Construction of beta spectrum on the basis of experimental nuclear decay data

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CONSTRUCTION OF BETA SPECTRUM ON THE BASIS OF EXPERIMENTAL NUCLEAR DECAY DATA

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ABSTRACT. A computer code for β-spectrum construction of 1300+ radionuclei has been created. The tools use the experimental nuclear data adopted and compiled in the ENSDF file and experimental or estimated parameters of shape factors. The final uncertainty on the calculated β-spectrum branch tries to deal with uncertainties on the radiation probability, the endpoint energy, and shape factor parameters.

INTRODUCTION

When calculating the influence of ionizing radiations on living organisms in biological research and medical practice, calculation of absorbed doses using experimental radiation spectra of radionuclides plays a significant part. Recently, interest in the beta spectrum shape has grown again, especially for its use in applied (biomedical and tracer investigations, LSC calibrations, etc.) and/or fundamental research (tritium and reactor neutrino experiments, tests of the CVC predictions, etc.).

In this work, we create a computer program named "Beta Spectrum" (BS) that, to some extent, could reconstruct beta spectra from available experimental data. Parameters of the shape factors are required for calculation in addition to nuclear decay data. Unfortunately, up to now, experimental values of those parameters have been obtained only for about 100 beta radioactive nuclei. Nevertheless, it is often possible to construct approximate beta spectra with errors that include uncertainties on the shape factors.

Most β-spectrum shape experiments performed during 1950–1970 used magnetic and semiconductor spectrometers and radioactive sources, which were evaporated on solid coatings. The methodology of such experiments requires careful study of instrumental spectrum distortions, which can appear due to the scattering of particles in the spectrometer, as well as absorption in the source matter and the backscatter from the source holder. Methods of continuous β-spectrum deconvolutions that take into account the spectrometer response function were not used in earlier experiments. Different authors used their own methods to consider and correct these instrumental distortions. The lack of well-established data led us to analyze the published shape factors in order to evaluate a mean shape factor for each degree of forbidding.

Experimental Shape Factor Analysis

The number of beta particles with energy W in the interval W and $W+dW$ emitted per unit of time can be calculated by the following expression:

$$N(W)dW \sim pW\,(W_0-W)^2\,F(\pm Z,W)\,C(W)\,dW$$

where $W = E/m_0c^2 + 1$ is the total electron energy (in units of m_0c^2); m_0 is the mass of the electron; c is the velocity of light; $p = (W^2-1)^{1/2}$ is the electron momentum (in m_0c^2); W_0 is the maximum value of W; Z is the atomic number of the nucleus; and $F(\pm Z,W)$ is the Fermi function, which takes into consideration the Coulomb field of a nucleus. The plus sign applies for β^- decay, while the

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minus sign signifies β^+ decay. The results of calculation of the Fermi function can be found in the tables of Dzhelepov et al. (1969) and Behrens and Bühring (1982). In Dzhelepov et al. (1969), the function is calculated for the 5–10,000 keV interval of the beta particle kinetic energy and for 100 unequally spaced points. In Behrens and Bühring (1982), the kinetic energy interval is 2.5–25,000 keV and the number of unequally spaced points is 48. In both cases, calculations are performed for $0 < Z < 100$ and the corrections for screening are, in principle, included.

$C(W)$ is the shape factor. It determines the spectrum form, which depends on its degree of forbidding. Shape factors for allowed and various forbidden transitions can be expressed in the following way as described by Behrens and Szybisz (1976).

Allowed (a) Beta Transitions

For allowed transitions, the shape factor is $C(W) = 1$; small deviations can occur and $C(W)$ is expressed by:

$$C_a(W) = 1 + aW$$

Eighteen experimental shape factor values, recommended by Behrens and Szybisz (1976), are presented in Figure 1 versus the atomic number of parent radionucleus Z. Besides the point (without uncertainty) of ^{66}Ga, the scattering of the shape factor coefficients does not exceed 3% and there is no visible dependence with the atomic number Z. The average value (-0.15 ± 0.50) $(\%/m_0c^2)$ of the experimental values is close to zero and seems to be the result of the underestimation of instrumental distortion in the experiments. Thus, for allowed beta transitions the Beta Spectrum program uses $a = 0$, that is, $C_a(W) = 1$.

