River Flow 2020 Estimation of the transverse mixing coefficients in shallow gravel-bedded rivers

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DE LA RECHERCHE À L'INDUSTRIE

Estimation of the transverse mixing coefficients in shallow gravel-bedded rivers

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RiverFlow 2020

STUDYING THE MIXING IN A NATURAL STREAM

Goals:

- Understanding the contribution of mixing processes in a reach due to turbulent diffusion, secondary currents dispersion
- Determining a predicting formulation to estimate the mixing efficiency in a reach

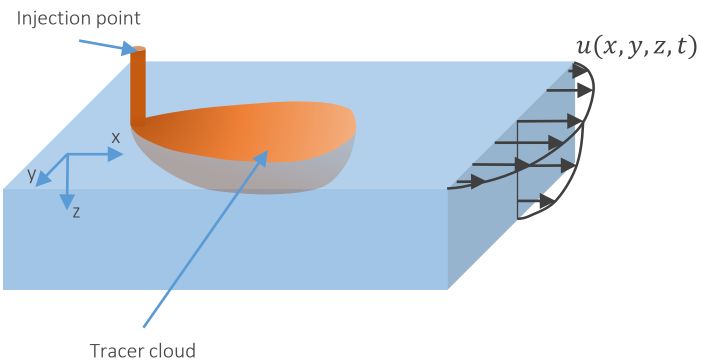
Applications:

- Predicting the spatial distribution of pollutants in a river downstream a disposal area
- Determining the «perfect transverse mixing length »

Fischer, 1979: $L_y = \alpha \frac{UW^2}{E_y}$

L_y: perfect transverse mixing length
U: bulk velocity of the stream
W: width of the stream
E_y: **reach-averaged transverse mixing coefficient**

When a cloud of particles is introduced in a stream, it spreads along the three spatial directions with increasing time:



For wide, shallow, uniform and stationary flows:

- velocity is mainly one directional ($u = u(y)$, $v = w = 0$)
- vertical mixing is rapidly complete
- streamwise mixing is negligible
- we focus on transverse mixing

→ Simplified 1D depth-averaged advection-diffusion equation:

$$\frac{\partial hc}{\partial t} + u\frac{\partial hc}{\partial x} = uh\frac{\partial\left(uh^2\varepsilon_y\frac{\partial c}{\partial q}\right)}{\partial q} \quad (1)$$

Yotsukura & Cobb (1972)

With:

- h = local flow depth (m)
- u = local depth and time-averaged velocity (m/s)
- $q = \int_0^y u(\delta)h(\delta)d\delta$ = cumulative discharge (m^3/s)
- ε_y = local mixing coefficient (m^2/s)
- c = local depth and time-averaged concentration (kg/m^3)

THEORETICAL BASIS

In the case of a slug injection of a mass of tracer, we can integrate Eq.1 over the time following Beltaos (1980, 1984):

$$\frac{\partial \theta}{\partial x} = E_y \psi U H^2 \frac{\partial^2 \theta}{\partial q^2} \quad (2)$$

With:

- $H =$ reach-averaged flow depth (m)
- $U =$ reach-averaged velocity (m/s)
- $E_y =$ reach-averaged coefficient (m²/s)
- $\psi = \frac{\frac{1}{Q}\int_0^Q \bar{u}(q) h(q)^2 dq}{UH^2}$ = velocity-shape profile
- $\theta =$ local depth-averaged dosage (kg.s/m³)

Solving Eq.2 in a reach from cross-section S_i to S_{i+1}:

$$\theta_0(x_{i+1}, q) = \int_0^Q \frac{\theta(x_i, q_0)}{\sqrt{4\pi E_{y_{i,i+1}} \psi U H^2 (x_{i+1} - x_i)}} exp\left[\frac{-(q \pm q_0)^2}{4 E_y \psi U H^2 (x_{i+1} - x_i)}\right] dq_0$$

Seo et al. (2006)

And using the mirror method to model the reflection at the banks:

$$\theta(x_{i+1}, q) = \theta_0(x_{i+1}, q) + \theta_0(x_{i+1}, q) + \sum_{n=1}^{\infty} \theta_0(x_{i+1}, 2n \pm q) \quad (3)$$

LITERATURE STUDIES

- Many field and laboratory studies since 1960 (Elder, 1959; Fischer, 1969; … ; Seo et al, 2016)

- Attempts to find correlation between the transverse mixing coefficient and parameters:
  $\frac{E_y}{Hu^*} = f(\frac{U}{u^*}, \frac{W}{H}, S_n)$ with U/u^* the friction factor, W/H the aspect ratio and S_n the sinuosity

