



Décomposition de domaine sur des formulations intégrales surfaciques en électromagnétisme

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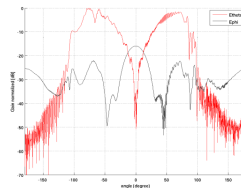
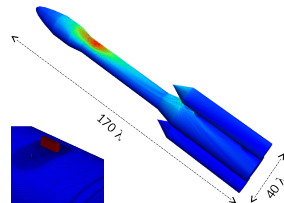
DE LA RECHERCHE À L'INDUSTRIE

Décomposition de domaine sur des formulations intégrales surfaciques en électromagnétisme

Francis Collino Justine Labat Agnès Pujols Muriel Sesques | CEA-CESTA

SMAI 2021

- Context : Solution of **electromagnetic** wave scattering problem by a **complex** object using a boundary element method
- Motivation : Accurate evaluation of **radar cross-sections**
 - ▶ Large-scale objects (in comparison with the wavelength)
 - ▶ Multi-scale phenomena

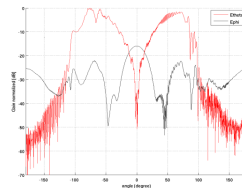
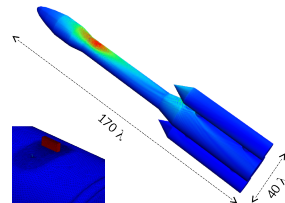


Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

- Context : Solution of **electromagnetic** wave scattering problem by a **complex** object using a boundary element method
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 - Large-scale objects (in comparison with the wavelength)
 - Multi-scale phenomena
- Difficulties : **High computational costs**
 - Number of degrees of freedom : 1 650 875
 - Approximately 100 000 000 in volume

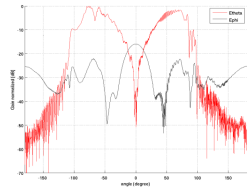
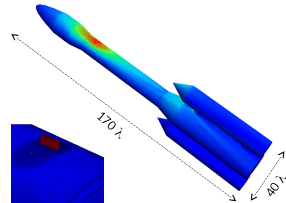
Solver	Number of CPU	Factorization (in CPU·H)
LL ^t (prediction)	15 000	151 435
Block low-rank	800	2 541
H-matrix	200	1 043

Hackbush (1999), Bebendorf (2008), . . .



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- Difficulties : **High computational costs**
- Objectives : Development of a **domain decomposition** method
 - ▶ Robust simulations for a **wide range** of frequencies
 - Solve very large problems (more than 10 000 000 of unknowns)
 - ▶ **Geometry-adaptative** strategies to handle multi-scale structures
 - Allow non-conformal meshes
 - ▶ Simulation-based engineering using **High Performance Computing**



Example : Monopole antenna in the presence of a dielectric object on a launcher (ISAE Workshop, 2016)

- Adaptation of volumical **discontinuous Galerkin** methods to surface problems

Peng, Hiptmair and Shao (2016), Messai and Pernet (2020)

	Volume	Surface	Comments
Locality	✓	✗	Iterative solution is inevitable
Restrictions	✓	✗	Restrictions of distributions are not easy to define
Traces	✓	✗	No trace theorem in $H_t^{-\frac{1}{2}}(\operatorname{div}_\Gamma, \Gamma)$

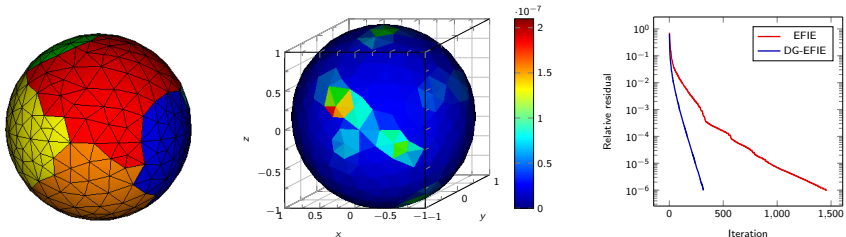
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- Why not to directly use an iterative solver on BEM?