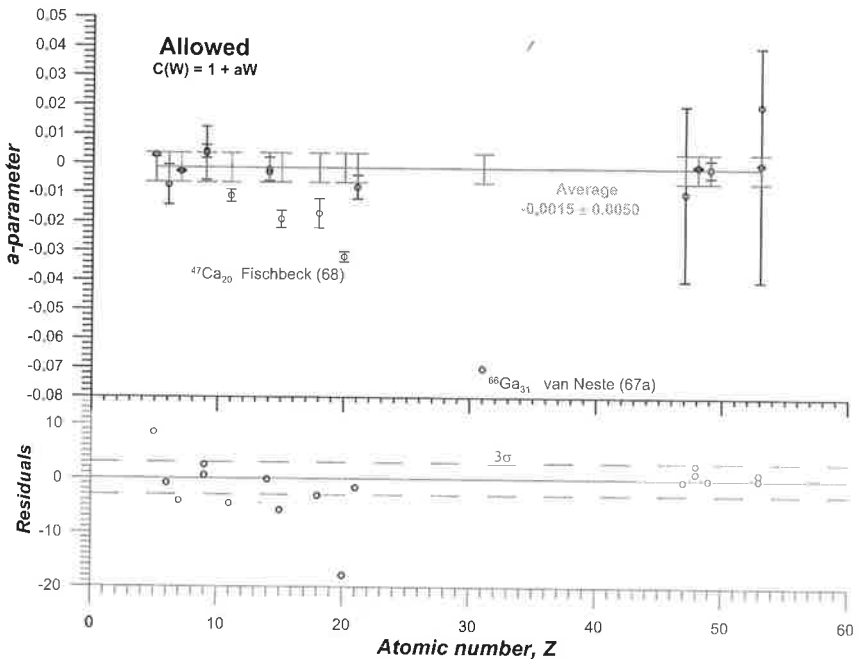


Figure 1 Experimental shape factor coefficients recommended by Behrens and Szybisz (1976) for allowed beta transitions.

First Forbidden Non-Unique (1nu) Beta Transitions

The shape factor is expressed by:

$$C_{1nu}(W) = 1 + aW + b/W + cW^2 + d(q^2 + \lambda_2 p^2)$$

where $q = W_0 - W$ is the neutrino momentum; λ_n is a Coulomb function; and a, b, c, and d are constants.

In 40 experimentally investigated 1nu beta spectra, 5 kinds of fitting expressions were used for the description of their shapes. Average values of $C(W)$ coefficients for each kind of expression are presented in Table 1. In 23 cases (Figure 2), the shape of the 1nu beta spectrum was approximated with good accuracy by $C(W) = 1 + aW$ ($a_{av} = 0.04 \pm 0.07$), and its shape is very close to the allowed one. At the same time, in 17 cases where other fitting expressions were used, a 100% spread is observed. It is therefore practically impossible to determine the predominant behavior of shapes of the 1nu beta spectra.

Table 1 Averaged values of a, b, c, d, and λ_2 constants and their spreads.

Beta transition type	Number of experiments	a	b	c	d	λ_2
Allowed	18	0.0015(50)	0	0	0	
1nu	23	–0.04(7)	0	0	0	
1nu	4	–0.016(8)	0	0.022(23)	0	
1nu	4	–0.36(10)	–0.04(26)	0.026(12)	0	
1nu	6	0	0	0	0.048(27)	
1nu	3				1	0.85(9)
1u	17	–0.011(3)	0	0	0	

