Challenges in MHD modelling in fusion devices

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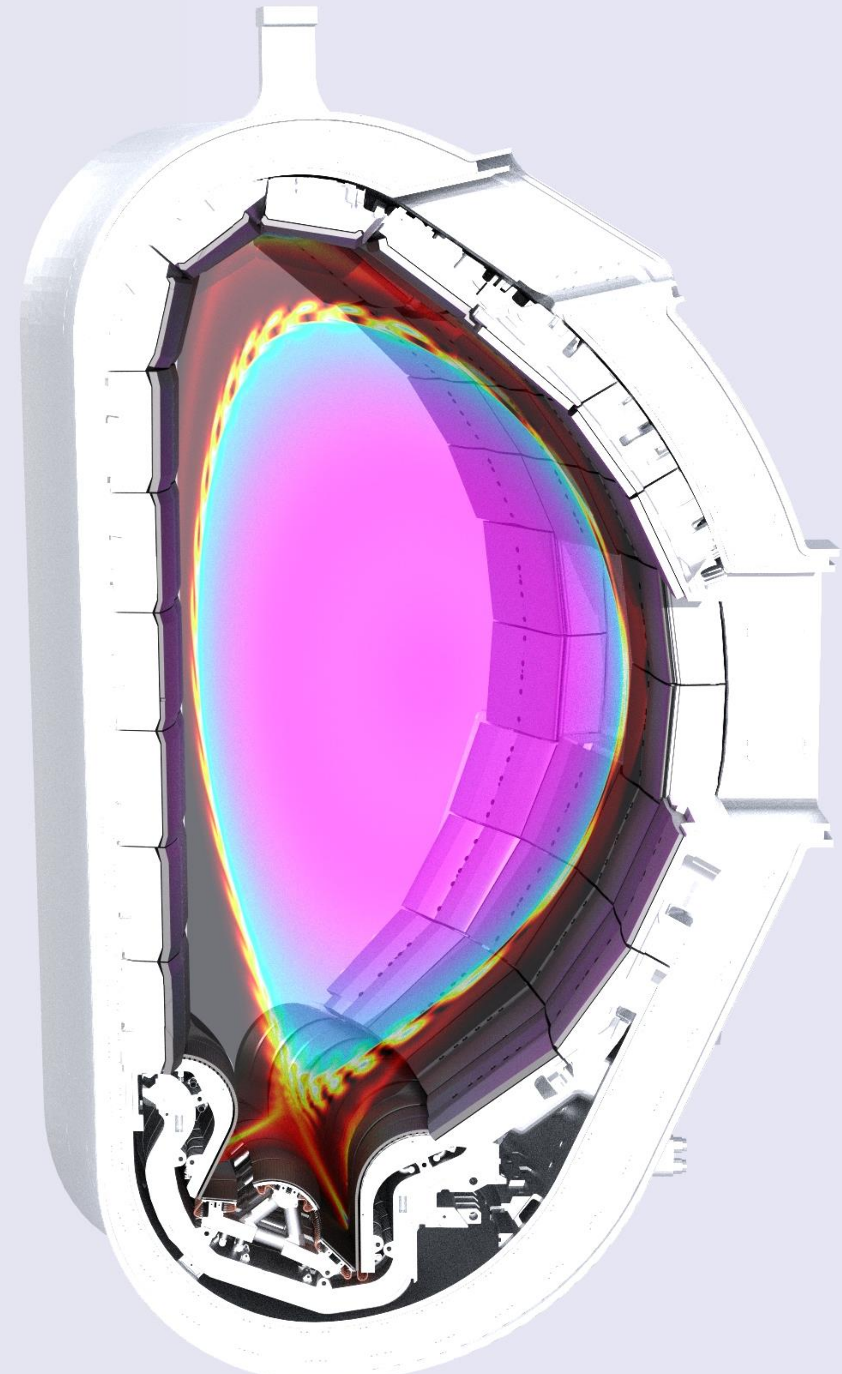


Outline

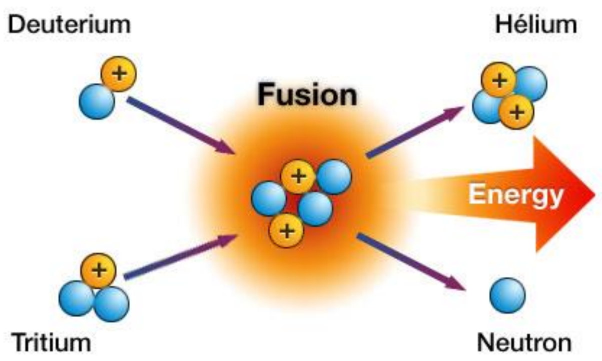
- Fusion
  –Tokamak, ITER
- Magnetohydrodynamic (MHD) Instabilities
  –Edge localised modes (ELMs)
- Nonlinear MHD code JOREK
  – finite elements
  – implicit time evolution
  – parallelisation
- Particles
  – neutrals, impurity transport
- Conclusion

Fusion Energy

- The fusion of light atoms release energy

$$^{2}D + ^{3}T = ^{4}He + n + 17.6\ \text{MeV}$$

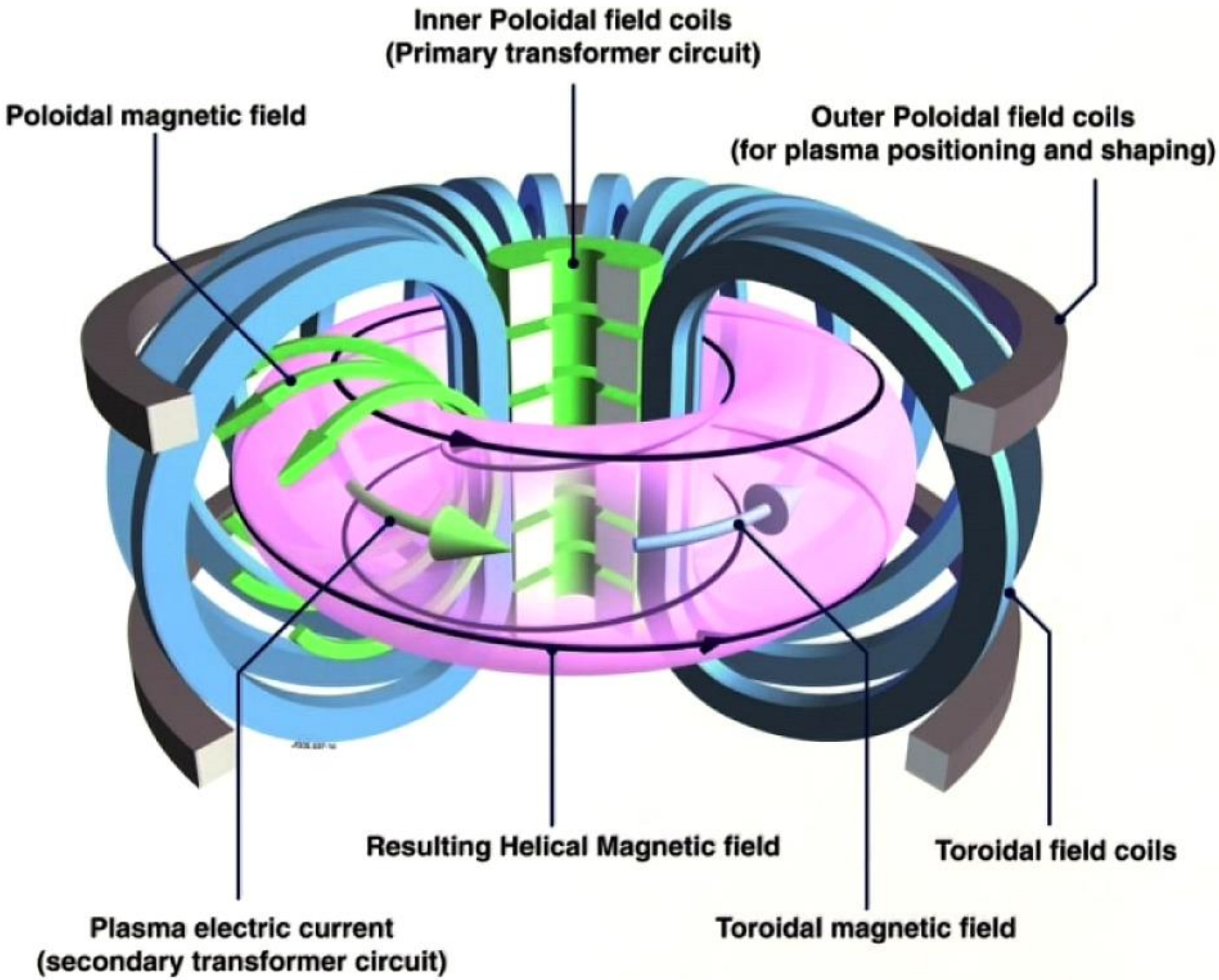
  – energy source of the stars
  – potential CO_2 free, safe energy source



- The reaction requires very high temperatures (150 M°)
  – gas is ionised and becomes plasma (free ions and electrons)

- The hot plasma can be contained by a magnetic field
  – tokamak is the most advanced approach
  – fusion power record in JET : 16 MW (in 1997)
  – next step: ITER

