Entropy and relaxation processes

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Motivation

- Open systems: find constitutive relations that link fluxes to gradients $\rightarrow$ transport matrix.
- Relaxation processes $\leftrightarrow$ transport equations. Consistency with second principle?
- Principle of minimum entropy production $\rightarrow$ second principle, relaxation processes, Onsager reciprocal relations. Can be done with quasi-linear theory - not always valid.
- Present status in magnetised plasmas?
- What can be done when this procedure fails?

Entropy production and Onsager symmetry

- Entropy production $\dot{S}$ vs "forces" $\mathbf{\Lambda}$ and "fluxes" $\mathbf{\Gamma}$

$$\dot{S} = \mathbf{\Lambda} \cdot \mathbf{\Gamma}$$

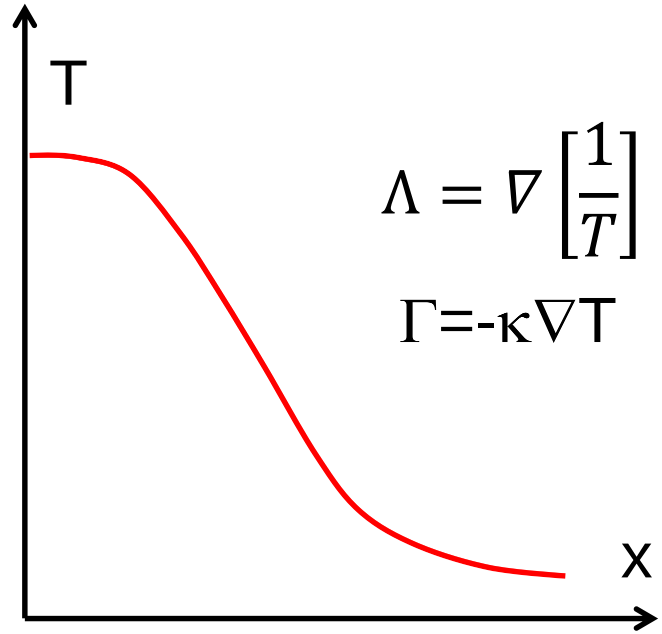
- Linear constitutive relationships

$$\mathbf{\Gamma} = \bar{\bar{\boldsymbol{L}}} \cdot \mathbf{\Lambda}$$

- $\bar{\bar{\boldsymbol{L}}}$ symmetric **B**$\rightarrow$-**B** Onsager 1931

- Compact form

$$\dot{S} = \mathbf{\Lambda} \cdot \bar{\bar{\boldsymbol{L}}} \cdot \mathbf{\Lambda}$$



$$\Lambda = \nabla\left[\frac{1}{T}\right]$$

Γ=-$\kappa\nabla$T

$$\dot{S} = \int dx\, \kappa \left(\frac{\nabla T}{T}\right)^2$$

$\rightarrow$ minimum of entropy production Prigogine 47

Boltzmann-Gibbs statistical mechanics

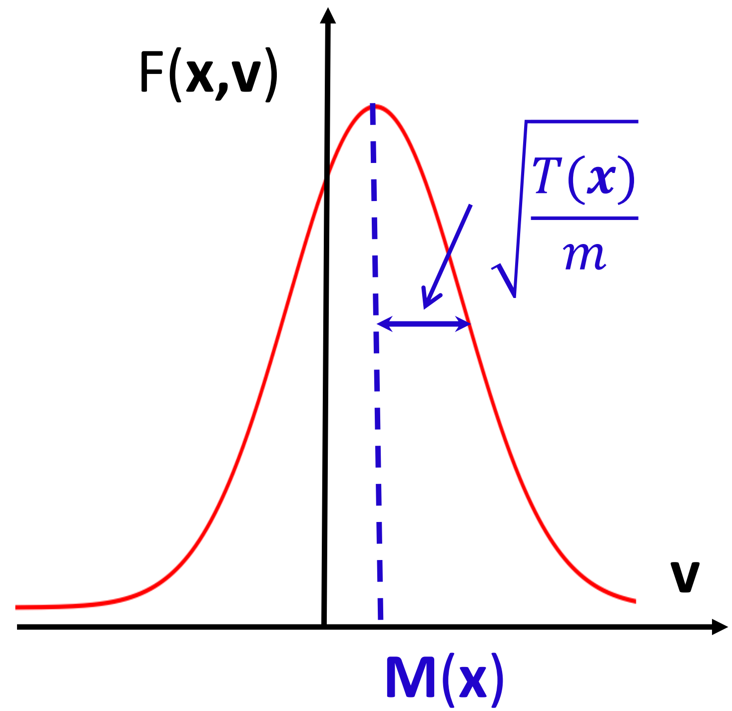
- Distribution function $F(\boldsymbol{x},\boldsymbol{v},t) \rightarrow$ entropy $S = -\int d\tau\, F\, lnF$
- Maximum of entropy under constraints $\rightarrow$ local Maxwellian

$$F = exp\left(-\frac{H}{T} + U(\boldsymbol{x})\right)$$

- F solution of a kinetic equation

$$\frac{dF}{dt} = C(F)$$

- Collisional transport well documented OVs Braginskii 65, Hinton 76, Balescu 87, Shaing 88 $\rightarrow$ Turbulent transport

