Unifying approach to hard diffraction

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We find a consistency between two different approaches of hard diffraction, namely the QCD dipole model and the Soft Colour Interaction approach. A theoretical interpretation in terms of S-Matrix and perturbative QCD properties in the small x_{Bj} regime is proposed.

1. In the present paper, we focus on two different theoretical approaches to hard diffraction, which are shown to be compatible and complementary. The first one is based on an extension [1] to hard diffractive processes of the "QCD dipole" approach in the small x_{Bj} regime of perturbative QCD. In this picture, the hard photon is supposed to probe the parton structure of the Pomeron considered as an hadronic particle. In the original Ingelman-Schlein formulation [2], this hard probe was formulated in the parton model, later complemented by QCD evolution equations [3]. In the dipole model, this hard interaction is described via the Balitsky Fadin Kuraev Lipatov (BFKL) resummation [4]. A major uncertainty for this model is the relative normalization of diffractive over non-diffractive cross-sections which stays beyond the perturbative framework.

A second approach [5] is the Soft Colour Interaction (SCI) model, where hard diffraction is described as the superposition of two processes. At short time, the hard probe initiates a typical deep-inelastic interaction with colour quantum numbers exchange. Then, at large times/distances, a "soft" colour interaction is assumed to rearrange the colour quantum numbers and gives rise to singlet exhanges -and thus diffraction- with a probability of order $\frac{\mathbf{1}}{\mathbf{N_c^2}}$, where N_c is the number of colours[1]. While in this approach, the relative normalization is fixed, the exact nature of the interplay between soft and hard components is not known.

In the present paper, we propose a way to relate these two approaches which allows one to consider them as compatible and complementary. This provides a definite prediction for both the normalization and the analytic form of the amplitude. Our main results are the following:

i) The interplay between hard and soft components of hard diffraction is expressed via "effective" parameters of a Pomeron interaction determined from leading log perturbative QCD resummation. It is found to depend not only on Q^2 but also on the ratio $\eta = (Y-y)/y$, where Y (resp. y) are the total (resp. gap) rapidity interval.

We obtain

$$F_{T,L}^{Diff}(Q^2, Y, y) = \frac{\mathbf{1}}{\mathbf{N_c^2}} \frac{\mathcal{N}^{tot}}{x_P} \frac{e^{2y\Delta}}{4\pi\Delta'' y} \sqrt{\frac{2}{1+2\eta}} \exp\{(Y-y)\,\epsilon_s\} \left(\frac{Q}{Q_0}\right)^{2\gamma_{\mathbf{s}}} \exp\left(-\frac{2\log^2\left(\frac{Q}{Q_0}\right)}{D_s(Y-y)}\right) , \tag{1}$$

with

$$\Delta(x) \equiv \frac{\alpha_s N_c}{\pi} \{2\psi(1) - \psi(x) - \psi(1-x)\} \sim \Delta + \frac{\Delta''}{2} (1/2 - x)^2 \tag{2}$$

is the BFKL evolution kernel [4] (together with its gaussian approximation near the minimum at $x = 1/2$) and Q_0 a non-perturbative scale associated with the proton. We have

$$\gamma_{\mathbf{s}} = \frac{\eta}{1+2\eta} \ ; \ \epsilon_s = \Delta + \frac{\Delta''}{8(1+2\eta)} \ ; \ D_s = \frac{1+2\eta}{\eta} \Delta'' \ . \tag{3}$$

Note that one may write $F_{T,L}^{Diff} \sim \sigma_{\gamma^*-P}^{tot} \times e^{2y\Delta}/(4\pi\Delta'' y)$, which is the known "triple Pomeron" formula [6] where $\sigma_{\gamma^*-P}^{tot}$ defines the effective interaction cross-section of a virtual photon with a BFKL Pomeron $e^{y\Delta}/\sqrt{4\pi\Delta'' y}$ derived from the QCD dipole formalism. By analogy with BFKL [4], γ_s, ϵ_s and D_s can be defined, respectively, as the anomalous dimension, intercept and diffusion parameter of an "effective" BFKL $\gamma^* - P$ cross-section.

The normalization, which remains unknown in the QCD dipole model description, is determined as the product of the factor $\frac{\mathbf{1}}{\mathbf{N_c^2}}$ and the normalization factor $\mathcal{N}^{tot}$ of the non-diffractive structure function, according to the SCI prescription, see below.