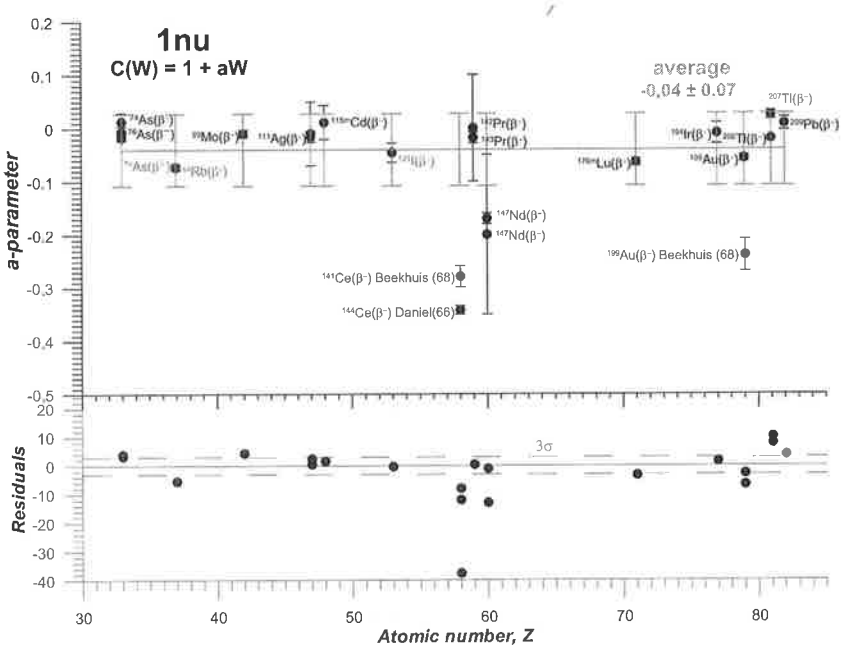


Figure 2 Experimental shape factor coefficients a recommended by Behrens and Szybisz (1976) for 23 first forbidden non-unique beta transitions.

The Beta Spectrum (BS) code calculates 1nu beta spectra by using the allowed shape factor. A possible 100% uncertainty is assumed in this case. Figure 3 demonstrates the spread obtained from all expressions of the experimental 1nu shape factors when the various possible parameters are used.

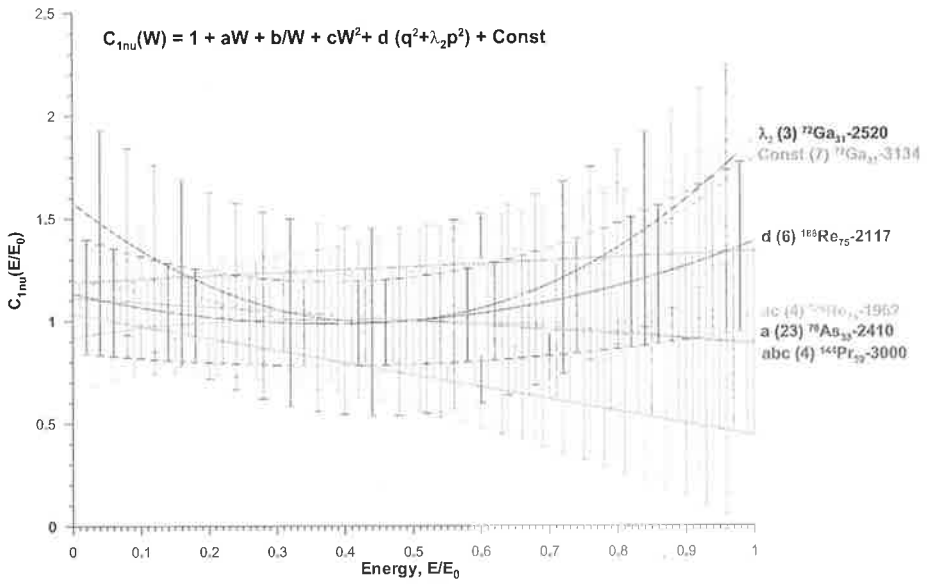


Figure 3 The spread of recommended 1nu shape factor curves for the analyzed experiments. Curves are normalized at a point that corresponds to the middle of the spectrum.

First Forbidden Unique (1u) Beta Transitions

For these transitions, the shape factor can be described by Behrens and Szybisz (1976):

$$C_{1u}(W) = (q^2 + \lambda_2 p_2)\,(1 + aW) = q^2(1 + aW) + \lambda_2 p_2(1 + aW)$$

In Figure 4, 17 recommended values (from Behrens and Szybisz 1976) are given for coefficients a for first forbidden unique (1u) beta transitions of 10 atomic numbers Z. As for allowed beta transitions, there is no visible dependence with the atomic number Z. In a first approximation, an average value of -0.011 ± 0.003 ($\%/m_0c^2$) is used for consequent correction in the BS code to calculate this type of spectrum.