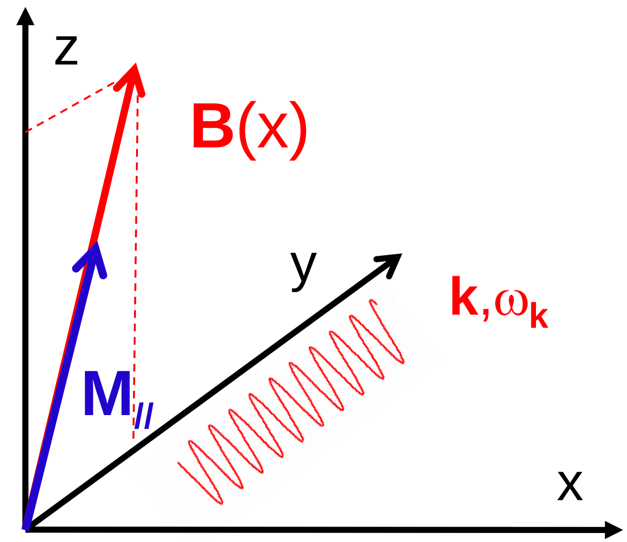


$$\int d^3\boldsymbol{v} F(\boldsymbol{x},\boldsymbol{v}) = N(\boldsymbol{x})$$

Quasi-linear theory provides fluxes for a given spectrum of fluctuations

Sheared B field, drift waves, chaotic

Hasegawa-Mima 77, Hasegawa-Wakatani 83



Fluctuations of ExB drift velocity $\boldsymbol{v}_E = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}$

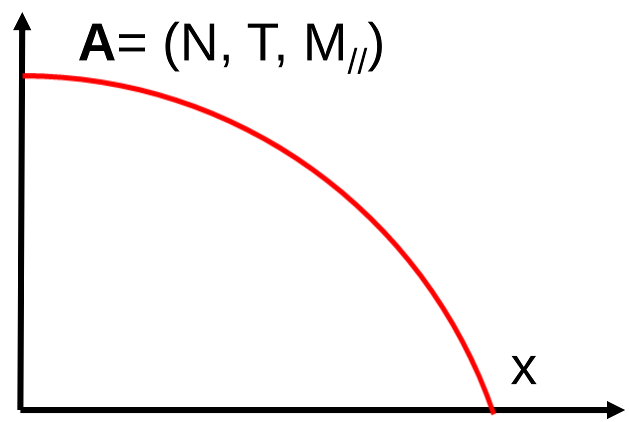
$\downarrow$

Plasma linear response

$$F_{\boldsymbol{k}} = R_{\boldsymbol{k}}(\boldsymbol{\Lambda})v_{E\boldsymbol{k}}$$

$\downarrow$

Flux $\Gamma = \sum_{\boldsymbol{k}} \int d^3\boldsymbol{v}\, F_{\boldsymbol{k}} v_{E\boldsymbol{k}}^*$



$$\boldsymbol{\Lambda} = \left(\frac{dN}{Ndx}, \frac{dT}{Tdx}, \frac{dM_\parallel}{dx}\right)$$

Drummond 62, Vedenov 62, OVs Krommes 02, Diamond 10

Entropy production is explicit

- Entropy production rate Horton 80, Itoh 82, Sugama 96, XG 13

$$\dot{S} = \frac{1}{2}\sum_{\boldsymbol{k}} \int d\tau\, F D_{\boldsymbol{k}} \left[\frac{\partial U}{\partial x} - \frac{eB}{k_y T}\left(\omega_{\boldsymbol{k}} - k_\parallel M_\parallel\right)\right]^2$$