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[1]More recent versions of the model consider a probability factor of the same order but not necessarily connected with colour.

ii) In SCI models, the following relation between the total structure function and the overall contribution of hard diffraction at fixed value reads of x_{Bj} :

$$F_{T,L}^{Diff/tot} \equiv \int_{x_{Bj}}^{x_{gap}} dx_P \; F_{T,L}^{Diff} = \frac{\mathbf{1}}{\mathbf{N_c^2}} \; F_{T,L}^{tot} \tag{4}$$

where $\log 1/x_{gap}$ is the minimal rapidity gap. If we insert the QCD dipole prediction for $F_{T,L}^{Diff}$ in formula (4), we find

$$F_{L,T}^{tot} = \frac{\mathcal{N}}{\mathbf{N_c^2}} \left(\frac{Q}{Q_0}\right)^{2\gamma^*} \frac{\exp\left(Y\Delta(\gamma^*)\right)}{\sqrt{2\pi\Delta'' \; Y}} \; , \tag{5}$$

which is equivalent to a canonical BFKL expression for non-diffractive structure functions, apart the substitution of the BFKL effective anomalous dimension γ_{BFKL} by γ^*, namely

$$\gamma_{BFKL} = \; 1/2 - 2\frac{\log\left(\frac{Q}{Q_0}\right)}{\Delta'' \; Y} \;\; \rightarrow \;\; \gamma^* = cst. \sim 0.175 \; , \tag{6}$$

where the "universal" value γ^* is solution (for $0 < \gamma < 1/2$) of the implicit equation $2\Delta\left(\frac{1-\gamma}{2}\right) - \Delta(\gamma) = 0$. Here, it is interesting to note that the shift (6) may be useful to avoid the objection to SCI models [7] based on "Low's theorem" following which soft colour radiation cannot be emmitted from inside a partonic process. The differences we find with the original model means that, in our dipole formulation, the soft colour interaction indeed seems to modify the initial parton kinematics.

iii) Using S-Matrix properties of triple-Regge contributions, a relation is found between discontinuities of a $3 \rightarrow 3$ amplitude and the two approaches to hard diffraction we consider. Following old results of S-Matrix theory in the Regge domain [8], and as sketched in Fig.1, one may consider three types of discontinuities of a $3 \rightarrow 3$ amplitude representing hard diffraction. A single discontinuity over the diffractive $mass^2$ describing the hard Pomeron interaction, a double discontinuity taking into account the analytic discontinuity in the subenergy variable of one of the incident Pomeron exchanges (and its complex conjugate) for the SCI model and the full triple discontinuity including those of the two Pomeron exchanges, which is characteristic of the QCD dipole model description [1]. An interesting new feature is thus the S-Matrix interpretation of the SCI approach as a specific double discontinuity of the $3 \rightarrow 3$ forward amplitude, which formulates the model in terms of simultaneous exchanges of a soft and a hard Pomeron.

2. Let us sketch the derivation of our results.

Our starting point is a triple-Regge formula for the diffractive structure function for longitudinal and transverse photon in the QCD dipole formalism:

$$F_{T,L}^{Diff}(Q^2,Y,y) \sim \frac{\mathcal{N}^{Diff}}{x_P} \int_{c-i\infty}^{c+i\infty} \frac{d\gamma_1}{2i\pi}\frac{d\gamma_2}{2i\pi}\frac{d\gamma}{2i\pi} \; \delta(1-\gamma_1-\gamma_2-\gamma) \; \left(\frac{Q}{Q_0}\right)^{2\gamma} \; \exp\left\{y(\Delta(\gamma_1)+\Delta(\gamma_2)) + (Y-y) \; \Delta(\gamma)\right\} \; , \tag{7}$$

where $\Delta(\gamma)$ is the BFKL evolution kernel (2) and $\mathcal{N}^{Diff}$ is a normalization containing both QCD perturbative and non-perturbative factors [9]. Strictly speaking [9] the δ-function is the unique contribution in the differential diffractive structure function at momentum transfer $t = 0$. However, it can be shown that this is the dominant perturbative contribution even at non zero transfer due to specific properties [10] of the analytic QCD triple-Pomeron couplings [11] .