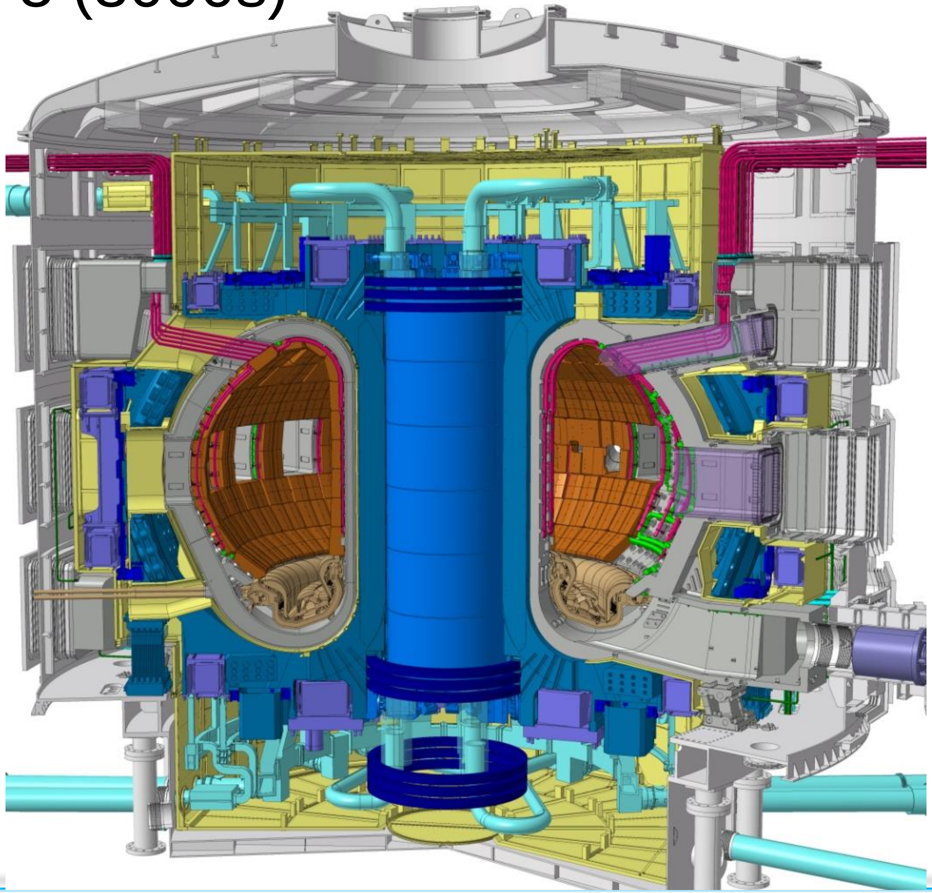
Tokamak



Inner Poloidal field coils
(Primary transformer circuit)
Poloidal magnetic field
Outer Poloidal field coils
(for plasma positioning and shaping)
Resulting Helical Magnetic field
Toroidal field coils
Plasma electric current
(secondary transformer circuit)
Toroidal magnetic field

The ITER Project

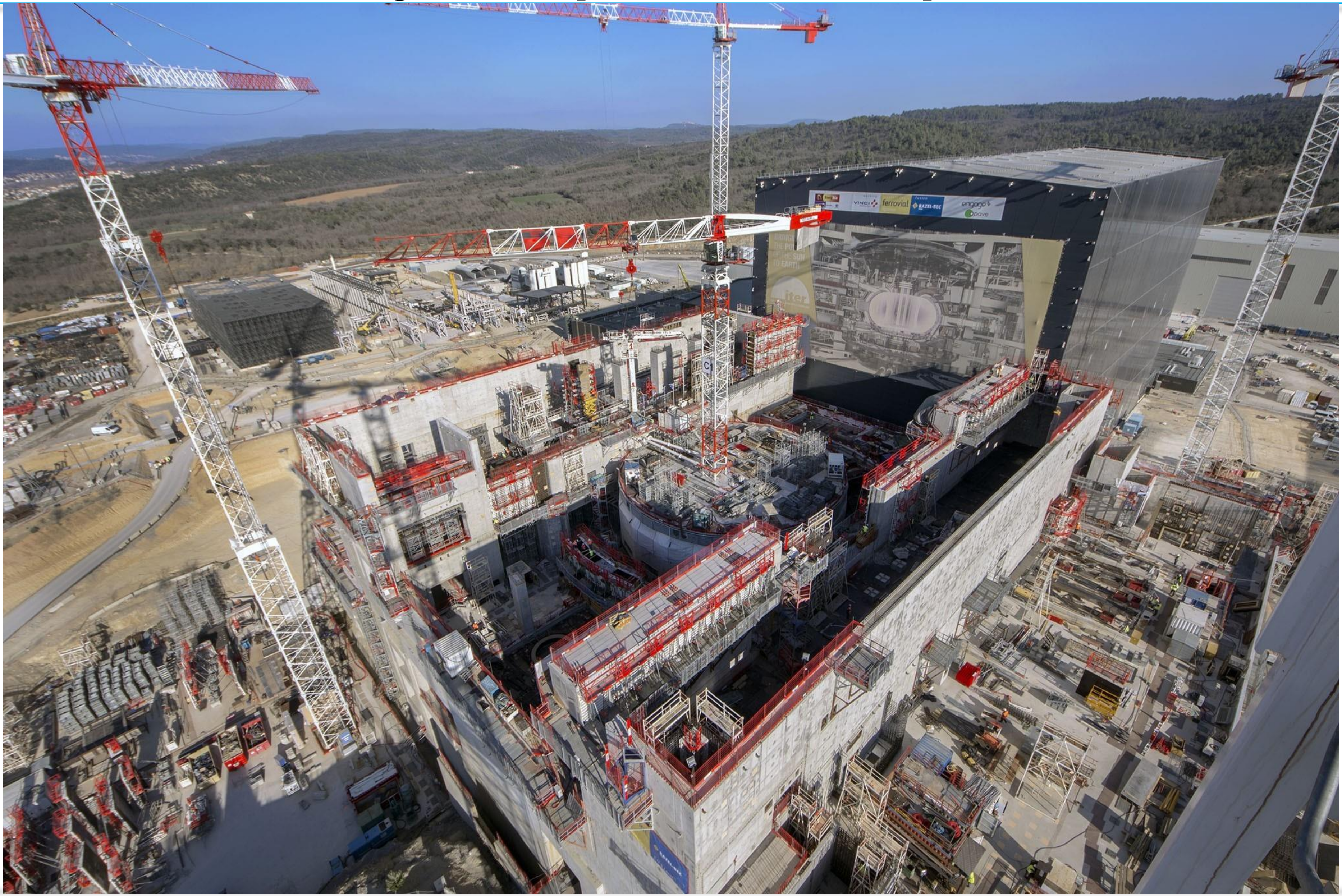
- Goals:
  - Fusion power amplification factor Q=10 for 300-500s
    - Fusion power ~500 MW
  - Steady state operation at Q=5 (3000s)



7 ITER Partners
EU, China, India, Japan,
Korea, Russia, USA

ITER Tokamak
Major radius : 6.2m
Minor radius : 2.0m
Toroidal field : 5.3T
Plasma current : 15 MA
First plasma : end 2025
Fusion power operation: end 2035

ITER Building Site (march 2019)

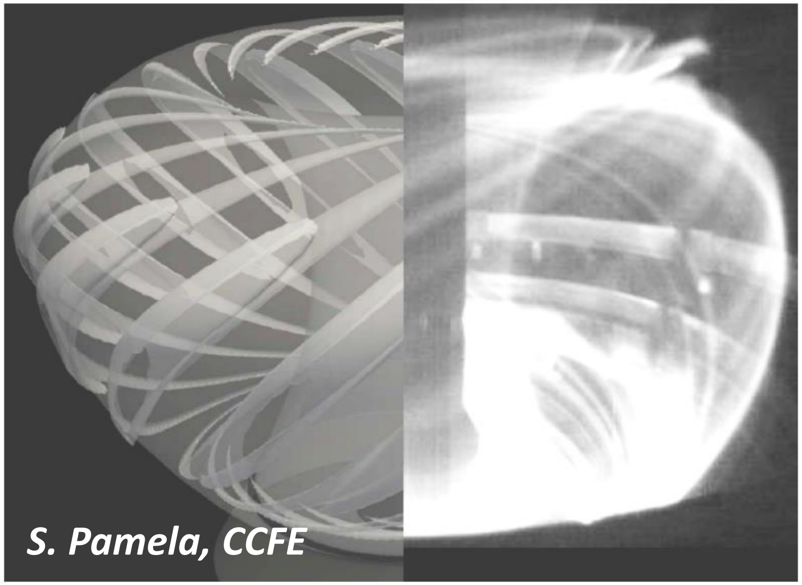


MHD Instabilities

- Large scale magnetic instabilities of the plasma
  - Driven unstable by pressure gradients and plasma current
    - **Limit global pressure, current and local pressure, current gradients**
  - Typical time scales : 1 millisecond
  - Leading to fast losses of plasma energy and density
    - Large transitory energy fluxes to the plasma facing components

- Disruptions:
  - Global MHD instability leads to fast plasma termination
- Edge Localised Modes (ELMs):
  - Local MHD instability in outer 20% of the plasma leading to repetitive energy and density losses of 1-10% (at 1-100Hz)