Diffusion due to ExB velocity fluctuations

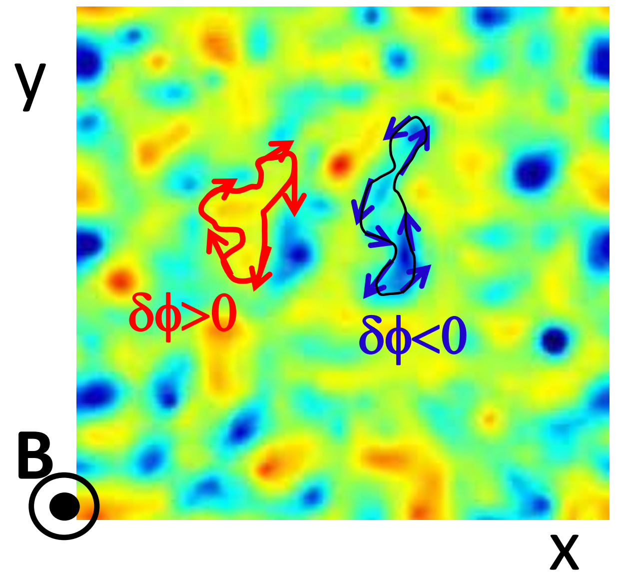
Forces

$$\boldsymbol{\Lambda} = \left(\frac{dN}{Ndx}, \frac{dT}{Tdx}, \frac{dM_\parallel}{dx}\right)$$

wave/particle energy & momentum transfer

$$\boldsymbol{V}_E = \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2}$$

$$D_{\boldsymbol{k}} = 2\pi |v_{E\boldsymbol{k}}|^2 R(\omega_k - k_\parallel v_\parallel)$$



Numata 06

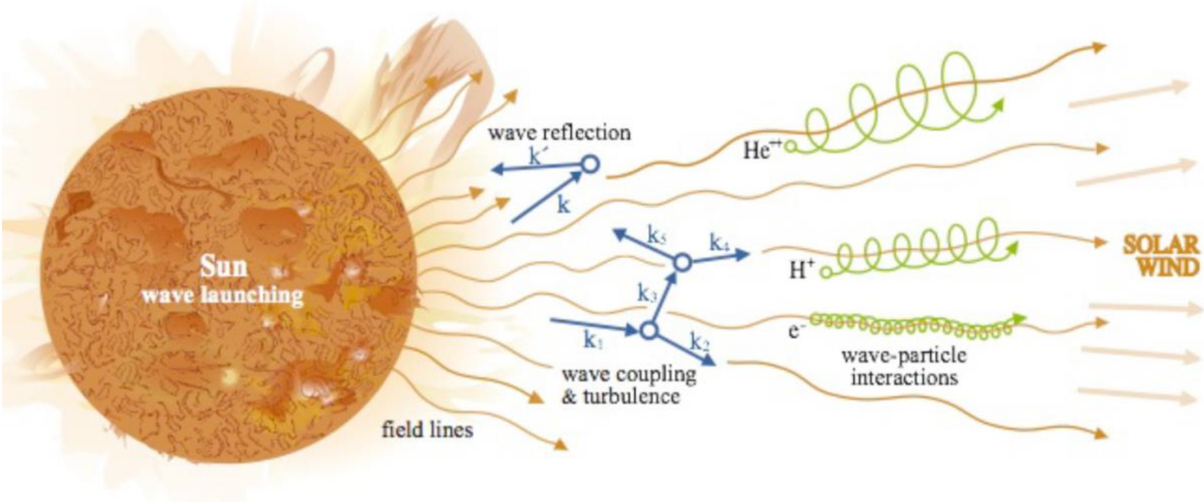
Transport equations bear a puzzling shape

- Evolution of thermodynamical variables **A**=(N,T,$M_{//}$)

$$\frac{\partial A}{\partial t} + \nabla \cdot \mathbf{\Gamma} = \mathbf{\Sigma} \qquad \mathbf{\Gamma} = \bar{\bar{\boldsymbol{L}}} \cdot \mathbf{\Lambda} + \mathbf{\Gamma}_{\mathrm{res}}$$

- Transport matrix $\bar{\bar{L}}$ Onsager symmetric. However :

- "Residual" momentum and energy fluxes $\mathbf{\Gamma}_{\mathbf{res}}$
- Sources **Σ**= turbulent heating and acceleration Rudakov 71, Ott 72



