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# UNCERTAINTY QUANTIFICATION OF DELAYED NEUTRON FRACTION OF U235 BASED FUEL CORES

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## ABSTRACT

The analysis of nuclear reactors behavior in transients depends among others on the effective delayed neutron fraction ( $\beta_{\text{eff}}$ ). The present paper aims at describing how to provide uncertainties for the effective delayed neutron fraction ( $\beta_{\text{eff}}$ ) to be used for safety studies. The use of the Iterated Fission Probability method in the Monte Carlo code TRIPOLI4 gives credit to deterministic codes such as ERANOS for calculating  $\beta_{\text{eff}}$ . The use of the Monte Carlo code TRIPOLI4 enables a better representation of experimental cores, especially the R2 experimental core which exhibit more heterogeneities for hosting experimental devices. Its use for evaluating the calculated parts of the  $\beta_{\text{eff}}$  has been found essential. The nuclear data uncertainty propagation has been leading to a 3.5% uncertainty. This 3.5% uncertainty is confirmed by the  $\beta_{\text{eff}}$  C-E bias for BERENICE R2 cores.

*Key Words:* **Delayed Neutron Fraction, IFP, TRIPOLI4, ERANOS.**

## 1. INTRODUCTION

The analysis of nuclear reactors behavior in transients depends among others on the effective delayed neutron fraction ( $\beta_{\text{eff}}$ ). Since the early days of civil nuclear power, the conservative approach has been used for the design and licensing of nuclear power plants (NPPs) and is still widely used today. However, the desire to maximize the economic potential of NPPs without compromising their safety has led many countries to use best-estimate codes and data together with an evaluation of the uncertainties. The present paper aims at describing how to provide uncertainties for the effective delayed neutron fraction ( $\beta_{\text{eff}}$ ) to be used for safety studies.

## 2. METHODOLOGY

The uncertainty quantification of effective delayed neutron fractions has to go a series of actions: the first ones are to define the uncertainty due to nuclear data and fuel compositions; the second one is to compare calculated results with experimental ones.

## 3. CALCULATION AND ITS SENSITIVITY

The effective delayed neutron fraction corresponds to the proportion of neutrons being generated by precursors (fission products) in opposition to prompt neutrons generated during the fissioning process.

With the help of the adjoint flux as mentioned first by G.R. Keepin [1] and using the perturbation theory [2,3], the  $\beta_{\text{eff}}$  can be calculated as if the perturbation was due to the delayed neutrons fission

operator  $F^d$  :

$$\beta_{eff}^{i,n} = \frac{\langle \Phi^*, F^d \Phi \rangle}{\langle \Phi^*, F \Phi \rangle} = \frac{\iiint_r d^3r. (\sum_g \chi_g^d \cdot \Phi_g^*) \cdot (\sum_g \nu_g^d \cdot \Sigma_{f,g} \cdot \Phi_g)}{\iiint_r d^3r. (\sum_g \chi_g \cdot \Phi_g^*) \cdot (\sum_g \nu_g \cdot \Sigma_{f,g} \cdot \Phi_g)} \quad (1)$$

Table 1. Isotope break down of Uranium core

Isotope	Delayed Neutron Fraction $\beta_{eff}$ (pcm)
<b>U235</b>	558.5
<b>U238</b>	182.3
<b>Total</b>	740.8

In Table 1, one can notice that U238 although with a fission threshold contributes significantly to  $\beta_{eff}$  since it exhibits more delayed neutrons than U235.

#### 4. $\beta_{EFF}$ UNCERTAINTY DUE TO NUCLEAR DATA

An accurate assessment of uncertainties on effective delayed neutron fractions must be estimated by considering the contributions of different nuclear data, including those from the delayed data (fission yield  $\nu_d$  and fission spectrum  $\chi_d$ ). The use of the generalized perturbation theory allows easy access to sensitivities of kinetic parameters to nuclear data. The generalized perturbation theory allows calculating sensitivities for bilinear functions of flux and adjoint flux by deriving the previous equation. The sensitivity is composed of two terms representing the direct and indirect effect. The direct effect corresponds to the variation of the integral parameter R with the cross-sections by which R is explicitly dependent. The indirect effect instead corresponds to the variation of the parameter R with other cross-sections by which R is implicitly dependent through the direct and the adjoint flux. The indirect effect needs the calculation of ‘‘importance functions’’.