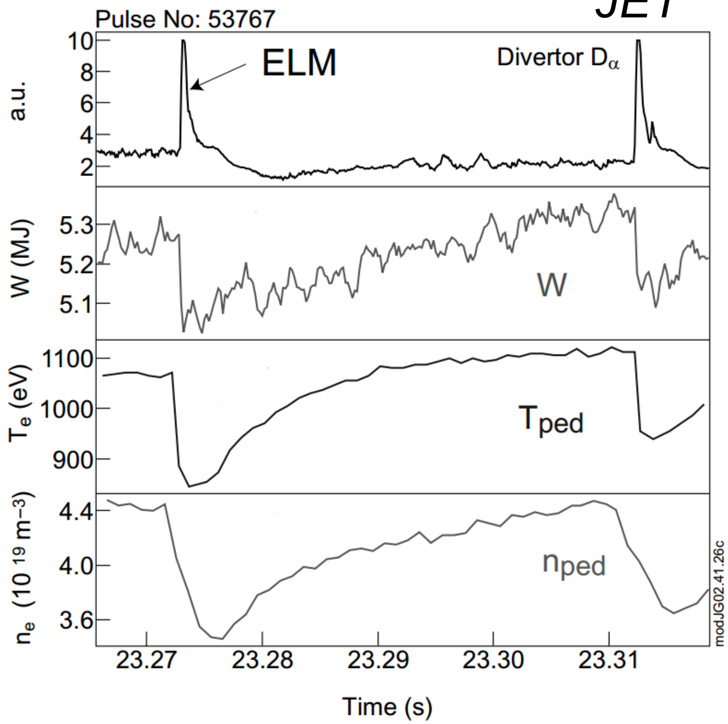
JOREK simulation MAST



Edge Localised Modes (ELMs)

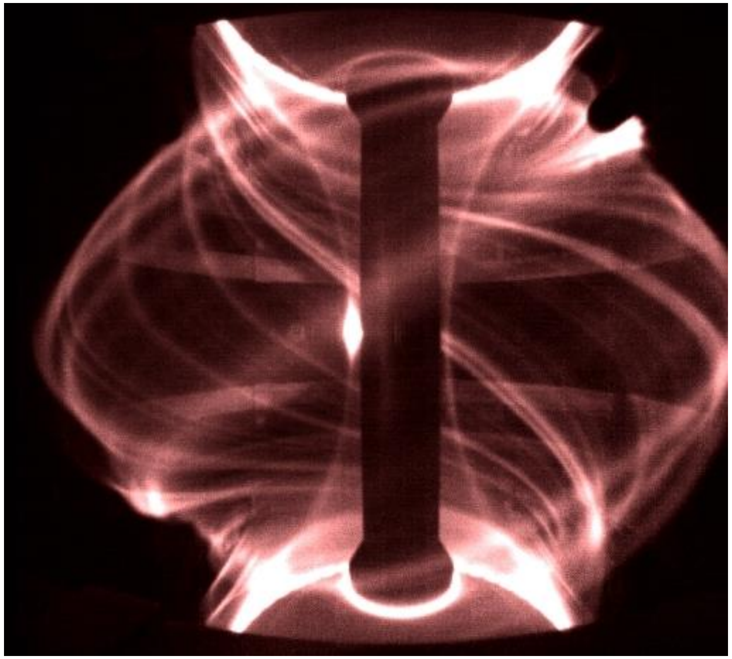
- H-mode regime: spontaneous stabilisation of turbulence at the plasma edge leads to large pressure gradients in the outer ~5% of the plasma
  - standard operating regime in ITER
- Edge pressure gradient is not limited by transport but by an MHD instability
  - Edge Localised Mode (ELM) removes up to 10% of the plasma energy in several hundred microseconds

JET



[Loarte, PPCF2002]

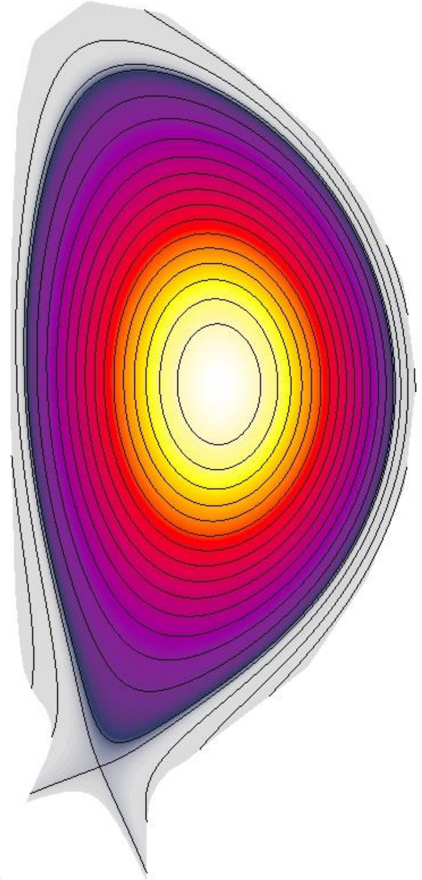
MAST



[Kirk, MAST]

Magneto-Hydro-Dynamics (MHD)

- Theoretical model (H. Alfven, 1942)
  - description of the plasma as a electrically conducting fluid embedded in a time varying magnetic field
    - Combining Navier-Stokes fluid equations with (pre-) Maxwell's equations
      - Conservation of mass, momentum, energy and magnetic flux
  - applications: plasmas, solar physics, astrophysics, dynamos



$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v}) + \nabla \cdot D \nabla \rho + S_\rho$$

$$\rho \frac{\partial \vec{v}}{\partial t} = -\rho \vec{v} \cdot \nabla \vec{v} - \nabla(\rho T) + \vec{J} \times \vec{B}$$

$$\frac{\partial \rho T}{\partial t} = -\vec{v} \cdot \nabla \rho T - \gamma \rho T \nabla \cdot \vec{v} + \nabla \cdot \kappa \nabla T + S_P$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} - \eta \vec{J} \right)$$

$$\mu_0 \vec{J} = \nabla \times \vec{B}, \quad \nabla \cdot \vec{B} = 0$$

Challenges MHD Simulations

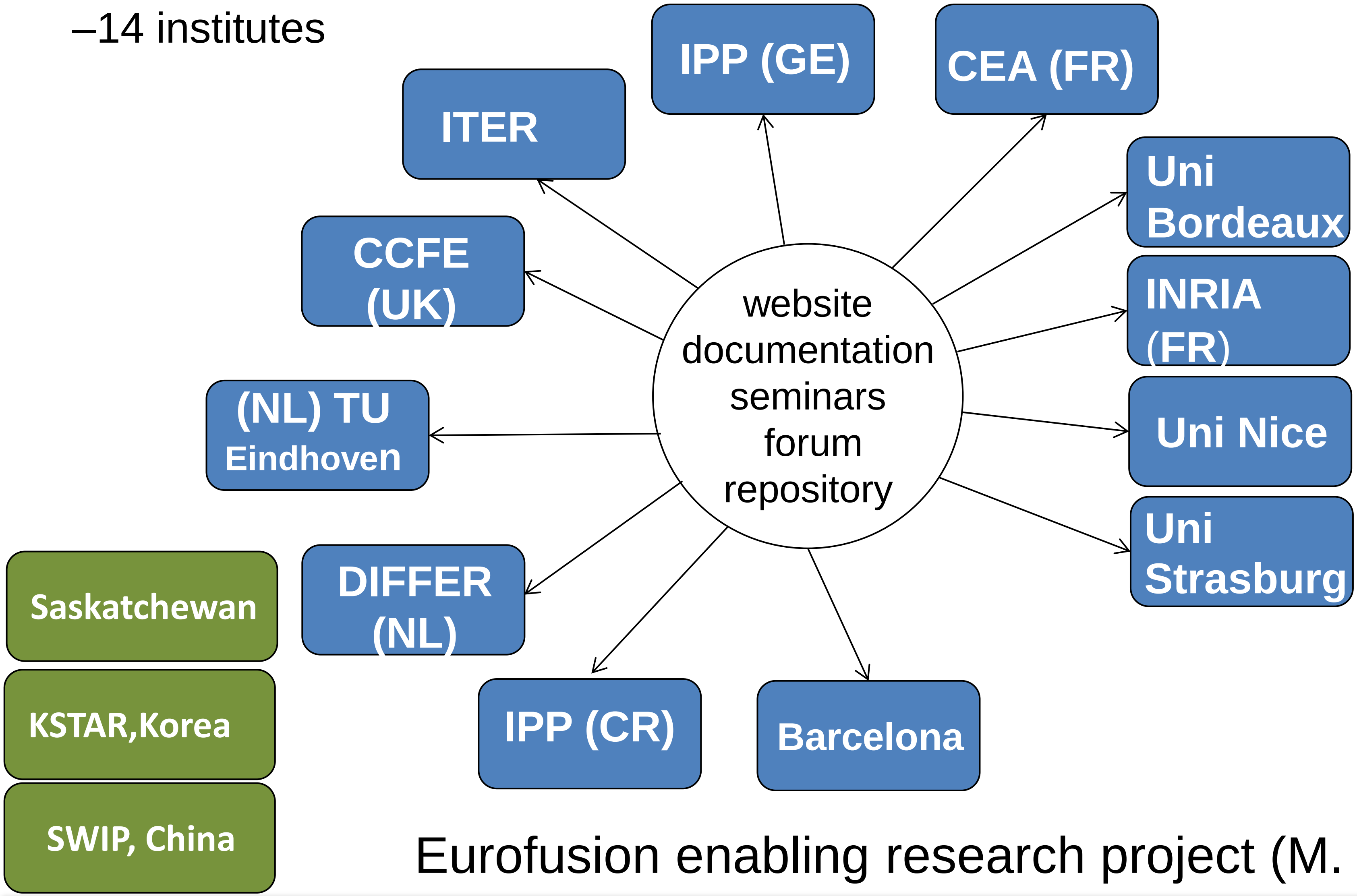
- Time scales:
  - – Depending on the application the time scale of interest varies from 1 microsecond to 100's milliseconds
    - Fast waves : 10^{-8} s, Alfven waves : 10^{-6} s, sound waves : 10^{-4} s
- Length scales:
  - – MHD instabilities develop thin current sheets
    - Magnetic Reynolds number S ~ 10^{8} -10^{10} (i.e. normalised resistivity^{-1})
    - Layer width scales as $S^{-1/2}$
- Anisotropy
  - – heat conduction is very much larger along a magnetic field line compared to the perpendicular direction
    - Conduction : $K_{//} / K_{\perp} \sim 10^{10}$