Bale 2012

Fluxes possess pinch and residual components

- Diffusion/convection flux structure

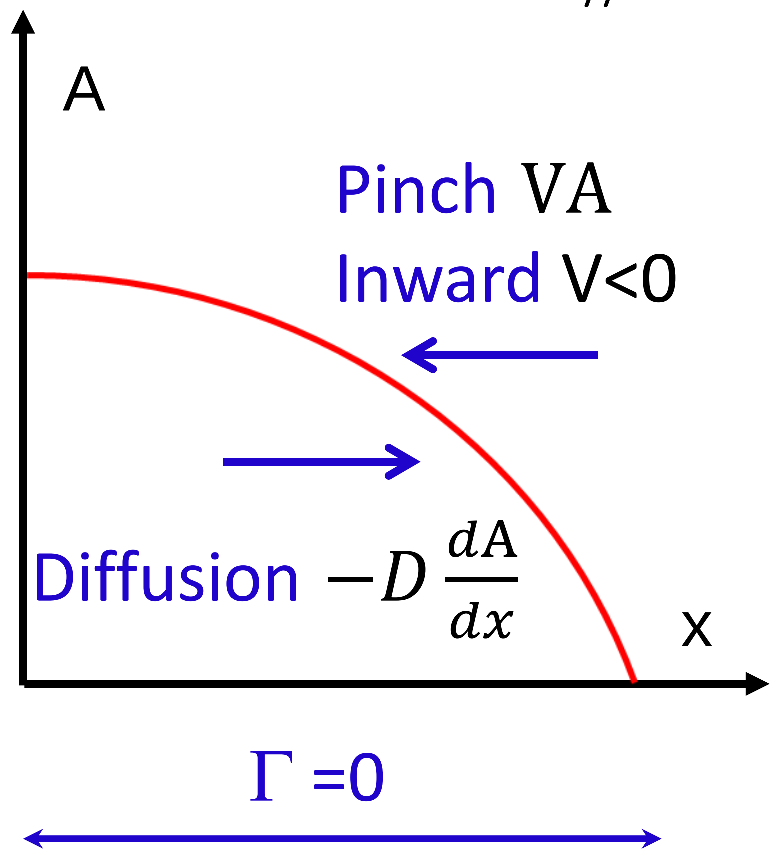
$$\Gamma = -D\frac{dA}{dx} + VA + \Gamma_{res}$$

- Diffusion = average of $\sum_{\boldsymbol{k}} D_{\boldsymbol{k}}$.

- Pinch velocity due forces other than $\frac{dA}{dx}$.

- Residual momentum and heat fluxes $\neq 0 \rightarrow$ requires symmetry breaking $\langle k_{\parallel} k_y \rangle \neq 0$.

A=(N,T,M$_{//}$)

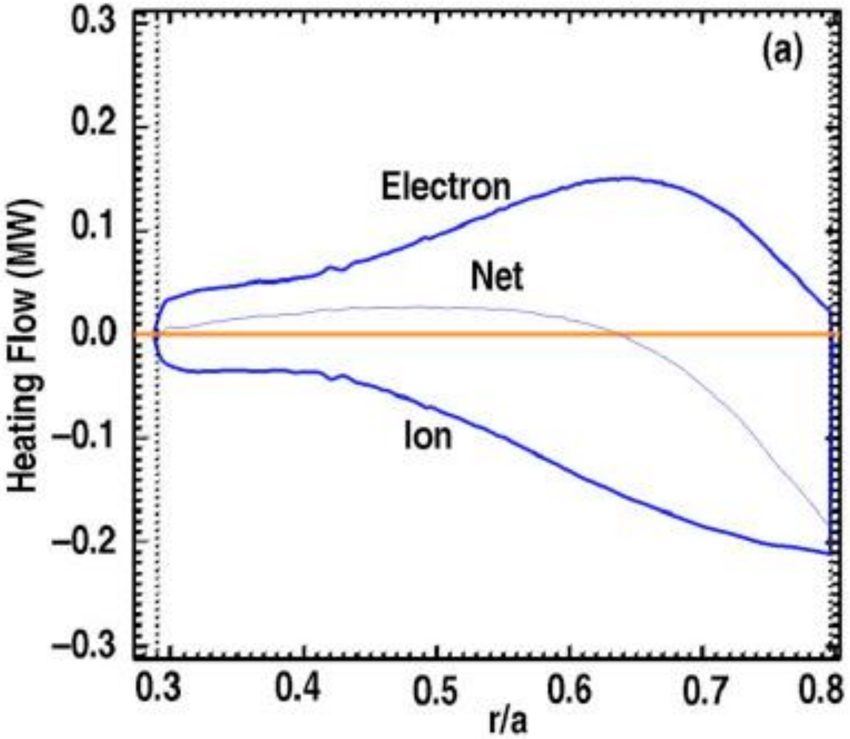


Plasma sped up and heated by turbulence

- Source terms Itoh 88, Hinton 06, Lu Wang 13, XG 13

$$\Sigma_{M\parallel} = \langle NeE_{\parallel} \rangle \qquad \Sigma_T = \langle Ne\boldsymbol{V}.\boldsymbol{E} \rangle$$