- ▶ Main motivation : **non-conformal** meshes
- ▶ EFIE simulation preconditioned using domain partitioning on a low-frequency case



- 1 Discontinuous Galerkin-based surface domain decomposition method
 - Model problem
 - Discontinuous formulation
 - Discrete formulation
 - Iterative procedure
- 2 Numerical results
 - Accuracy of solutions
 - Convergence of the iterative solver
 - Computational costs
- 3 Conclusion and perspectives

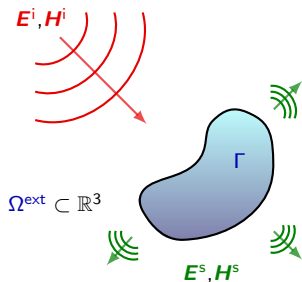
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3 Conclusion and perspectives



- Using a time-harmonic dependence in $\exp(+i\omega t)$

$$\nabla \times \mathbf{E}^s + i\kappa Z_0^{-1} \mathbf{H}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

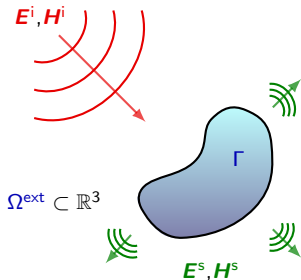
$$\nabla \times \mathbf{H}^s - i\kappa Z_0 \mathbf{E}^s = 0 \quad \text{in } \Omega^{\text{ext}}$$

$$\mathbf{n} \times \mathbf{E}^s = -\mathbf{n} \times \mathbf{E}^i \quad \text{on } \Gamma$$

$$\lim_{|\mathbf{x}| \rightarrow \infty} |\mathbf{x}| (Z_0 \mathbf{H}^s \times \hat{\mathbf{x}} - \mathbf{E}^s) = 0 \quad \text{unif. in } \hat{\mathbf{x}} = \frac{\mathbf{x}}{|\mathbf{x}|}$$

κ : wave-number, Z_0 : impedance coefficient in vacuum

- Ω^{ext} : Exterior domain
 - Γ : Scattering surface
 - $\mathbf{E}^i, \mathbf{H}^i$: Incident fields
 - $\mathbf{E}^s, \mathbf{H}^s$: Scattered fields
 - $\mathbf{E}, \mathbf{H}$: Total fields
- $$\mathbf{E} = \mathbf{E}^i + \mathbf{E}^s \quad \mathbf{H} = \mathbf{H}^i + \mathbf{H}^s$$



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- Using the Stratton-Chu formulas for $\mathbf{x} \in \Omega^{\text{ext}}$

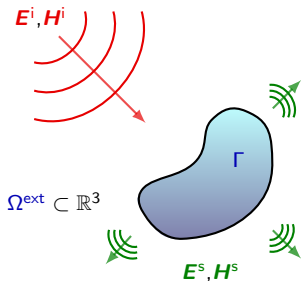
$$\mathbf{E}^s(\mathbf{x}) = -i\kappa \mathcal{T} \mathbf{J}(\mathbf{x}) \quad \text{and} \quad \mathbf{H}^s(\mathbf{x}) = \frac{1}{Z_0} \kappa \mathcal{J}(\mathbf{x})$$

$\mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H})$: total surface electric current

$$\mathcal{T} \mathbf{J} = \frac{1}{\kappa^2} \nabla (\mathcal{S} \operatorname{div}_{\Gamma} \mathbf{J}) + \mathcal{S} \mathbf{J}$$

$$\kappa \mathcal{J} = \nabla \times \mathcal{S} \mathbf{J} \quad \tilde{\mathcal{S}} \lambda(\mathbf{x}) = \int_{\Gamma} G(\mathbf{x}, \mathbf{y}) \lambda(\mathbf{y}) \, d\mathbf{y}$$

G : out-going Green function of the Helmholtz equation



- Ω^{ext} : Exterior domain
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- Jump relations give boundary integral equations

$$(\text{EFIE}) \quad i\kappa \mathcal{T} \mathbf{J} = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma$$

$$(\text{MFIE}) \quad \frac{1}{2} \mathbf{J} - \mathcal{K} \mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma$$

$$(\text{EFIE}) \quad i\kappa T\mathbf{J} = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma$$

$$(\text{MFIE}) \quad \frac{1}{2}\mathbf{J} - K\mathbf{J} = Z_0(\mathbf{n} \times \mathbf{H}^i) \quad \text{on } \Gamma$$

$$T : H_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow H_t^{-\frac{1}{2}}(\text{curl}_\Gamma, \Gamma)$$

$$T\mathbf{J} = \frac{1}{\kappa^2} \nabla_\Gamma (S \text{div}_\Gamma \mathbf{J}) + S\mathbf{J}$$

$$K : H_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma) \longrightarrow H_t^{-\frac{1}{2}}(\text{div}_\Gamma, \Gamma)$$

$$K\mathbf{J} = \text{p.v.} [\mathbf{n} \times (\nabla_\Gamma \times S\mathbf{J})]$$