Non-linear MHD code JOREK

- JOREK code solves the (extended) MHD equations in 3D general tokamak geometry
  - –Closed and open field lines, plasma wall interaction, conducting structures
  - –Main applications: ELMs and disruptions
- now developed with the EU fusion program
  - –14 Fusion Labs + Universities, ~25 persons
  - –GIT repository, wiki, decentralised development
- Characteristics
  - –Cubic (Bezier) Finite element representation in the poloidal plane
    - Extension of cubic Hermite elements
  - –Fourier series in the toroidal direction
  - –Fully implicit time evolution
  - –MPI/OpenMP parallelisation

JOREK Network (2019)

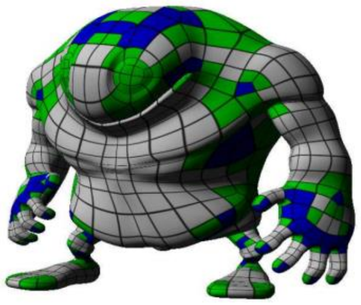
- Decentralised development
  - –14 institutes

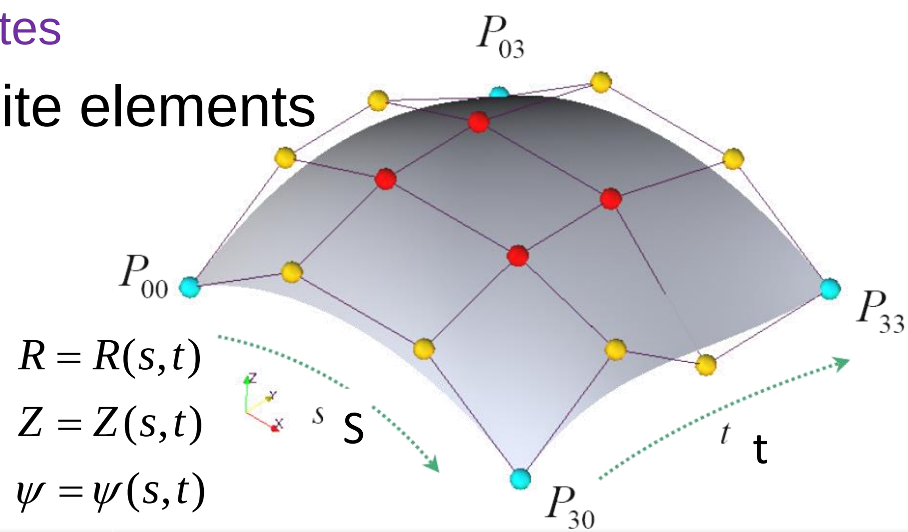


Eurofusion enabling research project (M. Hoelzl)

Bezier Finite Elements

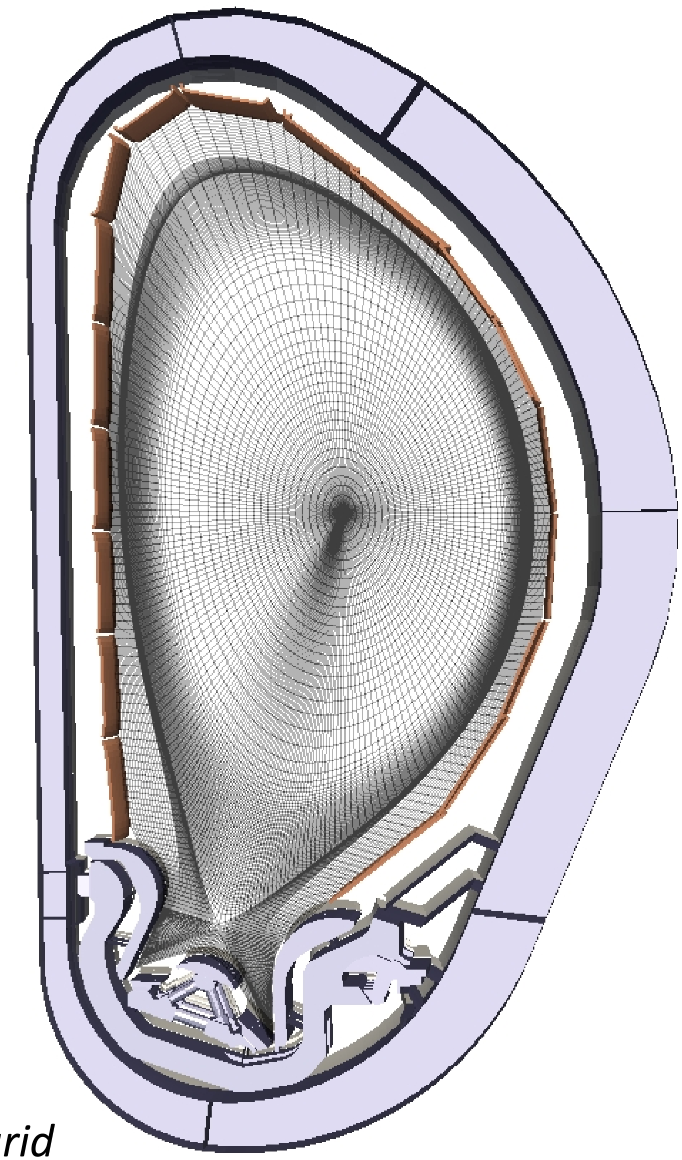
- Bezier patches are widely used in CAD, computer graphics
  - to describe 2D surface in 3D physical space
    - parametrised in local coordinates (s,t)
    - X(s,t), Y(s,t), Z(s,t)
    - Cubic bezier patch defined by 16 control points
  - here the height represents a physics variable
    - (R,Z) space and variables described by Bezier patches : Isoparametric representation: R(s,t), Z(s,t), ψ(s,t)
  - values and derivatives continuous in space (C^1)
    - but not continuous in local (s,t) coordinates
  - generalisation of cubic Hermite finite elements
    - Allows local refinement





JOREK Grids

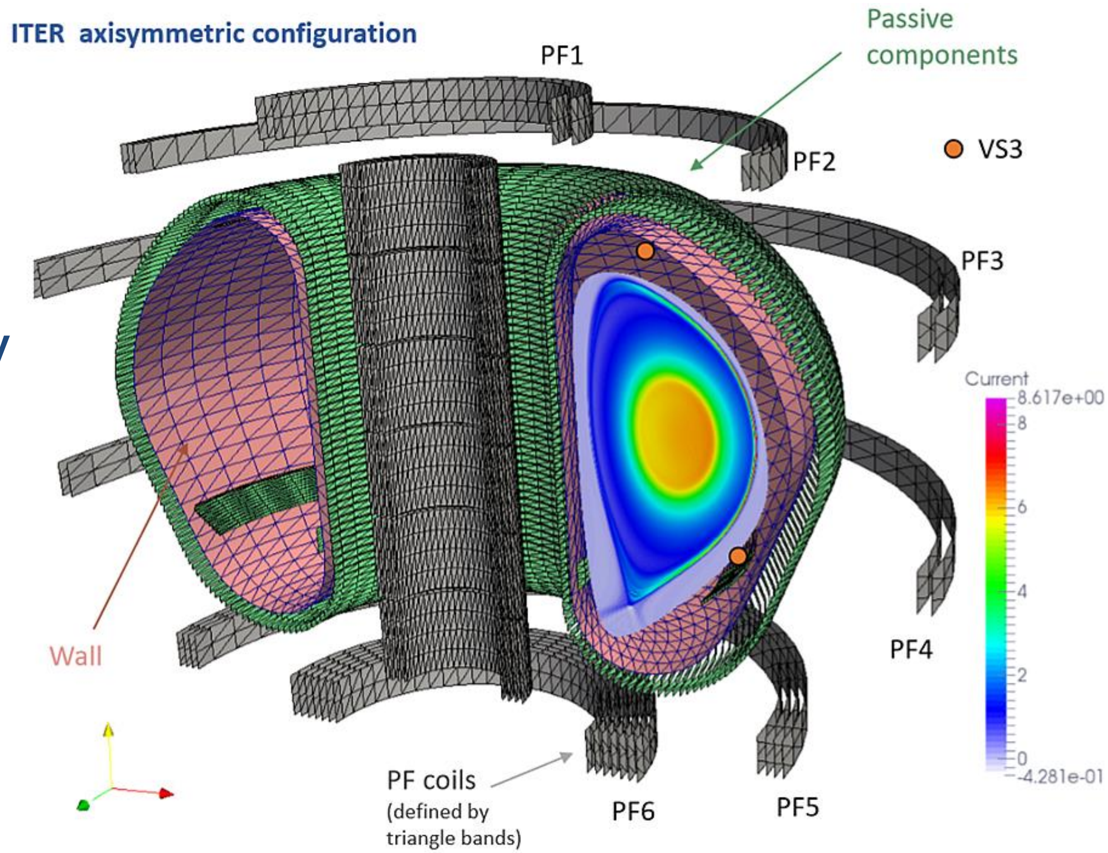
- Elements aligned to magnetic geometry
  - the curved boundaries of the Bezier patches can be accurately aligned with the magnetic geometry
  - alignment and higher order (cubic) elements allow an accurate description of the anisotropy of the heat conduction ($K_{//} / K_{\perp} = 10^{10}$)
- Aligned grids can be extended (non-aligned) up to the plasma facing components
  - essential for an accurate description of the plasma-wall interaction



example ITER grid

Boundary Conditions

- The evolution of magnetic fields and currents in coils and conducting structures can be solved independently from the main plasma
  - Solved (once) using the code **STARWALL** (boundary element method)