Waltz 11



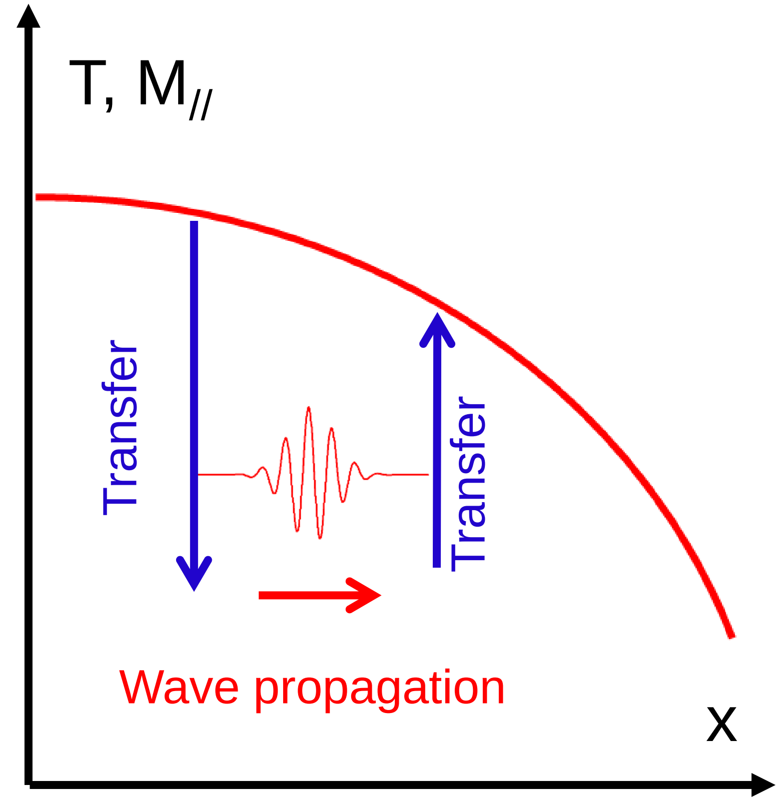
- Charge conservation → global conservation

$$\sum_{species} sources = 0$$

- Momentum and energy transfer between species Zhao 12

Residual fluxes and sources are Onsager symmetric

- Define new forces $\frac{1}{T}, \frac{M_\parallel}{T} \rightarrow$ transport matrix is symmetric Horton 80, Itoh 82, Sugama 96
- Total field + particle momentum/ energy is conserved $\rightarrow$ extended thermodynamics Boozer 92, Krommes 93, Watanabe 06, XG 12
- Sources = fluxes of momentum/ energy carried by waves Diamond 08



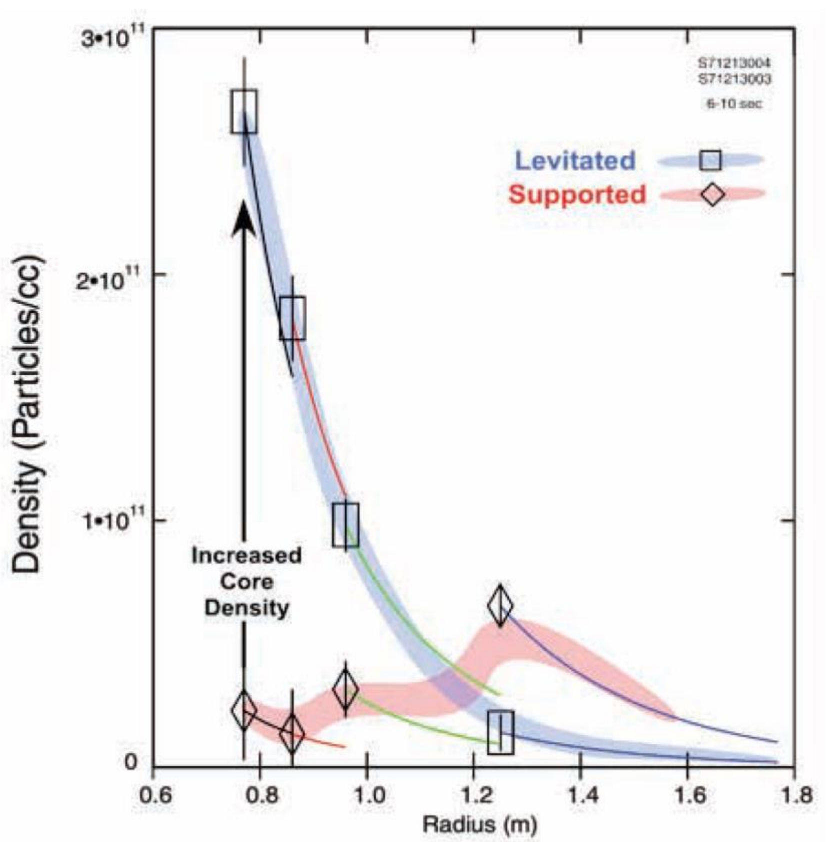
Magnetic drift contributes to pinch velocities

- Resonance $\omega = k_{\parallel}v_{\parallel} \rightarrow \omega = k_{\parallel}v_{\parallel} + \boldsymbol{k}\cdot\boldsymbol{v}_D \rightarrow$ introduces magnetic drift $\rightarrow$ pinch velocities proportional to $\nabla B/B$.