The first step of the computation of formula (7) is to use the saddle-point approximation [9] at large y to integrate over the difference $\gamma_1 - \gamma_2$. One easily gets

$$F_{T,L}^{Diff} = \mathcal{N}^{Diff}\frac{1}{x_P} \; \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2i\pi}\sqrt{\frac{1}{4\pi\Delta''(\frac{1-\gamma}{2}) \; y}}\exp\left\{2y \; \Delta\left(\frac{1-\gamma}{2}\right) + (Y-y) \; \Delta(\gamma) + 2\gamma\log\frac{Q}{Q_0}\right\} \; . \tag{8}$$

Using the gaussian approximation (2) for the BFKL kernels $\Delta(\gamma)$ and $\Delta\left(\frac{1-\gamma}{2}\right)$ in the relevant interval $0 < \gamma < 1/2$, and again a saddle-point approximation at large rapidity gap y, one obtains, up to a normalization factor, formula (1), with γ_s, ϵ_s and D_s defined as in (3). The normalization $\mathcal{N}^{Diff}$ is not yet specified at this stage.

The derivation of the normalization is coming from the comparison with the SCI approach. Inserting (7) in the integral (4), one is led to perform a two-dimensional saddle-point approximation in the y, γ complex plane. The saddle-point equations read:

$$-\,y\;\Delta^{'}\left(\frac{1\!-\!\gamma}{2}\right)+(Y\!-\!y)\;\Delta^{'}(\gamma)+2\log\frac{Q}{Q_0}=0$$

$$2\Delta\left(\frac{1-\gamma}{2}\right)-\Delta(\gamma)=0\ , \tag{9}$$

whose solution (y^*, γ^*) is

$$y^*=\left(Y+\frac{2\log\frac{Q}{Q_0}}{\Delta^{'}(\gamma^*)}\right)\;\left(1+\frac{\Delta^{'}\left(\frac{1-\gamma^*}{2}\right)}{\Delta^{'}(\gamma^*)}\right)^{-1}$$

$$\Delta(\gamma^*)=2\Delta\left(\frac{1-\gamma^*}{2}\right) \tag{10}$$

resulting in a value of $\gamma^* \simeq 0.175$, which is "universal", i.e. independent of the kinematics of the reaction.

After computation of the prefactors to the saddle-point approximation, one finds:

$$F_{T,L}^{Diff/tot}\!=\!\mathcal{N}^{Diff}\,\frac{1}{|\Delta^{'}(\frac{1-\gamma^*}{2})\!+\!\Delta^{'}(\gamma^*)|}\left(\frac{Q}{Q_0}\right)^{2\gamma^*}\frac{\exp\left(Y\Delta(\gamma^*)\right)}{\sqrt{4\pi\Delta^{''}\left(\frac{1-\gamma^*}{2}\right)y^*}}\ . \tag{11}$$

Note that it is the linearity in y of the saddle-point equation (9) which allows one to get such an elegant form for the integrals. Using (11) and by comparison with the canonical BFKL formula we identify the "hard" component of the SCI model by the substitution $\gamma_{BFKL} \to \gamma^*$, see (6). Then using the SCI ansatz (4), we obtain the relation

$$\mathcal{N}^{Diff}\approx\frac{\mathcal{N}^{tot}}{\mathbf{N_c^2}}\times\;|\Delta^{'}(\frac{1\!-\!\gamma^*}{2})\!+\!\Delta^{'}(\gamma^*)| \tag{12}$$

which fixes the relative normalization of the diffractive vs. non diffractive structure functions. This ends the derivation of formulae (1-6).

3. Let us finally come to the S-Matrix interpretation of our approach. For every fixed but arbitrary value of the parameters $\gamma, \gamma_1, \gamma_2$, the triple-Regge formula (7) can be obtained from the canonical formulalism[2] corresponding to the vertex of three Regge pole singularities[3] in the complex plane of angular momentum [6]. As such, one can make use of the important S-Matrix Mueller-Regge relation [8], valid in kinematical regions including the triple-Regge limit, between semi-inclusive amplitudes and specific discontinuity contributions of forward elastic $3 \to 3$ amplitudes. It naturally applies to hard diffraction initiated by a virtual photon, as sketched in Fig.1, namely

$$\gamma^*+p\to p+X\quad\Longleftrightarrow\quad Disc_1\left\{\gamma^*\bar{p}\;p\;\;\to\gamma^*\bar{p}\;p\right\}\ . \tag{13}$$