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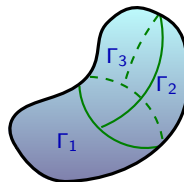
$$K\mathbf{J} = \text{p.v.} [\mathbf{n} \times (\nabla_\Gamma \times S\mathbf{J})]$$

- N : number of subdomains

$$\bullet \quad \Gamma = \bigcup_{n=1}^N \Gamma_n$$

$$\bullet \quad \gamma_{nm} = \Gamma_n \cap \Gamma_m$$

$$\bullet \quad \gamma_n = \partial\Gamma_n$$



- $\boldsymbol{\tau}_{nm}$: exterior normal vector to γ_{nm}

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$$i\kappa \sum_{m=1}^N T_{nm} \mathbf{J}_m = \mathbf{n} \times (\mathbf{E}^i \times \mathbf{n}) \quad \text{on } \Gamma_n$$

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1 Restriction to each subdomain

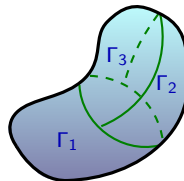
$$\mathbf{J}_m = \mathbf{J}|_{\Gamma_m} \in \tilde{H}_t^{-\frac{1}{2}}(\operatorname{div}_{\Gamma_m}, \Gamma_m)$$

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$$\mathbf{J}_m = \mathbf{J}|_{\Gamma_m} \in \tilde{H}_t^{-\frac{1}{2}}(\text{div}_{\Gamma_m}, \Gamma_m)$$

2 Obtention of variational formulations

- Multiplication by a test-function
- Integration on partial surfaces
- Integration by parts (EFIE only)

$$\langle \nabla_{\Gamma} (S_{nm} \text{div}_{\Gamma} \mathbf{J}_m), \mathbf{v}_n \rangle_{\Gamma_n} =$$

$$- \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \text{div}_{\Gamma} \mathbf{v}_n \rangle_{\Gamma_n} + \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n}$$

► which requires **more regularity** ($\varepsilon > 0$)

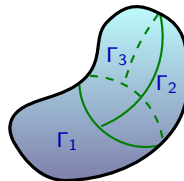
$$\text{div}_{\Gamma} \mathbf{J}_m \in H^{-\frac{1}{2}+\varepsilon}(\Gamma_m) \text{ and } \mathbf{v}_n \in H_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma}, \Gamma_n)$$

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2 Obtention of variational formulations

- Multiplication by a test-function
- Integration on partial surfaces
- Integration by parts (EFIE only)

3 Summation over all the subdomains

$$\sum_{n=1}^N \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, \mathbf{v}_n \cdot \boldsymbol{\tau}_n \rangle_{\gamma_n} = \sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

$$[\mathbf{v}]_{\gamma_{nm}} = \boldsymbol{\tau}_{nm} \cdot \mathbf{v}_m + \boldsymbol{\tau}_{mn} \cdot \mathbf{v}_n : \text{jump across } \gamma_{nm}$$

- N : number of subdomains

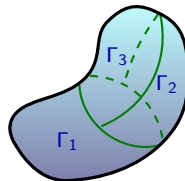
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Find $\mathbf{J} \in \bigoplus_{m=1}^N H_t^{-\frac{1}{2}+\varepsilon}(\text{div}_{\Gamma_m}, \Gamma_m)$ with $\varepsilon > 0$ such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N H_t^{\frac{1}{2}-\varepsilon}(\text{div}_{\Gamma_n}, \Gamma_n)$$

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) = \sum_{n=1}^N \sum_{m=1}^N \left\{ \frac{1}{i\kappa} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, \text{div}_{\Gamma_n} \mathbf{v}_n \rangle_{\Gamma_n} + i\kappa \langle S_{nm} \mathbf{J}_m, \mathbf{v}_n \rangle_{\Gamma_n} \right\} \quad (\text{continuous EFIE})$$

$$a_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{m=1}^N \sum_{\gamma_{nm}} \langle S_{nm} \text{div}_{\Gamma_m} \mathbf{J}_m, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}} + \tilde{a}_{\gamma}^{\pm}(\mathbf{J}, \mathbf{v}) \quad (\text{symmetrization})$$

$$p_{\gamma}^*(\mathbf{J}, \mathbf{v}) \text{ is function of } [\mathbf{J}]_{\gamma_{nm}} \text{ and } [\mathbf{v}]_{\gamma_{nm}} \quad (\text{penalization})$$