- Related to Lagrangian invariants $\rightarrow$ compressibility $\nabla.\mathbf{v}_E \neq 0$ Yankov 94, Isichenko 95&97, Baker 01, XG 04, Gürcan 10

$$\frac{d}{dt}\left(\frac{N}{B^2}\right)=0$$

Boxer 10 - LDX



Turbulent pinch theory successfully tested in tokamaks

- Particle flux $\Gamma = -D\frac{dN}{dx} + VN$
- Pinch velocity XG 04, Angioni 04 & 06, Camenen 09

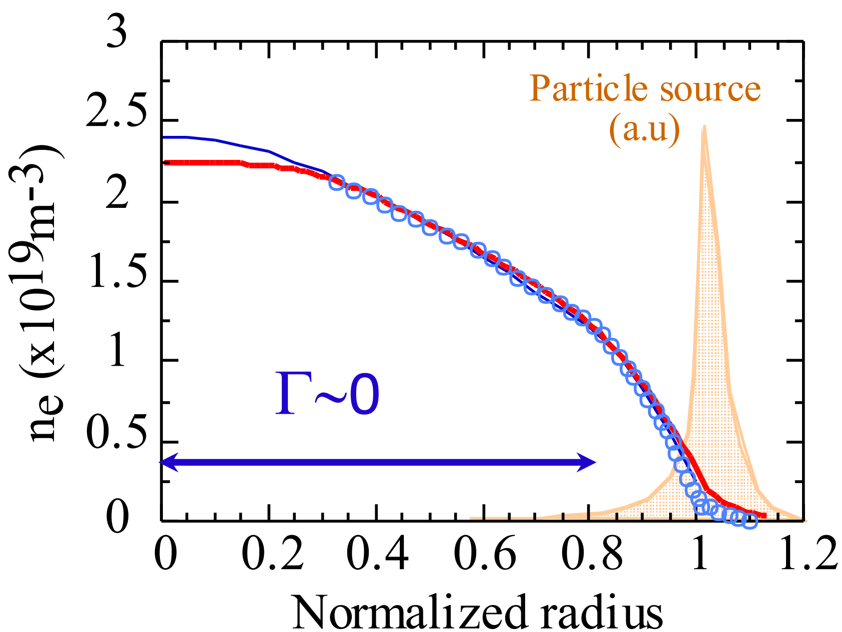
$$V = V_T\frac{dT}{Tdx} + V_{M\parallel}\frac{dM_\parallel}{dx} + V_B\frac{dB}{Bdx}$$

Thermo diffusion

Roto diffusion

compression

Hoang 04 - Tore Supra



- Onsager symmetry → thermal pinch Luce 92, Itoh 96, Mantica 05, Lu Wang 11

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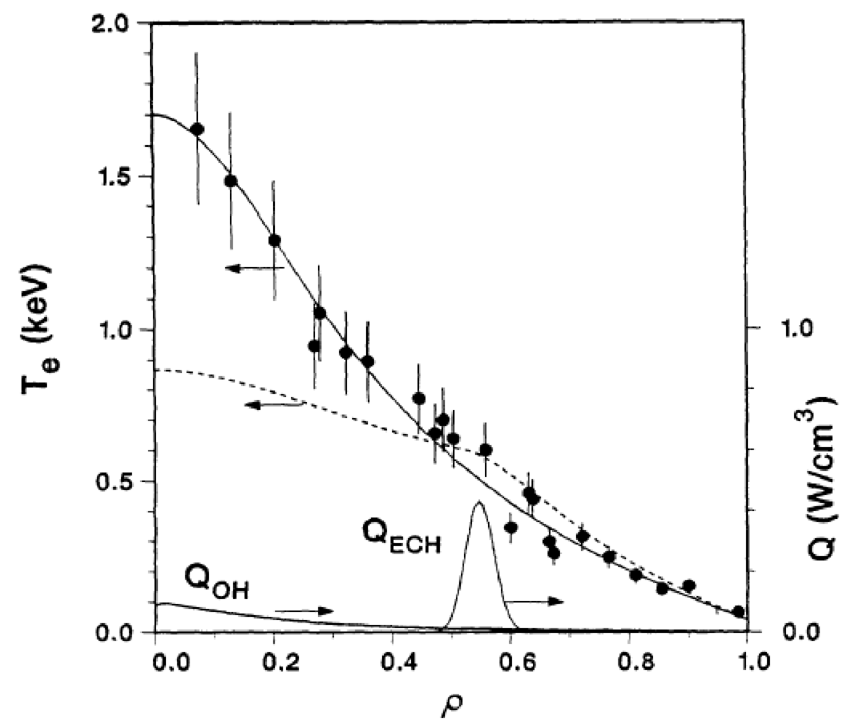
Luce 95 – DIII-D