- Problem:
  - no clear correlation is yet found and the quantitication of the transverse mixing coefficient remains an open question
  - most studies consider that the studied reaches follow the 1D model hypotheses, but natural reaches generally present significant longitudinal hydraulic and geometric variations (Jackson et al, 2013)

- Proposed solution:
  - Delineate a natural reach in smaller sub-reaches that would better follow the 1D hypotheses

FIELD STUDY: GOAL

→ Choose a longitudinally varied reach, delineate it in sub-reaches with quasi-homogeneous roughness elements by measuring concentration cross-profile at each two ends

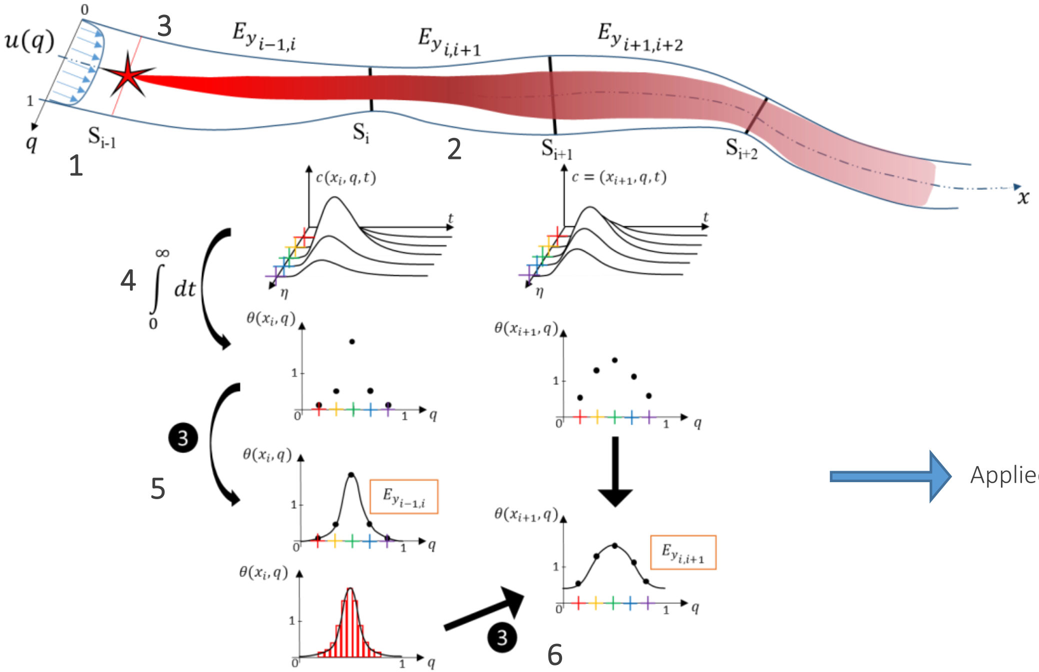
→ Highlight the variability of the mixing coefficient in a short reach usually disregarded in past studies

→ See if the delineation method gives useful correlations between the dimensionless mixing coefficient and hydraulic or geometric factors in order to use it for a predictive formulation