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$$(S) \quad \tilde{\mathbf{a}}_{\gamma}^{-}(\mathbf{J}, \mathbf{v}) = -\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}}$$

$$(AS) \quad \tilde{\mathbf{a}}_{\gamma}^{+}(\mathbf{J}, \mathbf{v}) = +\frac{1}{i\kappa} \sum_{n=1}^N \sum_{\gamma_{mn}} \langle S_{mn} \text{div}_{\Gamma_n} \mathbf{v}_n, [\mathbf{J}]_{\gamma_{mn}} \rangle_{\gamma_{mn}}$$

► Requires **more regularity** : $\varepsilon = \frac{1}{2}$

$$\mathbf{J} \text{ and } \mathbf{v} \in \bigoplus_{n=1}^N L_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

Find $\mathbf{J} \in \bigoplus_{m=1}^N L_t^2(\text{div}_{\Gamma_m}, \Gamma_m)$ such that

$$a_{\Gamma}(\mathbf{J}, \mathbf{v}) + a_{\gamma}^*(\mathbf{J}, \mathbf{v}) + p_{\gamma}^*(\mathbf{J}, \mathbf{v}) = \ell(\mathbf{v}) \quad \text{for any } \mathbf{v} \in \bigoplus_{n=1}^N L_t^2(\text{div}_{\Gamma_n}, \Gamma_n)$$

- Empirical choice coming from Peng, Hiptmair and Shao (2016) : $L^2(\gamma)$ -inner product

$$(L^2) \quad p_{\gamma}^0(\mathbf{J}, \mathbf{v}) = \frac{\beta}{\kappa} \sum_{\gamma_{nm}} \langle [\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

where β have to be without dimension

- requires **more regularity** (one more time!) to make sense : $\mathbf{J}$ and $\mathbf{v} \in \bigoplus_{n=1}^N H_t^{\frac{1}{2}}(\text{div}_{\Gamma_n}, \Gamma_n)$

- A new penalization : $H^{-\frac{1}{2}}(\gamma)$ -inner product (positive definite bilinear form)

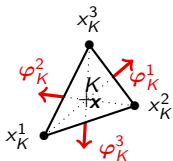
$$(H^{-\frac{1}{2}}) \quad p_{\gamma}^{-\frac{1}{2}}(\mathbf{J}, \mathbf{v}) = \beta \sum_{\gamma_{nm}} \langle S_{\gamma_{nm}}[\mathbf{J}]_{\gamma_{nm}}, [\mathbf{v}]_{\gamma_{nm}} \rangle_{\gamma_{nm}}$$

where $\tilde{S}_{\gamma} \lambda(\mathbf{x}) = \frac{1}{2\pi} \int_{\gamma} K_0(\kappa|\mathbf{x} - \mathbf{y}|) \lambda(\mathbf{y}) d\sigma_{\mathbf{y}}$ and β have to be without dimension

- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

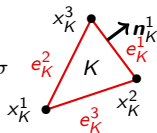
$$\mathbf{J} \approx \mathbf{J}^h \in \mathcal{V}^h \quad \Longleftrightarrow \quad \mathbf{J}^h = \sum_{m=1}^N \mathbf{J}_m^h \quad \mathbf{J}_m^h \in \mathcal{V}_m^h \subset \mathbf{L}_t^2(\operatorname{div}_{\Gamma_m}, \Gamma_m)$$

where $\mathbf{J}_m^h(\mathbf{x}) = \sum_{K \in \Gamma_m^h} J_{K,1}^{h,m} \boldsymbol{\varphi}_K^1(\mathbf{x}) + J_{K,2}^{h,m} \boldsymbol{\varphi}_K^2(\mathbf{x}) + J_{K,3}^{h,m} \boldsymbol{\varphi}_K^3(\mathbf{x}) \quad \mathbf{x} \in \Gamma_m^h$ (triangulation of Γ_m)



with $\boldsymbol{\varphi}_K^i(\mathbf{x}) = \frac{1}{2|K|} (\mathbf{x} - \mathbf{x}_K^i)$