$$S_{R,\sigma} = \frac{\sigma}{R} \cdot \left\{ \frac{dR}{d\sigma} - \left\langle \Psi^* \left| \left( \frac{\delta A}{\delta \sigma} - \frac{1}{K} \frac{\delta F}{\delta \sigma} \right) \cdot \Phi \right. \right\rangle - \left\langle \Psi \left| \left( \left( \frac{\delta A^*}{\delta \sigma} - \frac{1}{K} \frac{\delta F^*}{\delta \sigma} \right) \right) \cdot \Phi^* \right. \right\rangle \right\} \quad (2)$$

The contributions of direct term and indirect terms ( $\sigma_{fission}$ ,  $\nu$  and  $\nu_d$ ,  $\chi$  and  $\chi_d$ ) are calculated using the most recent data covariance suggested in COMAC (COVariance Matrices from Cadarache, version 0.1) [4].

The contribution of the direct effect to  $\beta_{eff}$  uncertainty is 2.5% as seen in Table 2. The main contributors are the fission and delayed neutron yield of U238 and delayed neutron yield of U235.

Table 2. Uncertainties (direct terms) on the effective delayed neutron fraction in %.

Isotope	Fis.	$\nu$	$\nu_d$	Total
U235	0.06	0.07	2.18	2.18
U238	0.88	0.03	0.82	1.20
<b>Total</b>	0.88	0.07	2.33	2.49

The contribution of the indirect effect to uncertainty is 2.7% (Table 3). The main contributors are the capture and fission of U235 and capture, fission and inelastic of U238. This is linked to the ratio fission U238 over fission U235 which drives the contribution of these 2 isotopes.

Table 3. Uncertainties on indirect-term in %.

Isotope	Cap.	Fis.	Elas.	Inel.	n,xn	nu	Tot.
O16	0.06	0.00	0.07	0.02	0.00	0.00	0.09
Na23	0.02	0.00	0.13	0.03	0.01	0.00	0.13
SS	0.09	0.00	0.11	0.03	0.00	0.00	0.15
U235	<b>2.31</b>	<b>0.42</b>	0.17	0.05	0.09	0.30	2.37
U238	<b>0.92</b>	<b>0.48</b>	0.11	<b>0.53</b>	0.15	0.09	1.18
<b>Total</b>	2.48	0.64	0.27	0.54	0.17	0.31	2.66

The result of these calculations is leading to a 3.5% uncertainty with 2.5% due to direct terms and 2.7% due to indirect terms.

## 5. BERENICE EXPERIMENTS

The BERENICE experimental programme (Beta Effective Reactor Experiment for a New International Collaborative Evaluation) took place at the zero power critical facility MASURCA from January 1993 to March 1994 [5]. Two R2 configurations have been loaded in the course of this programme both U235 based cores: a clean core and an experimental core. This last one exhibits more holes to host fission chambers and sources and hence has a stronger heterogeneity which was difficult to model with deterministic codes at the time. Different  $\beta_{\text{eff}}$  measurement techniques have been used:

- A Californium 252 source (spontaneous fission) inserted in different locations of the core in order to derive the experimental neutron importance.
- A noise method based on spectral analysis of neutron fluctuations in a "stationary" subcritical reactor.
- the Alpha-Rossi method similar to the noise one but looking at the statistical behavior of prompt neutrons over short periods of time (<1 ms).