- Provides boundary conditions for the magnetic fields at the JOREK domain boundary
  - $B_{normal} / B_{tan} = M$

Implicit time evolution

- The time scales of interest can vary from the Alfven time (microsecond) to slow growing resistive instabilities (10s of seconds in ITER)
- Explicit schemes would be limited by the fast wave frequency (full MHD) or by Alfven waves (reduced MHD)
  - Not practical for most MHD tokamak applications
- Implicit scheme allows large time steps resolving the phenomena of interest
  - Linearised Crank-Nicholson or BDF (Gear's) schemes

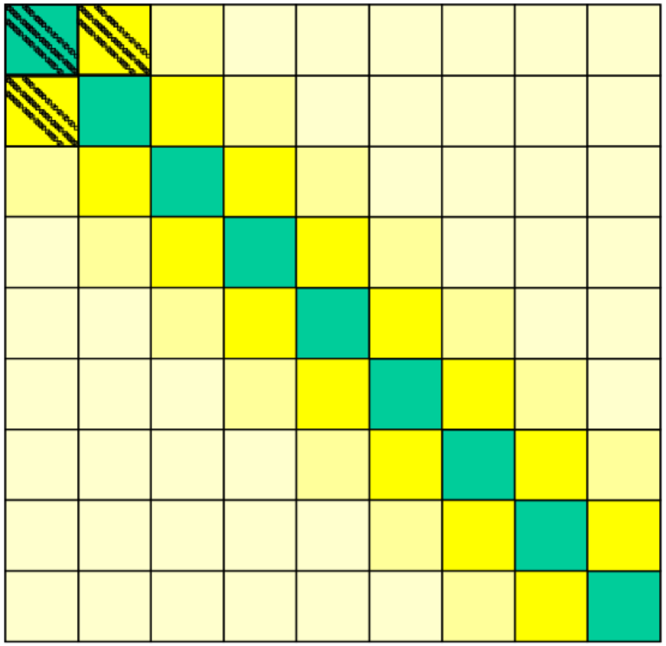
$$\frac{\partial A(\vec{y})}{\partial t} = B(\vec{y}) \quad \Rightarrow \quad \left(\frac{\partial A(\vec{y}_n)}{\partial y} - \tfrac{1}{2}\delta t \frac{\partial B(\vec{y}_n)}{\partial y} \right) \delta \vec{y} = B(\vec{y}_n)\delta t$$

- Implicit schemes yields large matrix problem at each time step
  - 5×10^4 nodes, 8 variables, 4 dof per node, 15 Fourier modes: $N = 5 \times 10^7$
  - Challenging for large scale parallelisation

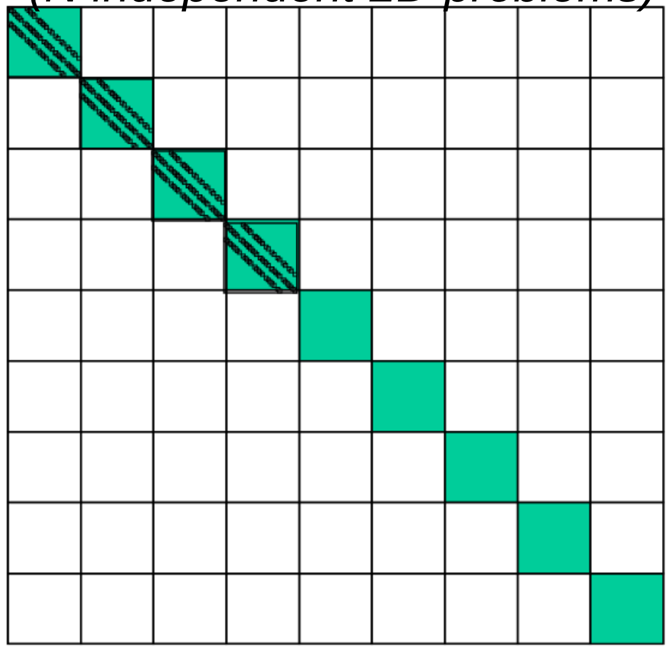
Preconditioned GMRES

- Matrix problem (~10^7unknowns) solved using preconditioned GMRES
- standard preconditioners not satisfactory for MHD
  - – condition number MHD matrices very large
- use physics-based preconditioner
  - – ignore non-linear coupling of N toroidal Fourier modes in preconditioning matrix
  - – N independent, sparse, blocks solved using **PaStiX** library

full matrix 3D problem

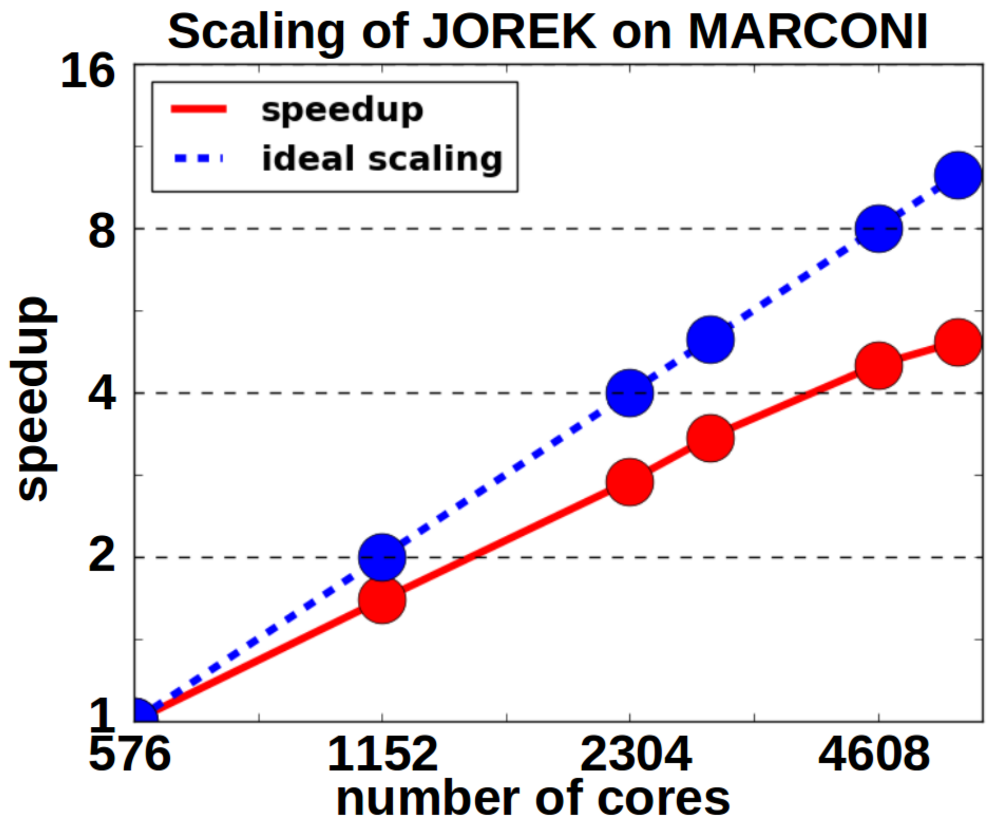


preconditioning matrix (N independent 2D problems)



JOREK Parallelisation

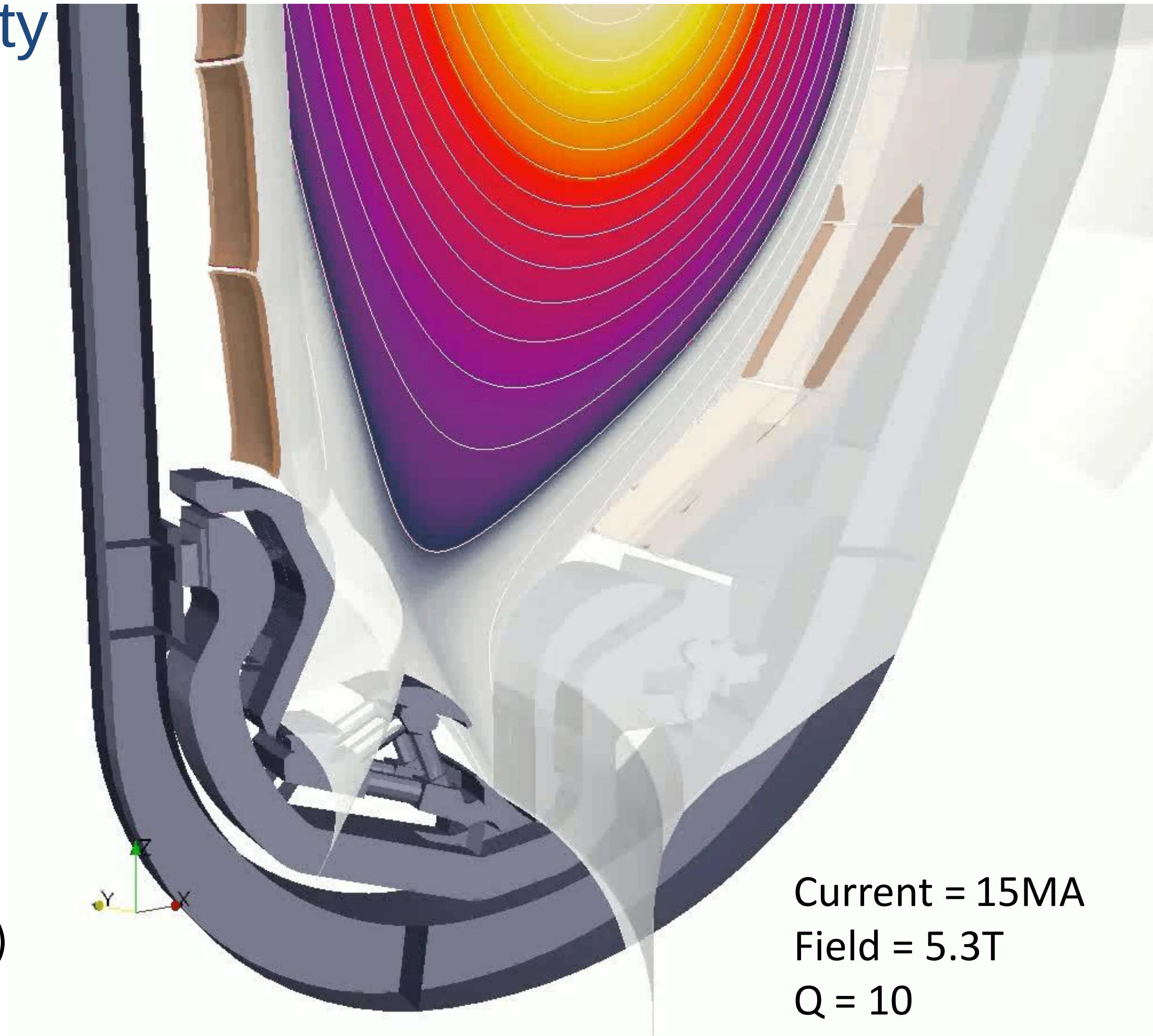
- Parallelisation : MPI+openMP,
  - # CPU : mostly due to memory requirements
- 3 main parts:
  - Matrix construction, factorisation preconditioner and GMRES solve