Second Forbidden Unique (2u) and Non-Unique (2nu) Beta Transitions

Unfortunately, only a few experimental results are available: 7 experimental values are retained by Behrens and Szybisz (1976) for 2nu transitions and 4 values for 2u transitions. Hence, it is not possible to derive acceptable experimental values for the shape factors. The BS program, therefore, uses C(W) = 1, which corresponds to allowed transitions, and an uncertainty of 100%.

Conclusion for Experimental Shape Factor Analysis

To conclude the investigations of the experimental shape factor results, the 1nu transitions can be treated as allowed ones; a small correction was determined and is applied for 1u transitions. However, for 2u, 2nu, and transitions of higher order, the lack of experimental results does not allow deriving possible correction factors. So, these transitions (2, 3, etc.) are treated as allowed in the BS program. Table 1 lists these results and the proposed values for the various corrective factors.

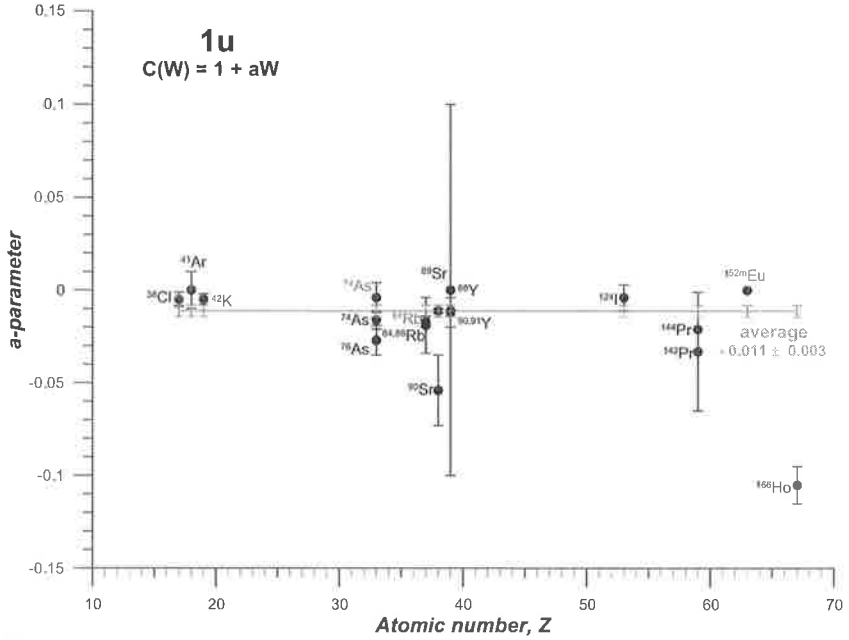


Figure 4 Experimental shape factor coefficients recommended by Behrens and Szybisz (1976) for first forbidden unique beta transitions.

FEATURES OF THE BETA SPECTRUM PROGRAM

Nuclear Decay Data

The Beta Spectrum program uses the nuclear data adopted from ENSDF files. These files are free for download from the Brookhaven National Laboratory web site (http://www.nndc.bnl.gov/ensdf). Beta Spectrum is supplied with the "default" data file containing only data for $\beta^{\pm}$ decay for all available isotopes in 2008. Any other ENSDF file can be simply loaded via the program menu.

Reading the data from the database file, the BS code recognizes the type of decay (electron or positron) and determines the following characteristics of each beta decay branch in the nucleus:

Endpoint energy Q–E_f for the electron decay (or Q–E_f–$2mc^2$ for the positron decay), where Q is the decay energy and E_f is the energy of the daughter excited level to which this branch is pointed;

Probability ε of the decay branch;

Momenta and **parities** of the parent and daughter excited levels, from which the beta transition forbidding order is determined. Sometimes the level momentum (or parity) has a few possible meanings (in the ENSDF file they are usually given in parentheses). In this case, the BS program uses the first value listed.