↳ Usually $\frac{E_y}{Hu^*} = f(\frac{U}{u^*}, \frac{W}{H})$

With:
- U/u^*: friction factor
- W/H: aspect ratio

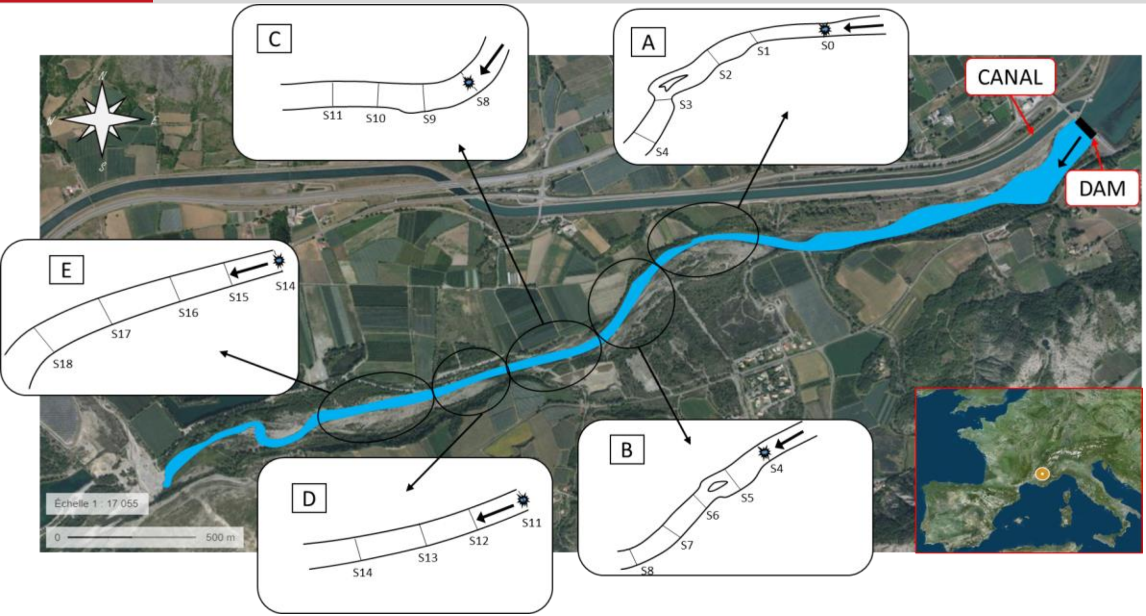
FIELD STUDY: METHOD



Applied to the Durance river, France

1. Position an injection point upstream
2. Measure hydraulics and position sensors along two successive cross-sections downstream
3. Inject a tracer
4. Gather data and integrate concentration profiles over time
5. Fit solution Eq.3 to determine E_y between the injection and the sub-reach's upstream cross-section
6. Discretize upstream profile and fit solution Eq.3 to determine E_y between the two cross-sections

FIELD STUDY: LAYOUTS



Location: La Saulce, France
Dates: September 23-27 2019

→ 5 injections zones, 18 cross-sections and sub-reaches

FIELD STUDY: VARIED BED MORPHOLOGY

Complexe succession of emerging blocks, riffle with island, and pool

Long sand bedded pool with vegetation

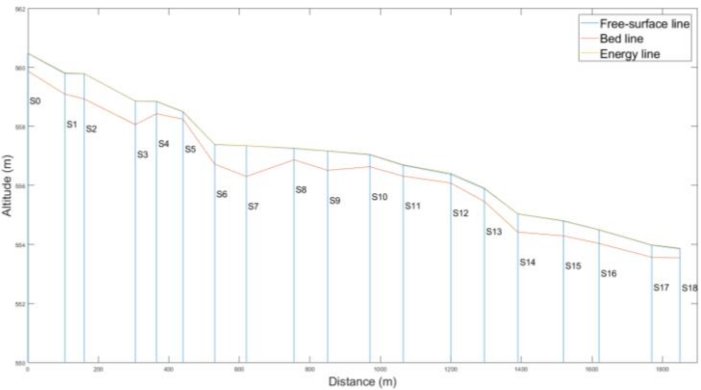
Patches of emerging blocks

FIELD STUDY: HYDRAULICS MEASUREMENTS

ADCP measurements at every cross-sections: velocity field + bathymetry + sensors positioning

Gives U, H, ψ and sensors position in cumulative discharge

Free surface elevation measurements with a DGPS Leica along the left bank



Gives $u^* = \sqrt{gHJ_e}$

With:

- g = gravitationnal acceleration (m²/s)
- J_e = energy line slope (m/m)

FIELD STUDY: CONCENTRATION MEASUREMENTS

Chosen tracer: Liquid rhodamine WT 20% → Fluorescent tracer, small quantities needed, low toxicity

Measured with:

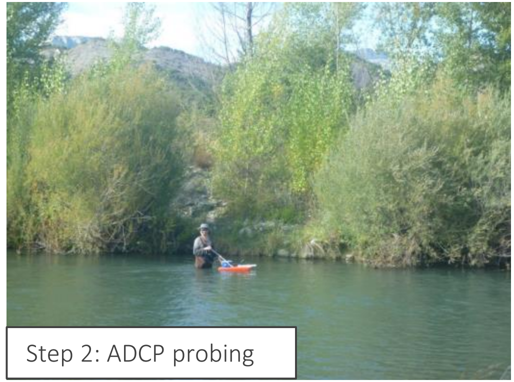
GGNU-FL30 Albillia

TQ-tracer (Sommer Messtechnik)