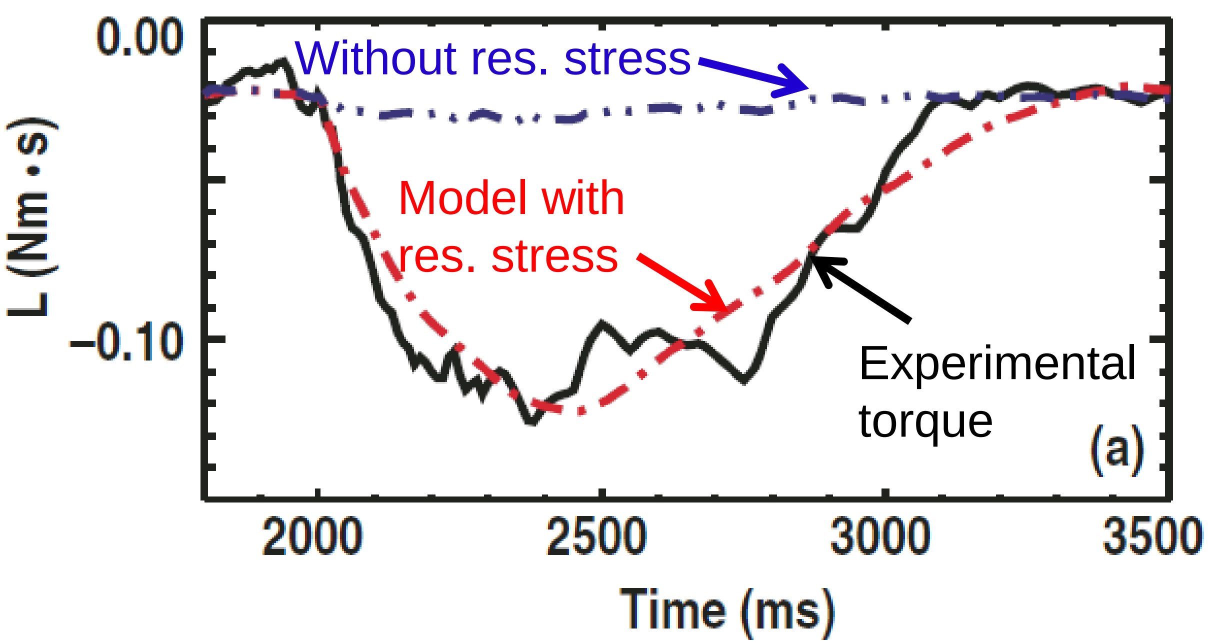


- Onsager symmetry → thermal pinch Luce 92, Itoh 96, Mantica 05, Lu Wang 11

Momentum flux has both pinch and residual components

- Plasma spin-up in tokamaks without external torque.
- Pinch and residual stress $\Gamma = -D\,\frac{dM_{\parallel}}{dx} + VM_{\parallel} + \Gamma_{res}$

OVs Diamond 09, Peeters 11, Ida 13, Tynan 19 CD-I9 entropy production Kosuga 10



Solomon 09 - DIII-D

Thermodynamics of non local transport is an open issue

- Non local transport Van Milligen 04, Del-Castillo-Negrete 05, Dif-Pradalier 10, OV Ida 15

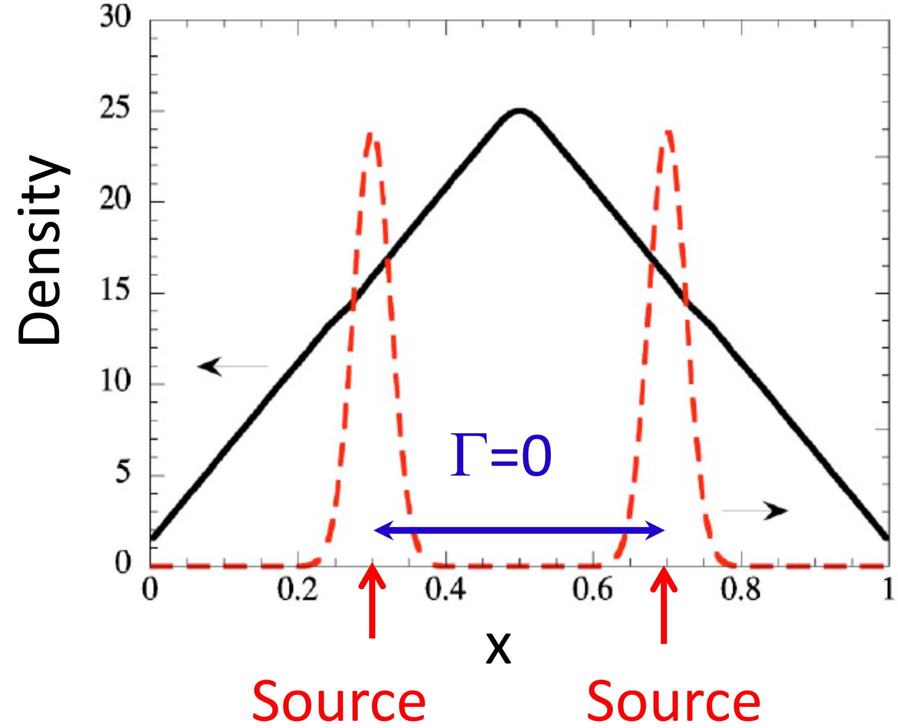
$$\Gamma(x) = -\int dx'\kappa(x - x')\nabla N\,(x')$$