Fermi Function

The Fermi function is calculated as spline interpolation of the tables from Dzhelepov et al. (1969) and Behrens and Bühring (1982). The choice of table is available in the program menu (Figure 5). In most cases, both tables lead to similar results, except for the wider energy interval in Behrens' tables. For a considerable number of radionuclides, the use of tables from Behrens and Bühring (1982) gives rise to oscillations in the shape of beta spectra (Figure 6). These oscillations come from the small numbers of interpolation points of the Fermi function in Behrens' tables.

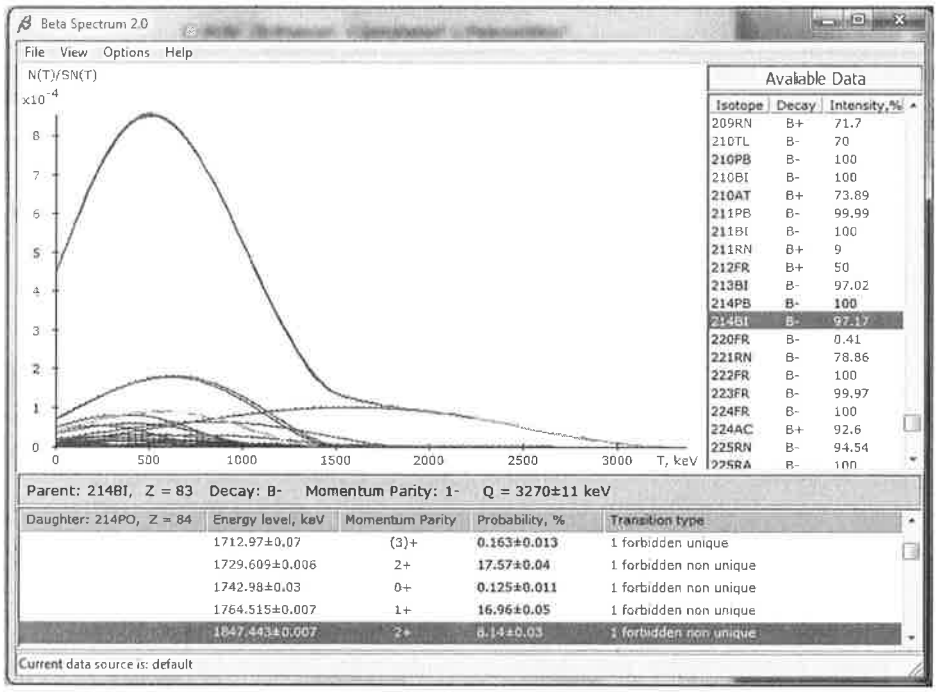


Figure 5 Main window of "Beta Spectrum" showing beta spectra of ^{214}Bi. In the right-hand section, all available data are shown from the current ENSDF file. In the bottom section, there is information about decay.

