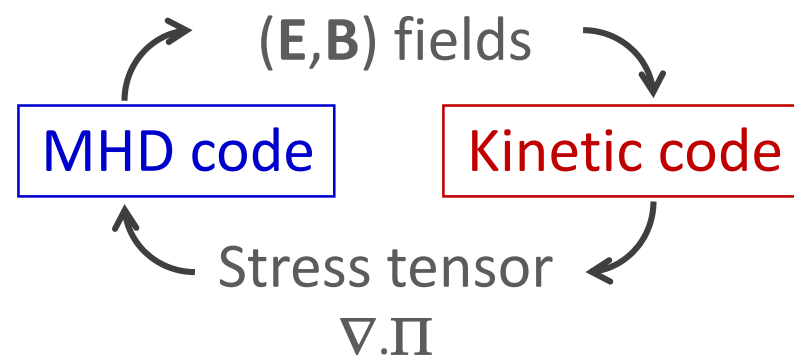


► Hybrid MHD codes:

Thermal particles → MHD

Fast particles → (gyro)kinetic

⇒ Coupling via stress tensor



E.g. CGL expression of Π :

[Chew, Goldberger, Low '56]

$F_h \rightarrow$ pressures

$$\Pi_{hot} = P_{\parallel} \mathbf{b}\mathbf{b} + P_{\perp} (\mathbf{I} - \mathbf{b}\mathbf{b})$$

$$P_{\parallel} = \int d^3\mathbf{v} F_h m v_{\parallel}^2$$

$$P_{\perp} = \int d^3\mathbf{v} F_h \frac{1}{2} m v_{\perp}^2$$

[Park '92; Briguglio '98;
Lütjens '15]

► Electrostatic **turbulence** mainly at **ion scale** (few $\rho_i \sim \text{cm}$)

► Yet, **kinetic electrons** are **mandatory**:

- Modify Ion Temperature Gradient (ITG) turbulence
- **Trapped electrons** → **Trapped Electron Modes**
- **Passing electrons** → **ETG turb., Electromagnetic effects**

► **Numerical issues**: $m_D/m_e \approx 3\,672$

$\Rightarrow \rho_e \sim \rho_i/60, v_{th,e} \sim v_{th,i} \times 60 \Rightarrow$ **memory, CPU time**

► **Staggered approach**:

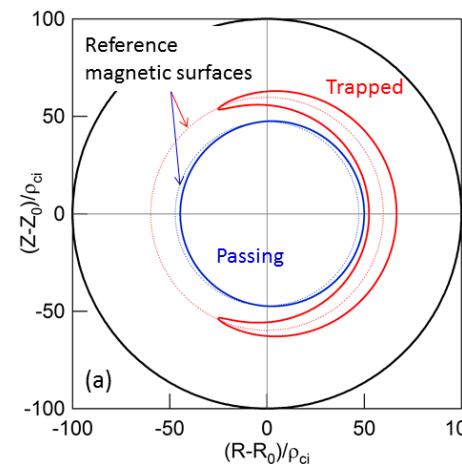
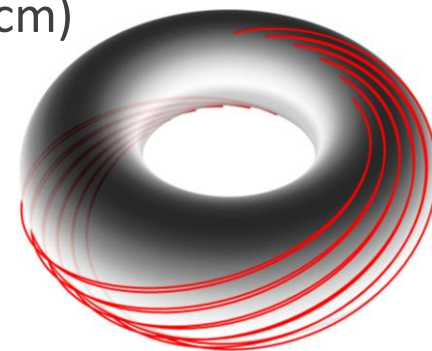
[Idomura 2016; Lanti 2020]

- **Trapped electrons** → **Kinetic**
- **Passing electrons** → **Boltzmann response (or fluid)**