- Pinch effect with single force Del-Castillo-Negrete 05, Bouzat 05.

- Second principle $\rightarrow$ Tsallis entropy Tsallis 88, Anderson 18

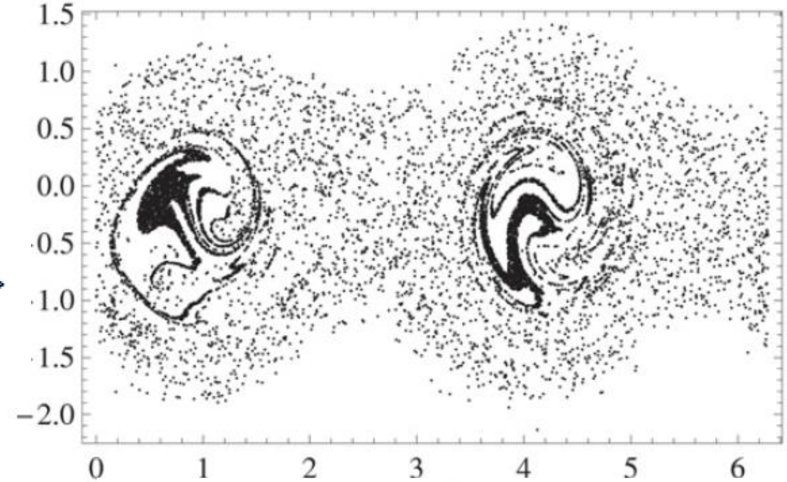
Onsager symmetry ?

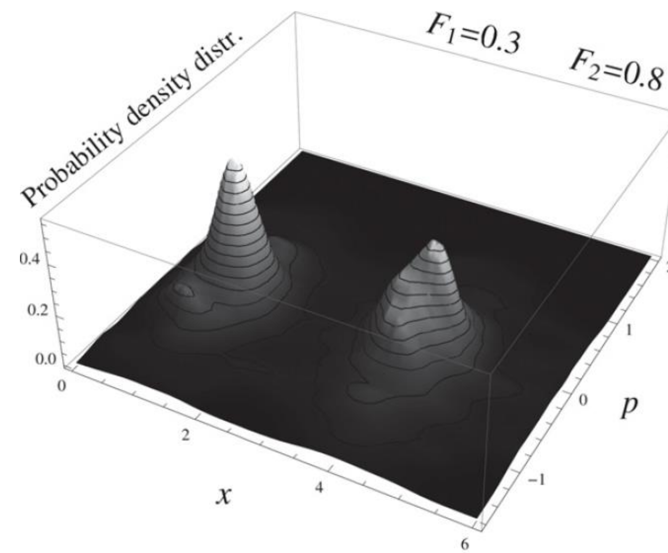
Van Milligen 04



Violent relaxation theory predict coherent states

Carlevaro 13





• Fast relaxation of a system with long range interactions. Entropy Lynden-Bell 67

$$S = -\int d\tau \left\{ \frac{F}{F_0} \ln \frac{F}{F_0} + \left(1 - \frac{F}{F_0}\right) \ln \left(1 - \frac{F}{F_0}\right) \right\}$$

• Maximum entropy with conservation constraints $\rightarrow$ Quasi-Stationary States Robert 92, Antoniazzi 08, Chavanis 06, Carlevaro 13

• Relaxation? Maximum entropy production principle? Martyushev 06

Conclusions

- Minimum of entropy production principle coupled to quasi-linear theory predicts fluxes vs forces in turbulent magnetised plasmas.
- Predicts pinches, residual contributions to energy and momentum fluxes, turbulent heating and acceleration.
- Onsager symmetry respected under conditions.
- May fail, typically in systems with long range interactions, with memory effects. Tsallis and Lynden Bell statistics offer alternatives.