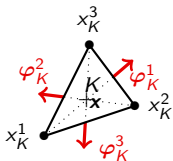
and $J_{K,i}^{h,m} = \int_{e_K^i} \mathbf{J}_m^h \cdot \mathbf{n}_K^i d\sigma$



- Discretization space : Restrictions of Raviart-Thomas boundary elements of smallest degree

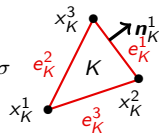
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and $J_{K,i}^h = \int_{e_K^i} \mathbf{J}_m^h \cdot \mathbf{n}_K^i d\sigma$



Find $\mathbf{J}^h \in \mathcal{V}^h$ such that for any $\mathbf{v}^h \in \mathcal{V}^h$

(DG-EFIE) $a_f^h(\mathbf{J}^h, \mathbf{v}^h) + a_\gamma^{*,h}(\mathbf{J}^h, \mathbf{v}^h) + p_\gamma^{*,h}(\mathbf{J}^h, \mathbf{v}^h) = \ell^h(\mathbf{v}^h)$

(DG-MFIE) $b_f^h(\mathbf{J}^h, \mathbf{v}^h) = \tilde{\ell}^h(\mathbf{v}^h)$

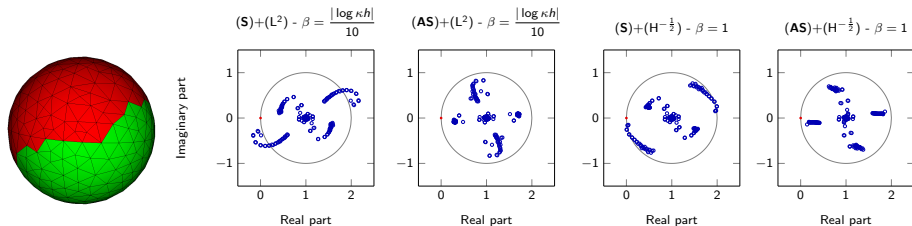
(from continuous MFIE)

(DG-CFIE) α (DG-EFIE) + $(1 - \alpha)$ (DG-MFIE)

(with $0 < \alpha < 1$)

- Use of **GMRes solver** (from CERFACS) with **block-diagonal Jacobi preconditioning**
 - ▶ Diagonal blocks correspond to matrices for individual subdomain
 - Flexibility in choosing subdomain solvers (LL^t , LU, Block low-rank, H-matrix, . . .)
 - ▶ Off-diagonal blocks correspond to interactions between subdomains
 - Could be sped-up by compression techniques

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- Eigenvalue distribution : **(DG-CFIE)** simulation at 95 MHz with CFIE parameter $\alpha = 0.5$



1 Discontinuous Galerkin-based surface domain decomposition method

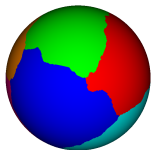
- Model problem
- Discontinuous formulation
- Discrete formulation
- Iterative procedure

2 Numerical results

- Accuracy of solutions
- Convergence of the iterative solver
- Computational costs

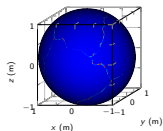
3 Conclusion and perspectives

- Difference modulus between electric currents : **(DG-CFIE)** at 1.52 GHz - $h = \frac{\lambda}{15}$



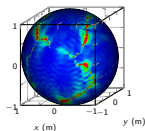
Maximal errors :

Formulation **(S)** + (L^2)



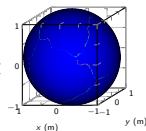
$8.681 \cdot 10^{-4}$

Formulation **(AS)** + (L^2)



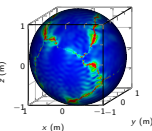
$4.920 \cdot 10^{-3}$

Formulation **(S)** + $(H^{-\frac{1}{2}})$

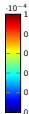


$1.269 \cdot 10^{-3}$

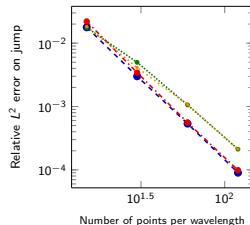
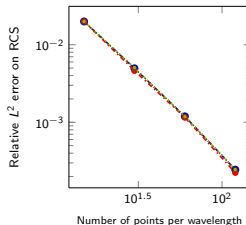
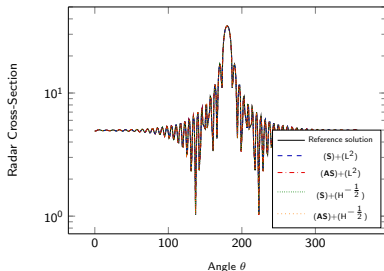
Formulation **(AS)** + $(H^{-\frac{1}{2}})$