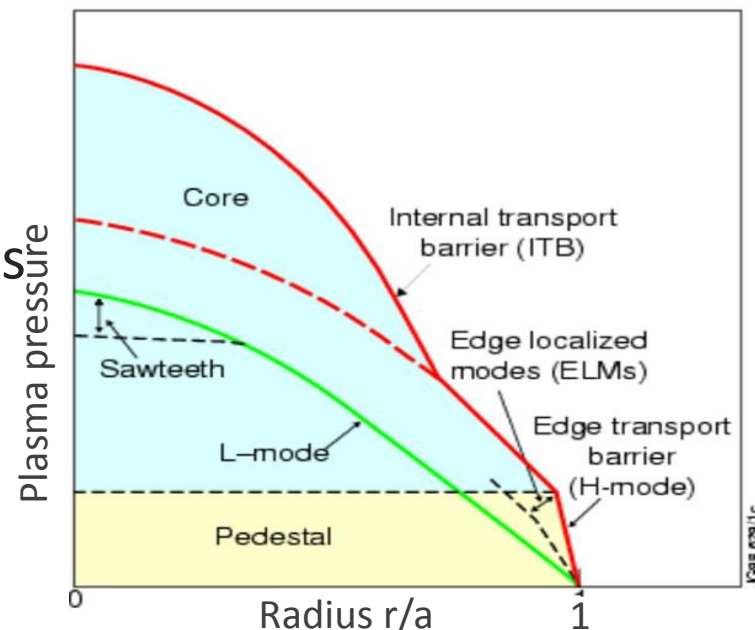
Particle conservation

$$\underbrace{n_{e,trapped}}_{\int d^3v F_e} + \underbrace{n_{e,passing}}_{\propto f_p \exp(e\delta\phi/T_e)} = \underbrace{n_i}_{\int d^3v J[F_i] + n_{pol}}$$



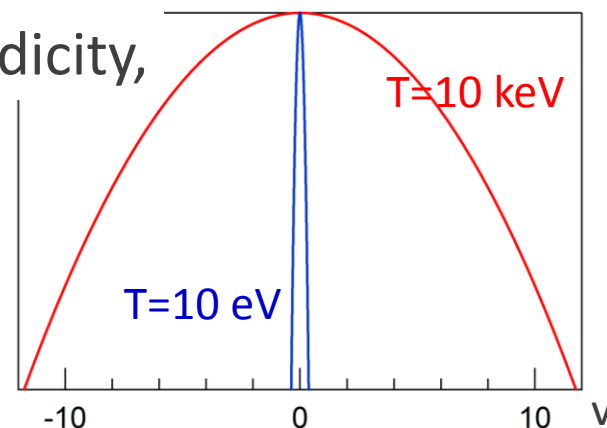
- In tokamaks, **global confinement** much dependent on **edge plasma**:

- **Edge transport barrier** = spontaneous reduced turbulent transport
- Critical role of **sheared electric field**
 $v_{E\theta} = -E_r/B \rightarrow$ vortex stretching
 (already in L-mode)



- Key challenges when **addressing core & edge plasma transport**:

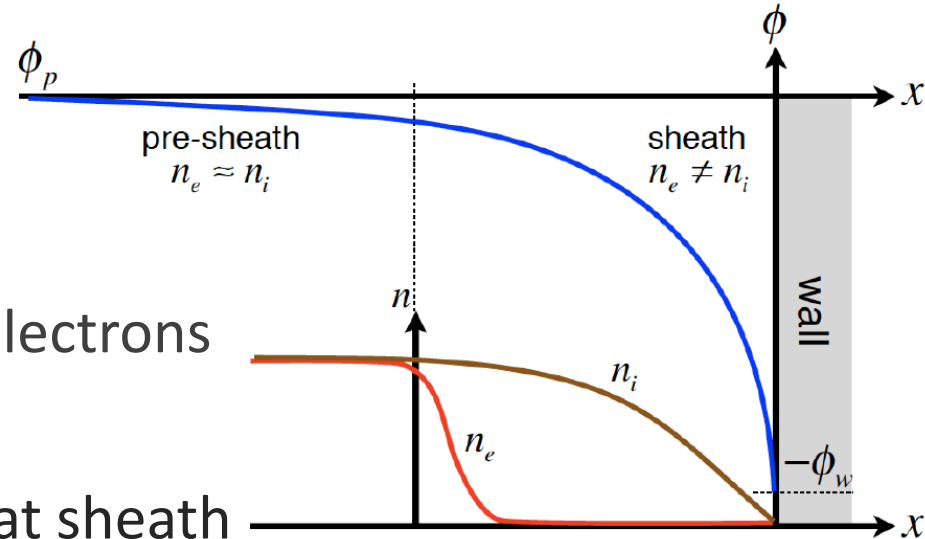
- **Closed vs. open field lines** $\Rightarrow$ loss of periodicity, **plasma-wall interaction**
- Complex 3D geometry
- Huge variations of temperature (10 keV $\rightarrow$ a few eV)