Quite interestingly, the existence of Regge phase factors allows one to relate other discontinuities of $A(3 \to 3)$ to $Disc_1 A$. As sketched in Fig.1, one may also consider a double discontinuity $Disc_2 A(3 \to 3)$ taking into account also the analytic discontinuity in the subenergy of one of the incident Pomeron exchanges and a triple discontinuity $Disc_3 A(3 \to 3)$ including the discontinuity over the two Pomeron exchanges. The expression of the discontinuities, through generalized unitarity relations, is obtained through the imaginary part of the relevant Regge phase factors [8]. Moreover, one finds an equality relation $Disc_1 A = Disc_2 A = Disc_3 A$ which is due to the fact that the discontinuity taken over the mass variable (corresponding to diffractively produced states) is common to all three cases in Fig.1 and factorizes the same $p\bar{p}$ vertex in $A(3\to3)$ (cf. the classical derivation in the last paper of Ref. [8]).

Let us now take advantage of the hard probe in the process, allowing one to introduce in the game the (resummed) perturbative QCD expansion at high energy (small x_{Bj}). In a generic S-Matrix approach, the analytic discontinuities of scattering amplitudes are related to a summation over a complete set of asymptotic *hadronic* final states. If however,

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[2]These triple Regge formulae, relevant for *inclusive cross-sections*, have to be distinguished from the AGK cutting rules which are relevant for the two and multi Pomeron contributions to the $2 \to 2$ *elastic* amplitudes in both the S-Matrix framework [12] and perturbative QCD at leading log level [13].

[3]We neglect at this stage the complications due to Regge cuts or other more sophisticated singularities and deal with simple effective Regge poles.

the underlying microscopic field theory is at work with small renormalized coupling constant due to the hard probe, it is possible in some cases to approximate the same discontinuity using a complete set of *partonic* states. In particular, at high energy and within the approximation of leading logs (and also large N_c), QCD dipoles can be identified as providing such a basis [1].

The discontinuity $Disc_3$ appears naturally in the dipole formulation of hard diffraction [1]. Indeed, hard diffraction calculations make use of the probability distribution for finding two dipoles in the wave-function of one initial dipole in the virtual photon through the exchange over a BFKL Pomeron exchange described by its discontinuity in the $Y - y$ rapidity range. Then, each of these dipoles interact with the target through two perturbative (BFKL) Pomeron exchanges described by their own discontinuities over the y range. Thus the calculation of scattering amplitudes implies the full discontinuity over three Pomerons described by intermediate dipole interactions, which is nothing but $Disc_3$.

The appearance of $Disc_2$ is natural in the Regge formalism [8]. However a physical interpretation involving perturbative QCD contributions has not been previously noticed. In the framework of our study, it corresponds to the superposition of a hard perturbative cut Pomeron interaction with a soft one described by a non-cut Pomeron (with complex conjugate contribution, see Fig.1). This is similar to the superposition of hard and soft interactions which characterizes the SCI approach. We are thus led to propose $Disc_2$ as a way to get quantitative predictions, in particular the relative normalization of diffractive over non diffractive cross-sections. Indeed, it appears as a "hard" partonic interaction very similar to the one describing ordinary deep-inelastic processes, in parallel with a "soft" correction evolving during a long time, corresponding to the uncut Pomeron singularity in the middle graphs (including complex conjugate) of Fig.1.

The equality between the different discontinuities allows the connection between the different models, leading to the results given in section **1**.

FIGURE

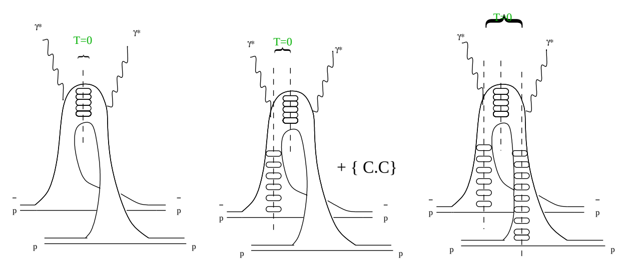

Figure 1

S-Matrix interpretation of the three approaches to hard diffraction. Upper graph: Description of $Disc_1 A(3 \to 3)$; Middle graph: Description of $Disc_2 A(3 \to 3)$ and its complex conjugate (candidates for the SCI approach); Lower graph: Description of $Disc_3 A(3 \to 3)$ (QCD dipole approach).

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