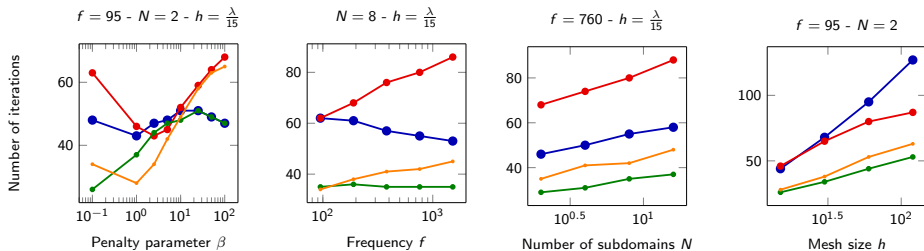
$7.004 \cdot 10^{-3}$



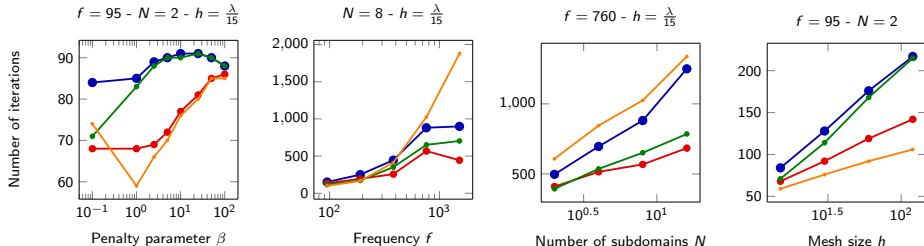
- Comparison of associated radar cross-sections and convergence (95 MHz - 2 subdomains)

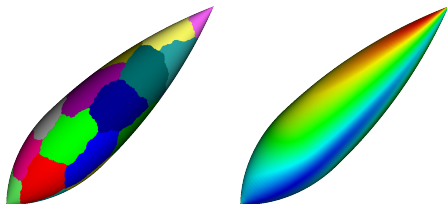


• (DG-CFIE) with $\alpha = 0.5$ • (S)+(L²) • (AS)+(L²) • (S)+(H^{-1/2}) • (AS)+(H^{-1/2})



• (DG-EFIE)





- Dimensions : $2\lambda \times 2\lambda \times 6\lambda$
- Number of degrees of freedom : 273 312
- Number of subdomains : 24
- (DG-CFIE) simulation with $\alpha = 0.9$

- Comparison between CFIE+LU, CFIE+GMRes and DG-CFIE+GMRes

Formulation	Solver	Number of CPU	Factorization (in CPU·H)	Memory (in GB/MPI process)
CFIE	LU	3 328	5 018	11.43
DG-CFIE	GMRes	2 560	45	18.81

Formulation	Solver	Relative residual	Number of iterations	Convergence (in CPU·H)
CFIE	GMRes	10^{-3}	202	5 697
DG-CFIE	GMRes	10^{-3}	49	1 382
DG-CFIE	GMRes	10^{-6}	149	4 203

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Conclusion

- Development and numerical analysis of a discontinuous Galerkin surface domain decomposition method for electromagnetic scattering by non-penetrable objects
 - ▶ Comparison of **symmetric** and **anti-symmetric** formulations
 - Symmetric formulation involves **better accuracy** on currents and **preserves symmetry** in EFIE
 - Anti-symmetric formulation involves **better conditioning number** in CFIE formulation
 - ▶ Comparison of L^2 and $H^{-\frac{1}{2}}$ interior penalty terms
 - L^2 penalization is robust with respect to the **frequency** but **parameter β have to be calibrated**
 - $H^{-\frac{1}{2}}$ penalization is more robust with respect to the **discretization size**
- Comparison with a boundary element method
 - ▶ Computational costs
 - Using a direct solver : **large gain** in factorization time
 - Using a preconditioned iterative solver : **faster convergence** for EFIE formulation
 - ▶ Accuracy : Jump **does not pollute** radar cross-sections

Perspectives

- Integration of **hp-refinement**
 - ▶ **Non-conformal** meshes
 - ▶ **High-order** boundary elements
- Integration of **H-matrix** formalism
- Extension to **dielectric** scatterers

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Thank you for your attention !