► **Mainly governed by // motion:**

[Bohm 1949; ...; Riemann 1991; Stangeby 2000; Ghendrih 2011]

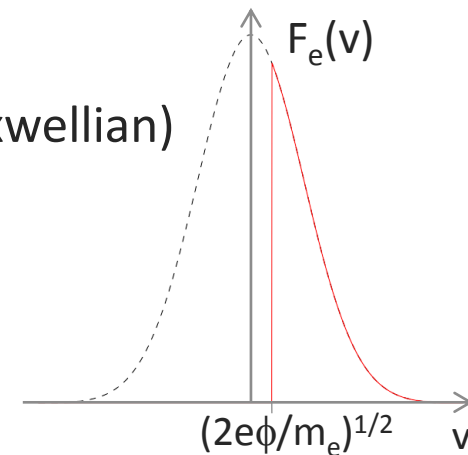
- Different electron & ion inertia
 $\Rightarrow$ Space electric charge @ wall
- Large $E_{//}$ in the sheath
 $\Rightarrow$ Accelerates ions / confines electrons
 $\Rightarrow$ Vanishing current in the wall
- Bohm (fluid) criterion: $v_i = \pm c_s$ at sheath entrance



[Martinez-Sanchez 2015]

► **Physical & numerical issues:**

- Strongly **distorted distribution functions** (non-Maxwellian)
- **Broad range of scales:** $L_{//} \approx 2\pi qR \gg \lambda_{\text{sheath}} \approx \lambda_D$
 $\approx 100\text{m}$ $\approx 10^{-4}\text{m}$
- **Kinetic Bohm criterion?**
- Important role of **reflected ions**



► Immersed boundary via penalization technique:

- Additional Krook terms: $dF/dt = -\nu(F - F_{\text{target}})$
- On-off mask (tanh) for wall / plasma
- Flexible, experience in fluid
- Unknown territory in kinetic

[Isoardi 2010;
Paredes 2014]

► Wall penalization:

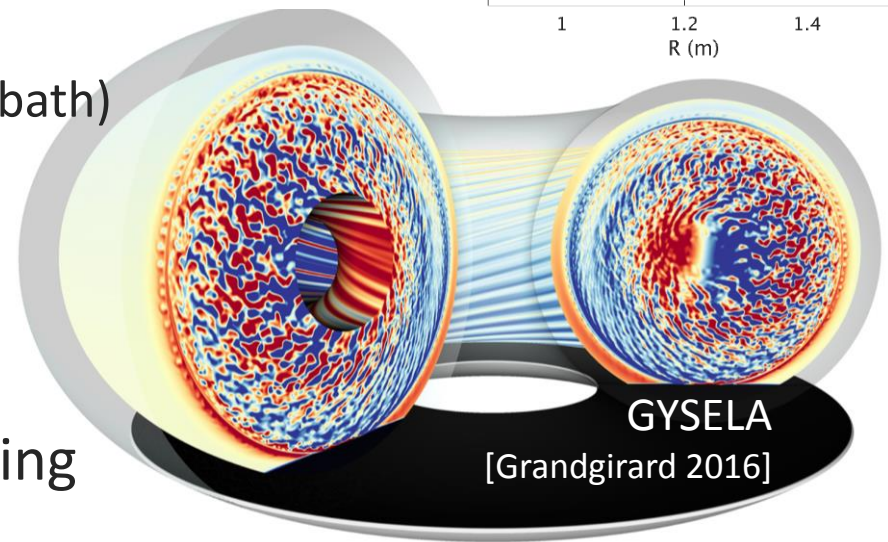
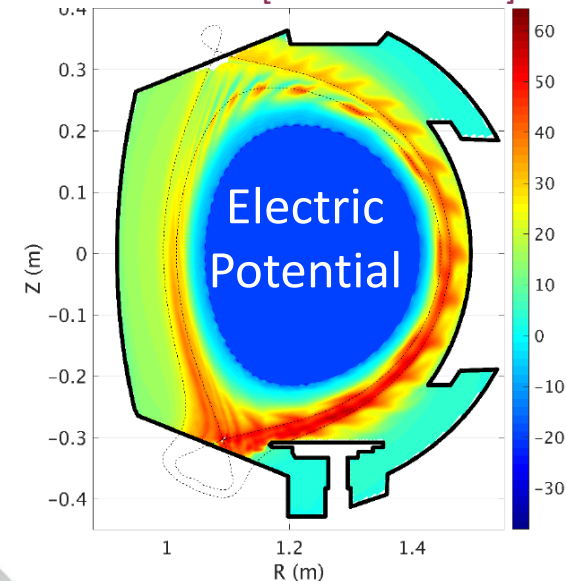
- ∞ reservoir of particles
- ∞ reservoir of energy (thermal bath)
- ∞ conductor

► Plasma in wall:

- ∞ / partial recombination
- $\phi=0$ grounded potential / biasing
- ∞ momentum sink

SOLEEDGE3X-EIRENE

[Bufferand 2021]



[Caschera 2018; Dif-Pradalier 2020]

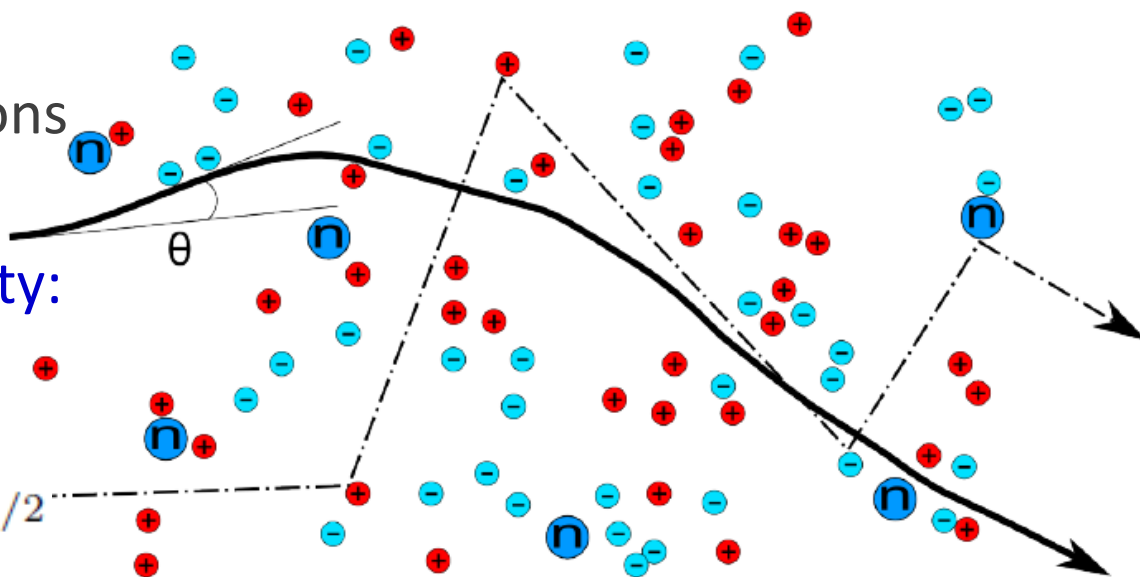
► Dilute & hot plasma:

- Most Coulomb collisions are **grazing**
- Weak core collisionality:

$$\nu_* = \nu_{coll} \tau_{||} \ll 1$$

$$\nu_{coll} \sim n T^{-3/2}$$

$$\tau_{||} \sim L_{||} T^{-1/2}$$



► Critical issues governed by collisions (NS vs. Euler):

- Relaxation towards Maxwellian
- Flow damping: friction on trapped particles (→ transition between turbulent regimes, ...), Zonal Flow damping (→ turbulence saturation)
- Impurity transport, especially high Z + possible synergy turb./coll.
- Momentum & Energy exchanges between species

- **Landau formulation** (1936) of the Fokker-Planck operator
(limit of small angle scattering events & cut-off at Debye shielding distances)

$$\frac{\partial f_a}{\partial t} = \sum_b C_{ab}(f_a, f_b)$$

Coulomb logarithm: $\ln \Lambda = \ln \left(\frac{\lambda_D}{b_0} \right) \gg 1$

$$C_{ab}(f_a, f_b) = \frac{\partial}{\partial \mathbf{v}} \cdot \left\{ \frac{2\pi e_a^2 e_b^2 \ln \Lambda}{m_a} \int d^3 \mathbf{v}' f_a(\mathbf{v}) f_b(\mathbf{v}') \right. \\ \left. \frac{\mathbb{I}u^2 - \mathbf{u} \otimes \mathbf{u}}{u^3} \cdot \left(\frac{1}{m_a} \frac{\partial \ln f_a(\mathbf{v})}{\partial \mathbf{v}} - \frac{1}{m_b} \frac{\partial \ln f_b(\mathbf{v}')}{\partial \mathbf{v}'} \right) \right\}$$

$$\mathbf{u} = \mathbf{v} - \mathbf{v}'$$

- Particle, momentum & energy **conservation** → exchanges between species
- Satisfies Boltzmann **H-theorem** (1872):
Entropy production & $C(f_{\text{Maxwell}}) = 0$

[Helander-Sigmar'05, Hirschman-Sigmar '76 '77 '81, Hinton-Hazeltine '76, Hinton '83, Rosenbluth-Hazeltine-Hinton '72, Abel '08, Barnes '09, Donnel '19]

- Usually **linearized**: $f_s = f_{M,s} + \delta f_s$ with $\delta f_s \ll f_{M,s}$

$$C_{ab}(f_a, f_b) \approx \underbrace{C_{ab}(f_{Ma}, f_{Mb})}_{\text{Test particle op.}} + \underbrace{C_{ab}(\delta f_a, f_{Mb}) + C_{ab}(f_{Ma}, \delta f_b)}_{\text{Field op.}}$$

- **Full operator required at the edge** (strong fluctuations, departure F_{Maxwell})

$$C_{ab}(f_a, f_b) = -\frac{4\pi e_a^2 e_b^2 \ln \Lambda}{m_a m_b} \frac{\partial}{\partial \mathbf{v}} \cdot \left(\frac{m_a}{m_b} \frac{\partial h_b}{\partial \mathbf{v}} f_a - \frac{1}{2} \frac{\partial^2 g_b}{\partial \mathbf{v} \partial \mathbf{v}} \cdot \frac{\partial f_a}{\partial \mathbf{v}} \right)$$

$$\nabla_v^2 h_b = -4\pi f_b \qquad \nabla_v^2 g_b = 2h_b$$

Rosenbluth potentials (1957)

- **Physical** issues:

- **Conservation** properties
- **Neoclassical** results $df_a/dt = C_{ab} \rightarrow \chi_{\text{neo}}, V_{\theta, \text{neo}}, \Gamma_{Z, \text{neo}}, \dots$

- **Numerical** issues:

- Whatever **mass ratio** m_a/m_b and **charge** states e_s
- To be adapted to actual **choice of variables**, e.g. $(v, v_{\parallel}/v), (v_{\parallel}, \mu)$
- Moderate **CPU time** $\rightarrow$ parallelization

- ▶ **First principal simulations** are crucial for ITER (pulse preparation, post-pulse analysis)
- ▶ At the **limits of current HPC** supercomputers (memory & CPU time)
- ▶ A hierarchy of **complementary approaches**

Reduced 1D models
(D,V), QuasiLinear, K- ϵ , ...

3D Fluid, MHD

5D Gyrokinetic
Eulerian, PIC/MC, SL

6D kinetic

- ▶ **Numerical bottlenecks**: coupling $\neq$ approaches, core $\rightarrow$ wall
- ▶ **Validity limitation of physical models** $\Rightarrow$ numerical issues
- ▶ Other issues: optimization of 3D magnetic configurations (stellarators), free boundary magnetic equil., ...

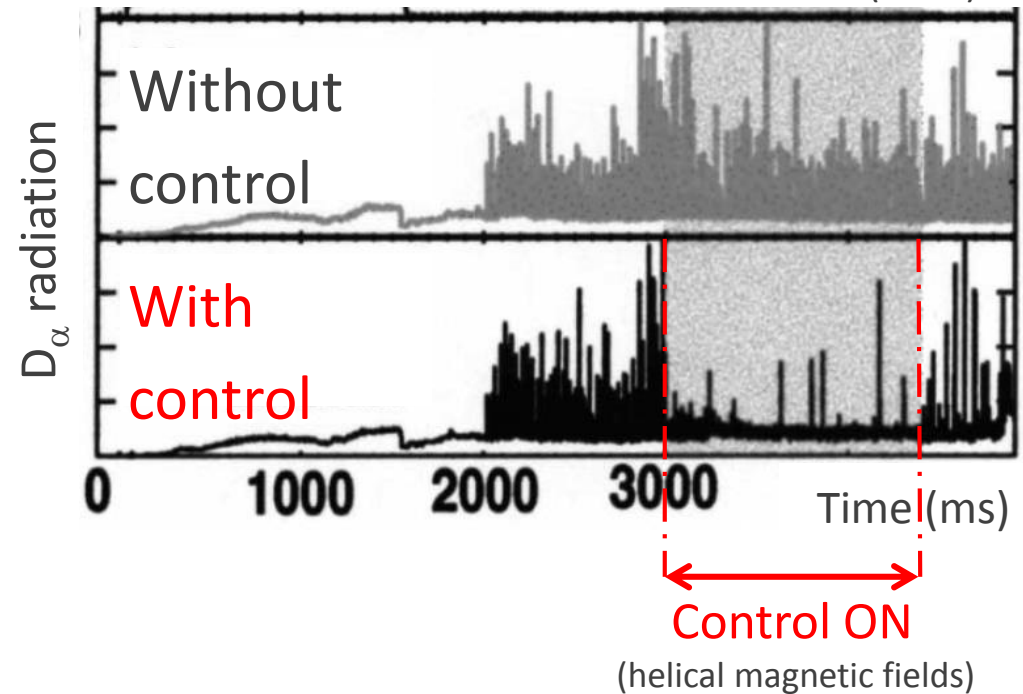
Inputs from Maths is essential!

Back-up slides

Relaxation oscillations in JET (ELMs)



Evans et al. PRL (2004)



- ▶ Pressure / current exceeds instability threshold → fast relaxation → recovery
- ▶ Bursts of particle & heat fluxes
- ▶ **Control is mandatory**: via external coils or localized heating / current