Strong scaling
(fixed problem size)

ITER ELM Simulation

- Ballooning instability
  - –n=10
- Losses:
  - –Energy loss : 1%
  - –Density loss : 2%
  - –ELM depth : ~12cm
  - –Duration 250 micro sec



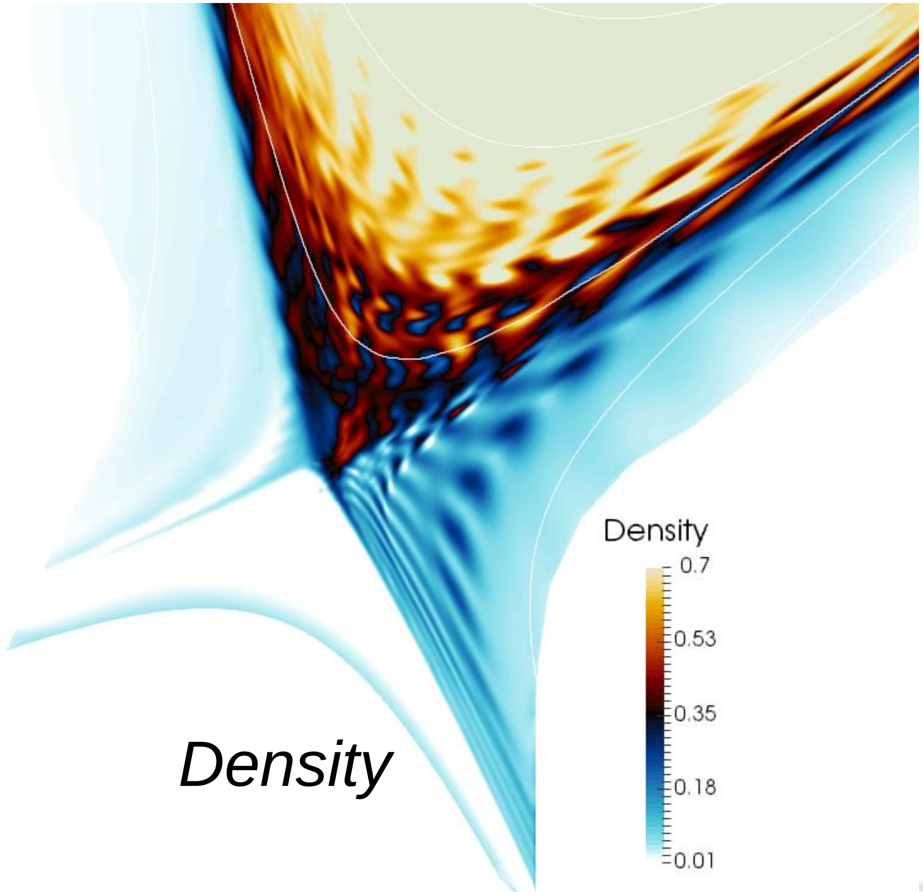
Plasma temperature (0-20keV)

Heat flux to divertor (0-3 GW/m^2)

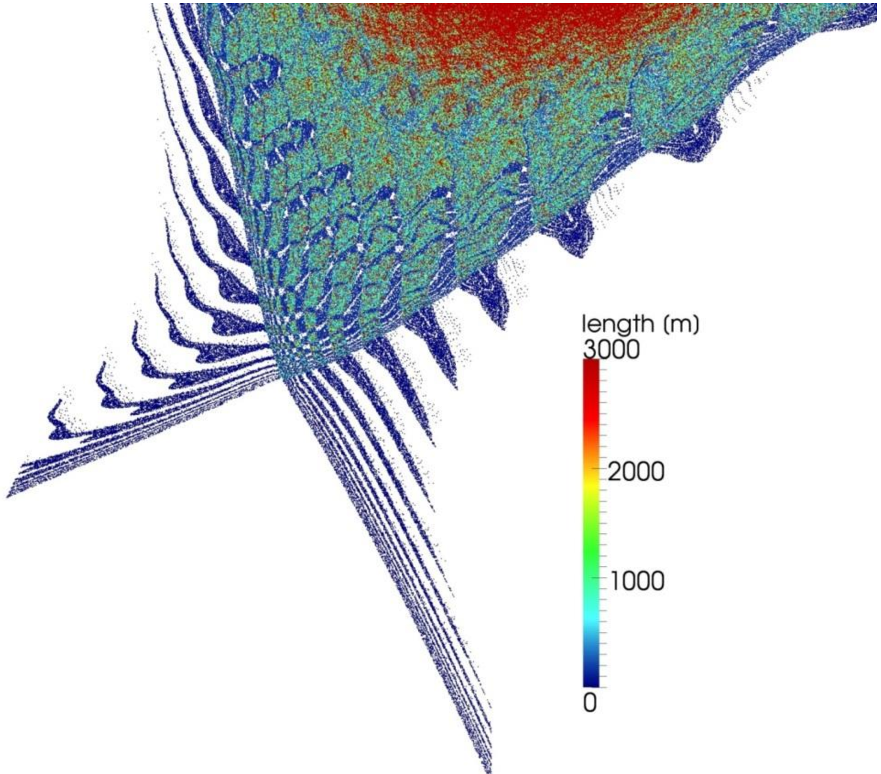
Current = 15MA
Field = 5.3T
Q = 10
$W_{thermal}$ = 380 MJ

ELM Energy Losses

- Energy loss during ELMs is due to two mechanisms:
  - Ejection of filaments : convective energy loss
  - Formation of magnetic tangles : conduction parallel to perturbed field lines
    - conductive – convective ELMs



Density

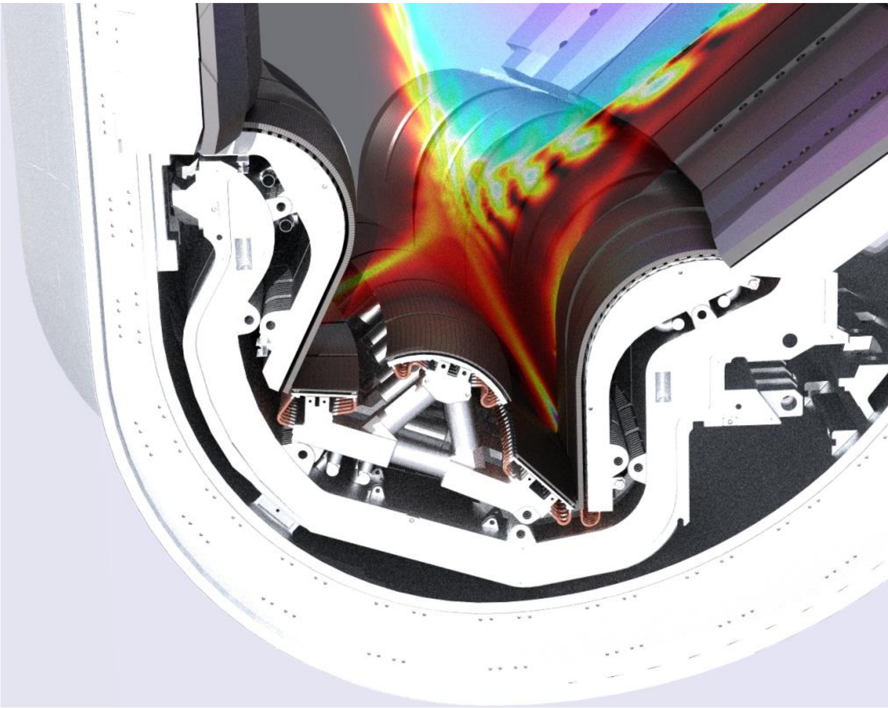


Poincare, connection length

Plasma-Wall Interaction

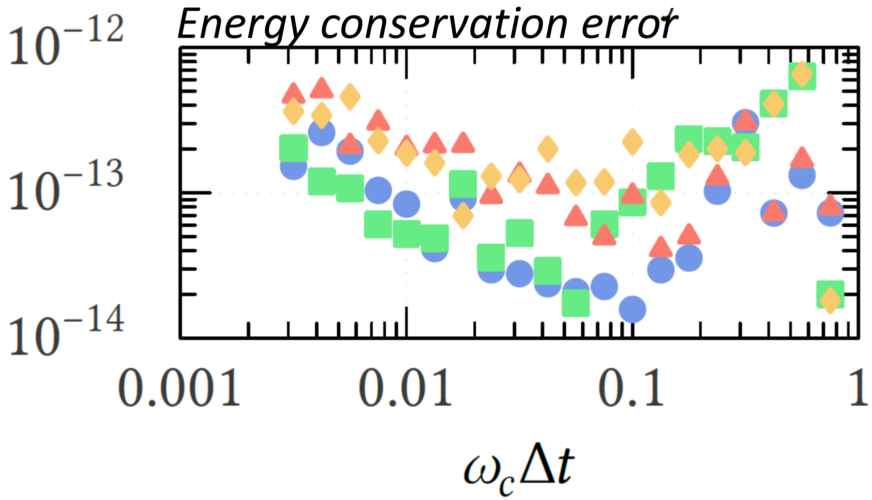
- To describe the consequences of the MHD instabilities (ELMs, disruptions), the plasma-wall interaction needs to be accurately modeled:
  – **MHD model extended with "divertor" boundary conditions**:
    - Plasma flows into the wall at Mach one (or larger)
    - Relation between conductive and convective energy fluxes: $K\nabla_{//}T = c\, n\, v_{//} T$