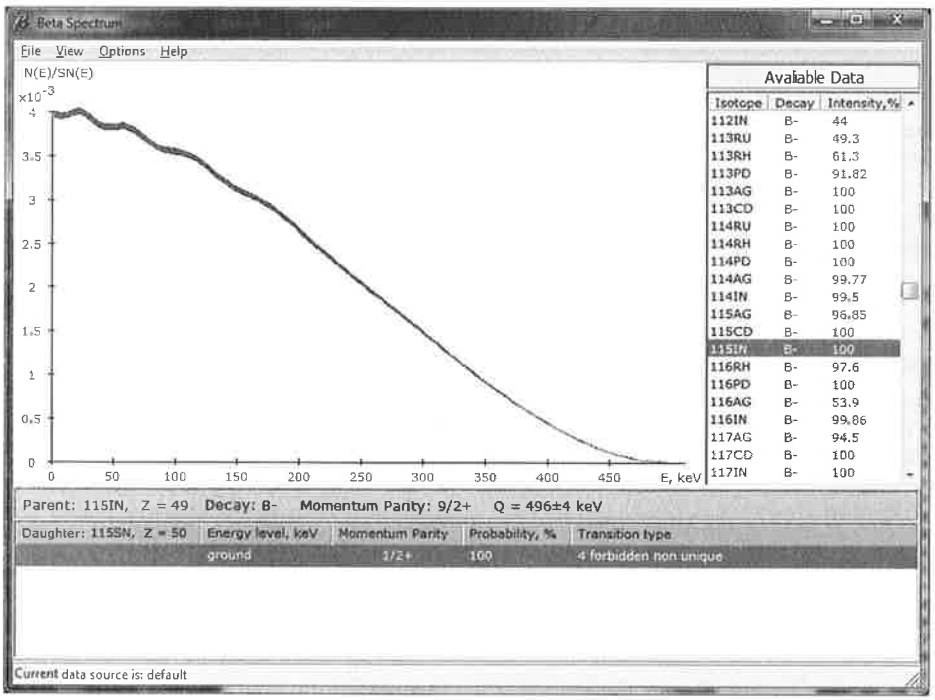


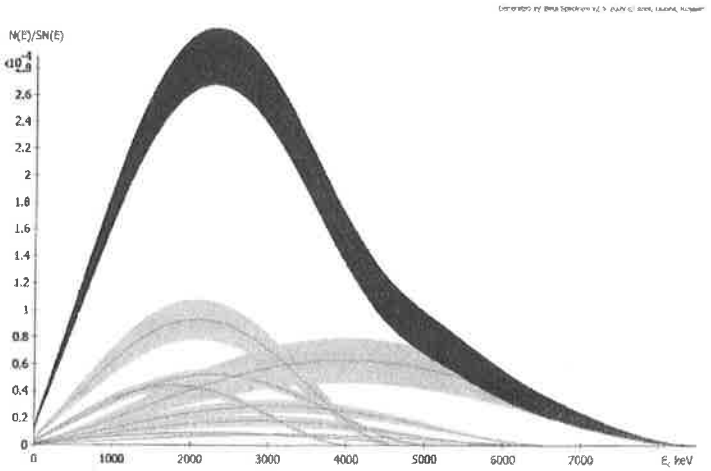
Figure 6 Beta spectrum of ^{115}In calculated with interpolated Behrens and Bühring (1982) table values for the Fermi function

Spectra Errors

The uncertainty of an individual beta spectrum branch includes uncertainties of radiation probability, endpoint energy, and uncertainties of parameters of shape factors as specified above.

Other Features of Beta Spectrum

The current version of Beta Spectrum calculates spectra (Fermi or Kurie) normalized to 1 for a given radionuclide with error estimation, saves it in ASCII text file, displays some nuclear decay data from the loaded ENSDF file, and prints spectra and decay information as shown in the Figure 7.



Parent: 23F

Decay	Level, keV	Momentum Parity	Decay Energy, keV
β-	ground	5/2+	8480±80

Daughter: 23NE

Level, keV	Momentum Parity	Rate, %	Transition Type
ground	5/2+	30±8	allowed
1016.9±0.4	1/2+	3±0.5	2 forbidden non unique $C(W)=q^2+B_0L_2p^2$
1701.7±0.6	(5/2,7/2+) using 5/2+	3±0.5	allowed
1822.29±0.21	3/2+	10.9±1.9	allowed
2314.8±0.5	5/2+	6.3±1.5	allowed
3431.8±0.3	3/2+	15.2±1.2	allowed
3830.7±0.4	3/2+	25±4	allowed
4436.1±0.7	(3/2,5/2,7/2)+ using 3/2+	10.4±1	allowed

Figure 7 Printed information about beta decay of ^{23}F

CONCLUSION

The computer program Beta Spectrum calculates beta spectra on the basis of available experimental data. The use of experimental nuclear data adopted and compiled in ENSDF files and experimental values of parameters of shape factors is the distinctive feature of the program. This makes it possible

to calculate the uncertainty of the beta spectrum based on experimental results. Probable prospective changes concern the way the Fermi function is computed as well as some improvements in the user interface.

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Dzhelepov BS, Zyryanova LN, Suslov YP. 1969. *Beta-Processes*. Leningrad: Nauka. In Russian.