Positioned along the cross-sections every 7th of the total discharge

FIELD STUDY: CONCENTRATION MEASUREMENTS

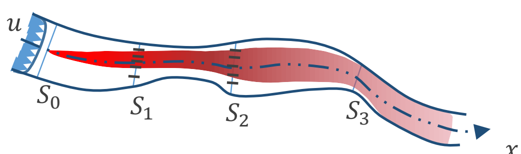
Step 1: Calibration



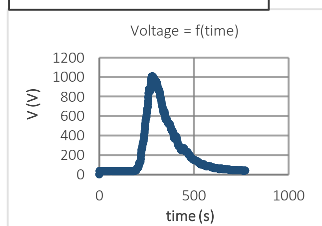
Step 2: ADCP probing

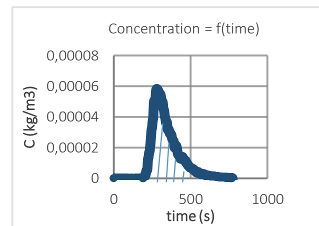
Step 3: Probes positioning on two consecutive cross-sections

Step 4: Instantaneous injection of tracer (Rhodamine WT) and re-do steps 2,3 and 4 for the next sub-reaches



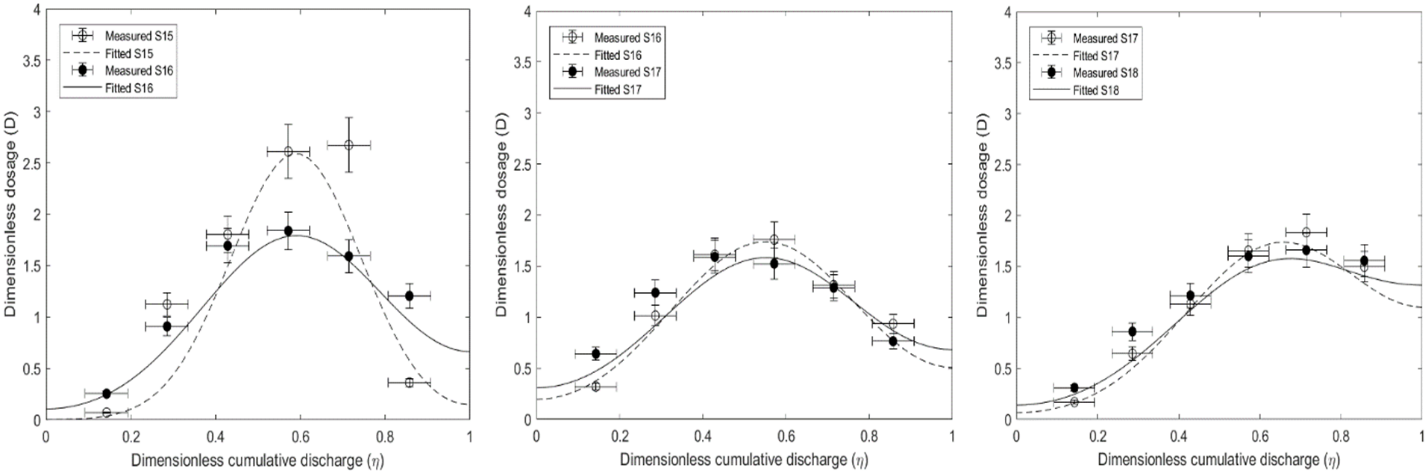
Step 5: Data analysis



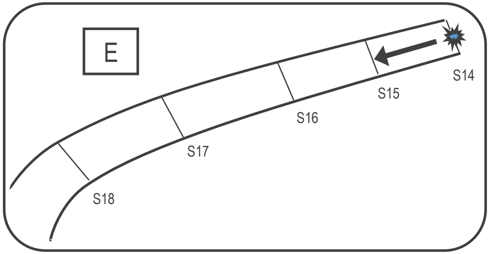


Integrate to get: $\theta = \int_{t=0}^{t=\infty} c dt$

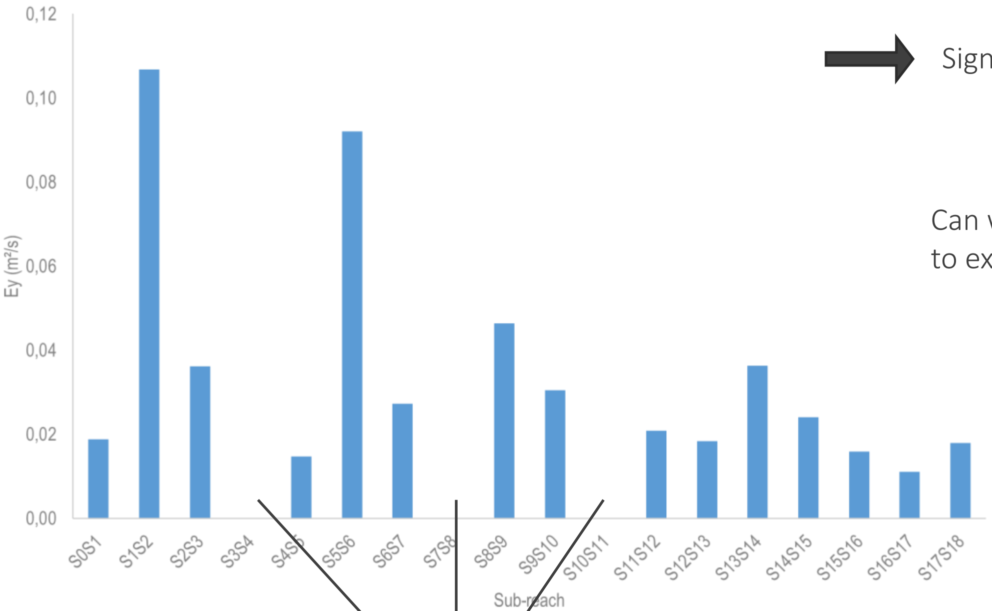
FIELD STUDY: DOSAGE PROFILE FITTING



Example for injection zone E:
(straight channel with patches of emerging blocks)



FIELD STUDY: RESULTS



Complete mixing reached

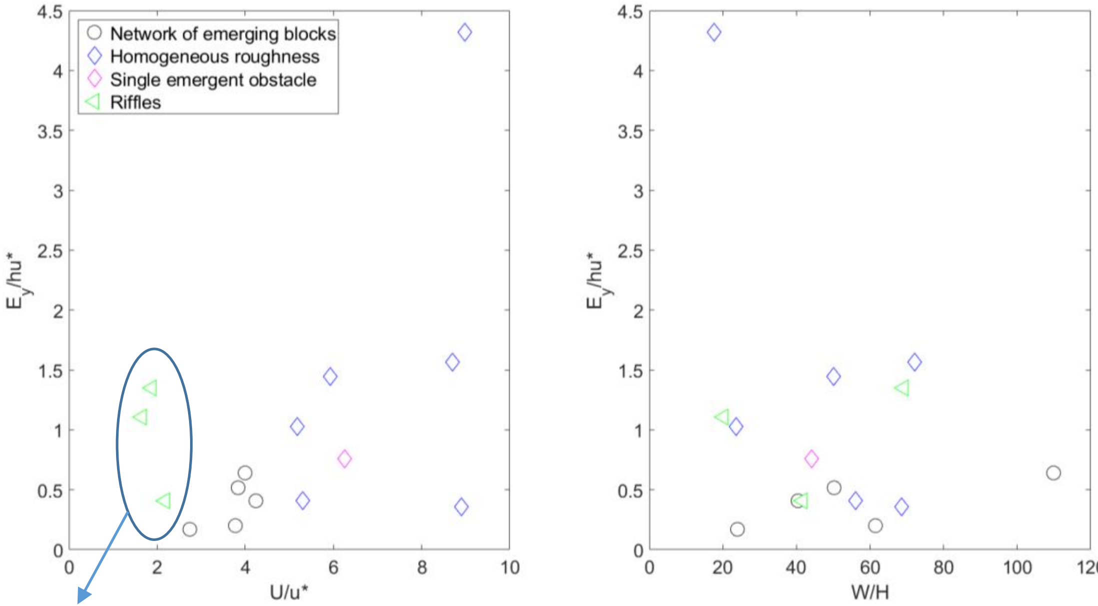
→ Significant longitudinal variability

↓

Can we find interesting correlations to explain it?

FIELD STUDY: RESULTS

$$\frac{E_y}{Hu^*} = f\left(\frac{U}{u^*}, \frac{W}{H}\right)$$



Values of Hu^* poorly representative of the riffles hydraulics (measurements upstream and downstream of it)

No clear correlation observed even when a reach is delineated in quasi-homogeneous sub-reaches

CONCLUSION

- We applied tracing experiments in a river presenting strong variations in bed morphology; we delineated the chosen reach in sub-reaches with quasi-homogeneous roughness elements
- We used a simple 1D-diffusion model with simplificative hypotheses to determine the transverse mixing coefficient of each sub-reaches and show significant variations of the mixing coefficient in a natural reach
- We found no clear correlation of the dimensionless mixing coefficient against the usual factors defined in the literature

Perspective:

- A new parameter seems needed to explain the variations in our data, and eventually explain those in the literature; a new analysis of the data considering the non-uniformity of the flow may reveal interesting results
- Laboratory measurements are planned to study the influence of the relative roughness height on the transverse mixing (including emerging obstacles)

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