  – **Discrete particles** to model
    - recycling of plasma into neutrals,
      – ionisation, recombination, charge exchange
    - Impurities : sputtering and spreading due to MHD instabilities

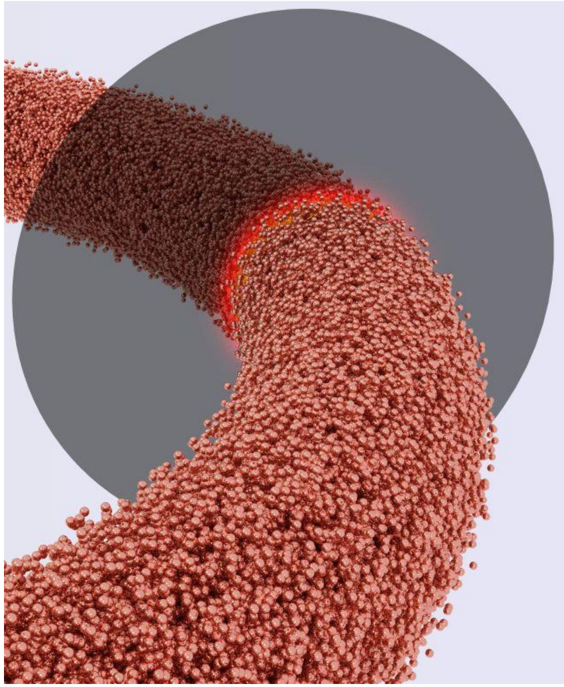


Particles

- Particles are followed in the time evolving magnetic and electric field of the MHD fluid
  - full orbit using well-known Boris method (with correction toroidal geometry)
  - particles are followed in real space (R,Z,φ) and finite element space (s,t,φ)
  - charge states are evolving in time (Ionisation, recombination, CX)
  - Particle-background collisions with binary collision model
  - sputtering model for impurity and neutral sources



Energy conservation independent of time step Δt, momentum conservation $\sim\Delta t^2$



Kinetic Neutrals

- Coupling of JOREK MHD fluid and particle
  - –Particle contributions become sources in MHD equations
    - Conservative form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_\rho$$

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v}\vec{v}) + \nabla \cdot \overline{\overline{P}} - \vec{J} \times \vec{B} = \vec{S}_V$$

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \tfrac{3}{2}\rho T\right) + \nabla \cdot \left(\tfrac{1}{2}\rho v^2 \vec{v} + \tfrac{3}{2}\rho T \vec{v} + \vec{v} \cdot \overline{\overline{P}}\right) = S_E$$

    - JOREK form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_\rho$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho (\nabla \cdot \vec{v})\vec{v} + \nabla \cdot \overline{\overline{P}} = \vec{S}_v - \vec{v} S_\rho$$

$$\frac{\partial \left(\tfrac{3}{2}\rho T\right)}{\partial t} + \vec{v} \cdot \nabla \left(\tfrac{3}{2}\rho T\right) + \tfrac{5}{2}\rho T (\nabla \cdot \vec{v}) = S_E - v S_v + \tfrac{1}{2} v^2 S_\rho$$

[D. van Vugt , S. Franssen]

Sources, Moments

- The sources are given by:

$$S_\rho(\vec{x}) = \int mf(\vec{x},\vec{v})\,d\vec{v}$$
$$S_v(\vec{x}) = \int m\vec{v}f(\vec{x},\vec{v})\,d\vec{v}$$
$$S_E(\vec{x}) = \int \tfrac{1}{2}mv^2 f(\vec{x},\vec{v})\,d\vec{v}$$

- Express sources in JOREK finite element representation to obtain a continuous function from a list of discrete particles

$$\tilde{S}_\rho(\vec{x}) = \sum_{ijk} p_{\rho,ijk} H_{ij}(s,t) H_{\varphi,k}(\varphi)$$

- Projection using weak form (system of equations to be solved)

$$\int v^*(\vec{x})\tilde{S}_\rho(\vec{x})\,d\vec{x} = \int v^* S_\rho(\vec{x})\,d\vec{x}$$
$$\int v^*(s,t,\varphi)\sum_{ijk} p_{\rho,ijk} H_{ij}(s,t) H_{\varphi,k}(\varphi)\,d\vec{x} = \int v^* mf(\vec{x},\vec{v})\,d\vec{v}\,d\vec{x}$$
$$\int v^*(s,t,\varphi)\sum_{ijk} p_{\rho,ijk} H_{ij}(s,t) H_{\varphi,k}(\varphi)\,d\vec{x} = \sum_n v^* m w_n \delta(\vec{x}-\vec{x}_n)$$

Moments: smoothing

- Some smoothing is required
  - depending on number of particles
- Modified projection equation:

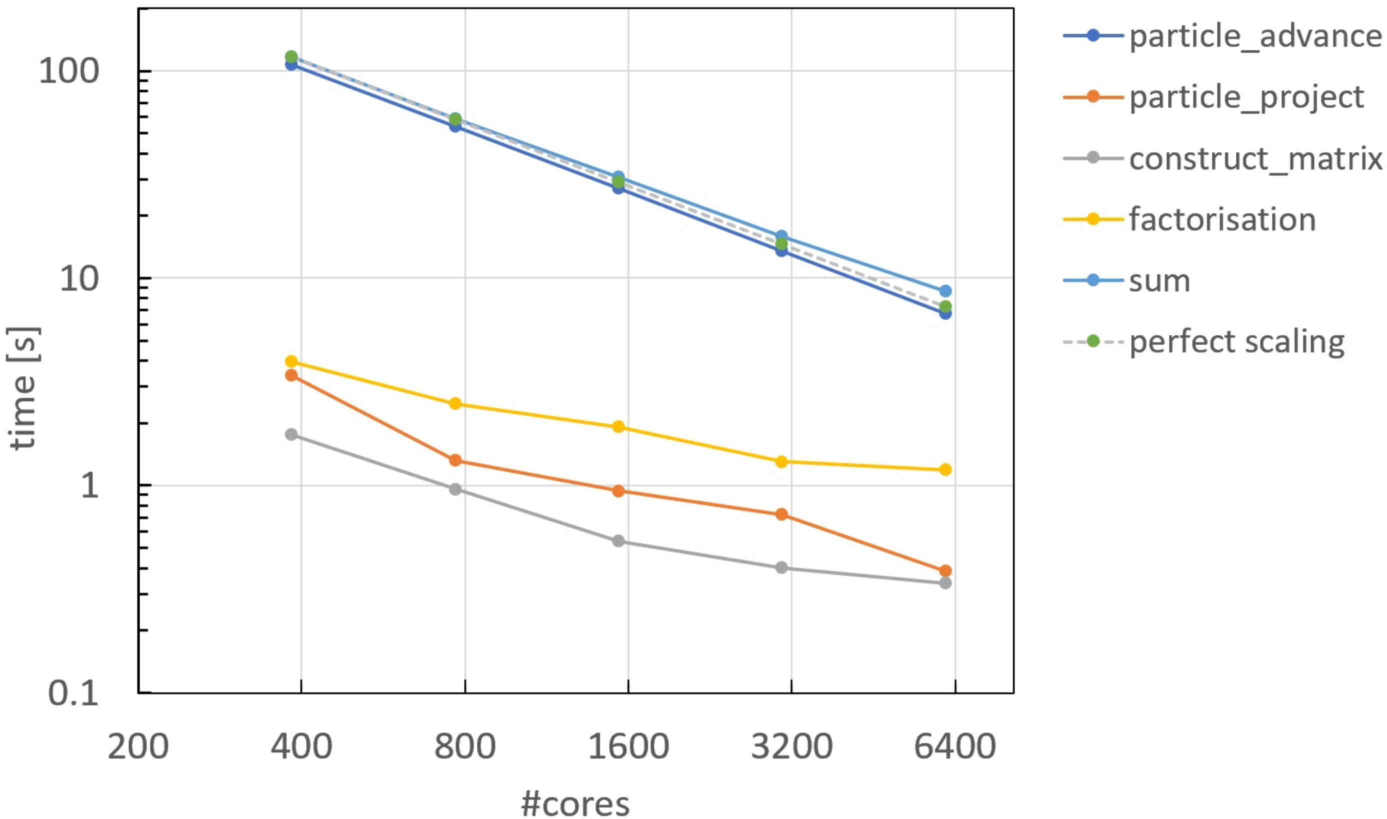
$$\int v^*(\vec{x})\left(1-\lambda\nabla^2-\varsigma\nabla^4\right)\tilde{S}_\rho(\vec{x})\,d\vec{x}=\int v^* S_\rho(\vec{x})\,d\vec{x}$$

$$\int\left(v^*\tilde{S}_\rho+\lambda\nabla v\cdot\nabla\tilde{S}_\rho+\varsigma\nabla^2 v^*\nabla^2\tilde{S}_\rho\right)d\vec{x}=\int v^* S_\rho(\vec{x})\,d\vec{x}$$

  - LHS matrix is calculated and factorised once (using MUMPS)
  - **RHS is collected at every particle step**
    - Important for conservation properties
  - Projection (i.e. MUMPS solve) at every JOREK fluid time step

Parallelisation Scaling

- Good parallel scaling as long as particles dominate total time
  - 4x10^7 particles, 1600 elements, 8 toroidal harmonics
  - Marconi, Skylake (1 mpi, 48 threads per node)
  - efficiency 84% at 6000 cores

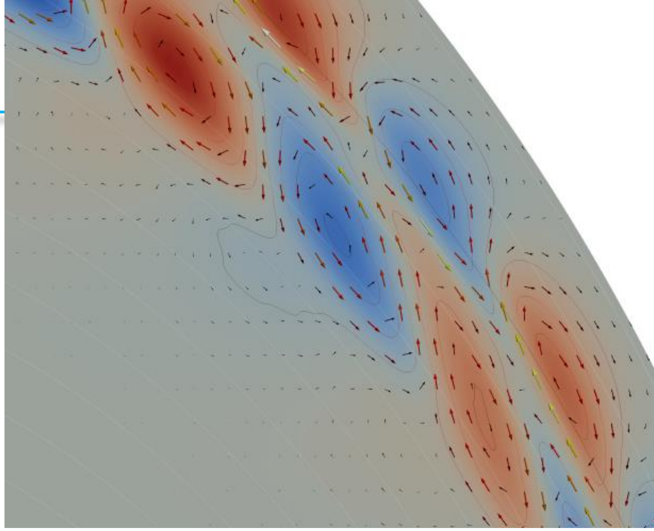


Tungsten (W) transport and ELMs

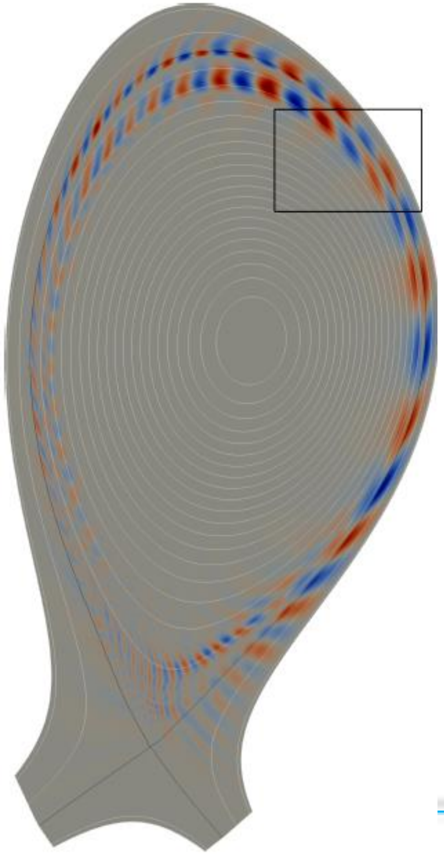
- Modern tokamaks (and ITER) have Tungsten divertors
  - high melting temperature
  - low retention of Tritium
- Even low concentrations of W lead to large radiation losses
  - leading to disruptions
- Main source of W is due to sputtering of the divertor due to large energy deposition due to ELMs
  - i.e. produces W in the divertor, outside main plasma
- However, ELMs also expel Tungsten from the main plasma
  - at least in current experiments

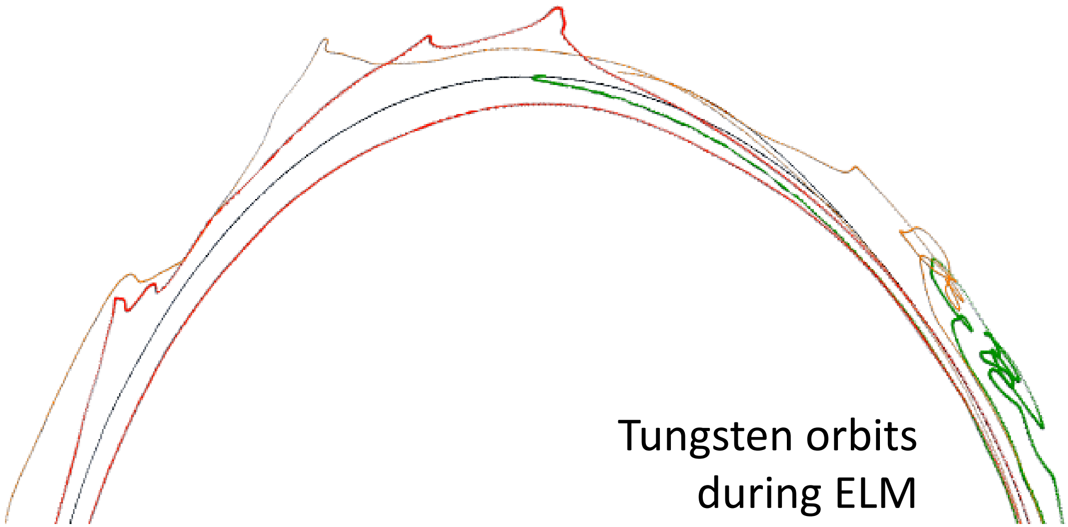
- JOREK MHD + W-particles simulations (D. van Vugt, TUe)
  - Study the ELM induced W transport
  - W production due to sputtering

ELM induced W Transport



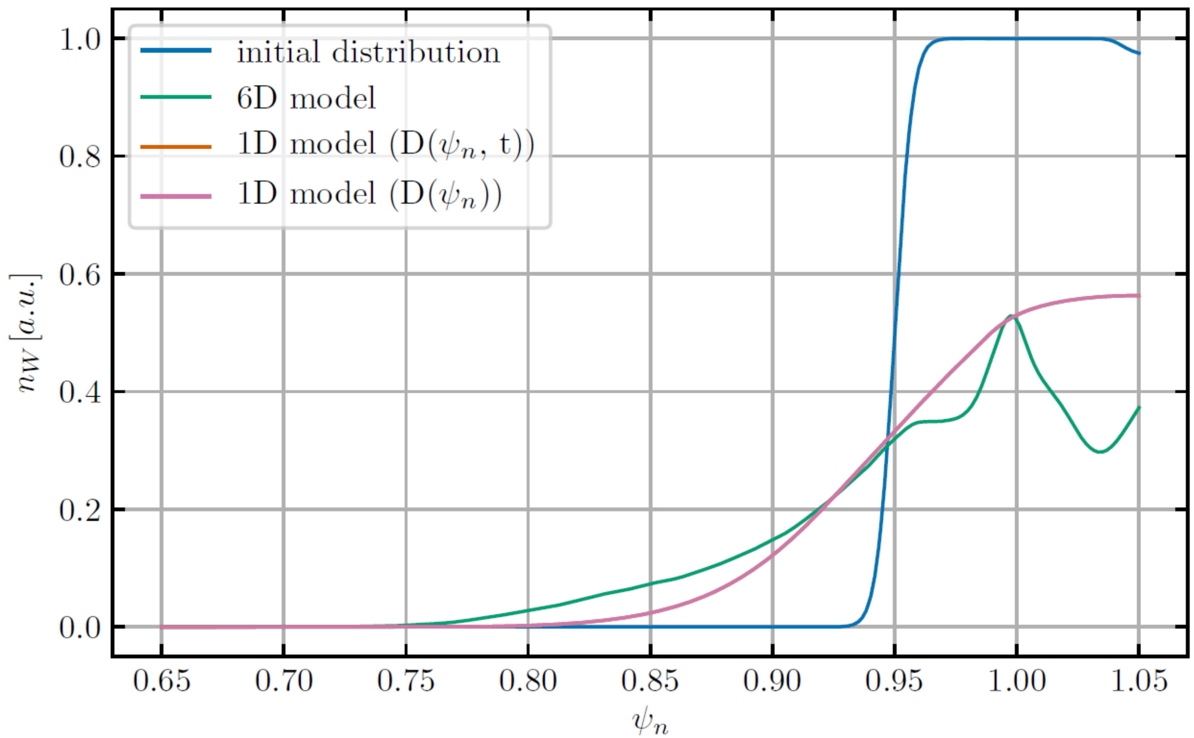
- 3D vortices of the ELM instability at the edge of the plasma
  - associated electric field perturbs W orbits and causes radial transport
  - magnetic field perturbation unimportant
- Transport is due 3D to interchange motion





ELM induced W transport

- Effective ELM induced W radial transport (in 1D) is diffusive
  - –In the case of W accumulation inside the plasma ELMs expel Tungsten
    - As in current experiments
  - –In ITER, ELMs are **predicted to move Tungsten into the plasma**
    - Due to inter-ELM profiles of Tungsten mainly outside the main plasma
    - Opposite to current experiments



Summary

- MHD instabilities are one of the main concerns for ITER operation
  - ELMS, disruptions
  - MHD instabilities must be controlled in ITER
- Nonlinear MHD code JOREK
  - simulation of MHD and its control in current and future fusion experiments
  - MHD+particle model for plasma wall interaction, fast particles
- HPC requirements
  - fully implicit scheme due to large range of time scales
    - Large sparse matrices ⇒ memory requirements, parallisation inefficiency beyond several 1000 cores (fluid model)
    - MHD+particle model scales much better
- Next: extending particle model
  - verification and validation: towards a predictive capability
  - fast α-particles, massive material injection