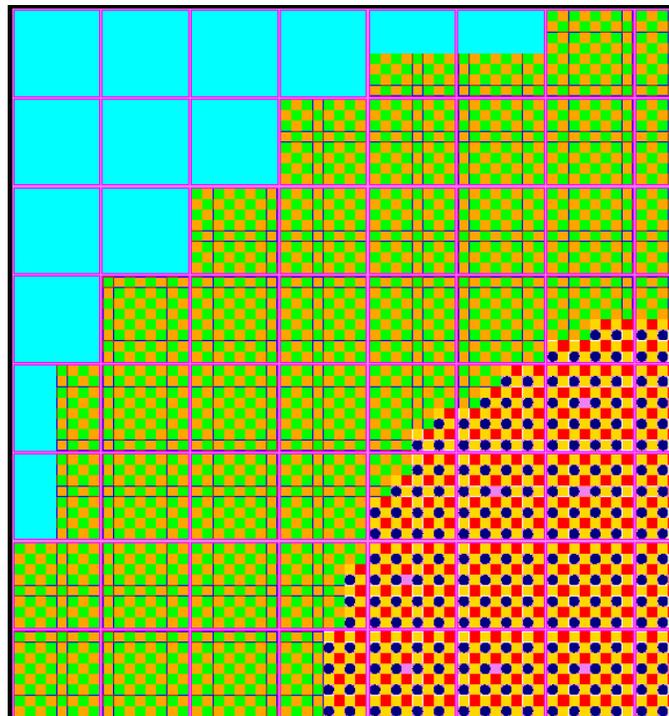
The experimental values (Table 4) have been derived from the raw measurements with some correction factors.

Table 4. Experimental results for BERENICE R2 cores

Method	Configuration	Source	Experimental values
<b>Californium Source</b>	R2	CEA	$761.5 \pm 7.7$
		JAERI	$747.3 \pm 4.5$
		IPPE	$753.1 \pm 31.9$
	R2 experimental	CEA	$780.1 \pm 9.3$
<b>Frequencies</b>	R2	-	$717.4 \pm 10.0$
<b><math>\alpha</math>-Rossi</b>	R2 experimental	-	$769.3 \pm 9.2$

Correction factors have been calculated with ERANOS, a deterministic code widely used for fast reactor studies.

With TRIPOLI4, it is possible to model very precisely both R2 cores: the clean one (Figure 1) and the experimental one. In particular, fuel end caps, experimental channels, steel blocks can be better modelled.



*The blue cylinders are representing the Uranium enriched pins, the yellow and orange squares the sodium boxes, red and green ones stainless steel rodlets*

Figure 1. R2 clean core as modeled with TRIPOLI4

As a consequence of this better modeling,  $K_{\text{eff}}$  C-E bias using JEFF3.1.1 [6] are more consistent with TRIPOLI4 than with ERANOS as one can see on Table 5.

Table 5.  $K_{\text{eff}}$  C-E comparisons for BERENICE R2 cores

Configuration	R2 clean core	R2 experimental core
<b>keff with ERANOS</b>	1.00635	1.01063
<b>keff with TRIPOLI4</b>	1.00083	0.99919
<b>keff experiment</b>	0.99883	0.99705
<b>C-E (ERANOS) in pcm</b>	752	1358
<b>C-E (TRIPOLI4) in pcm</b>	200±28	214±28

With Monte Carlo codes, the solution of the adjoint neutron transport equation is much more difficult because of the continuous-energy treatment of nuclear data. Consequently, alternative methods [7], which do not require the explicit calculation of the adjoint neutron flux, have been proposed. At first, Bretscher evaluated the  $\beta_{\text{eff}}$  as the ratio between the delayed and total multiplication factors (the k-ratio method). Because this method needs two different runs to produce a value it had been improved in TRIPOLI-4 to calculate  $\beta_{\text{eff}}$  in only one run [8]. Nauchi [9] introduced an evaluation of the importance of neutrons by evaluating the chance for each neutron to give rise to fission whereas Meulekamp's method is using the next fission probability event.

The Iterated Fission Probability Method [10] (IFP) definitely closed that pioneering approach by defining a function  $F(r,u)$ , called the iterated fission probability, according to the occurrences induced by a neutron in a reactor which is just critical: A neutron being introduced in the assembly at point  $r$  and with lethargy  $u$  will produce further fissions, each succeeding generation having a distribution closer to the actual power distribution in operating assembly. TRIPOLI4 has been using also to calculate  $\beta_{\text{eff}}$  with the newly IFP method [11, 12] (Table 6). Values are in good agreement with deterministic ones. K-ratio and Nauchi's  $\beta_{\text{eff}}$  however have been found insufficient.

Table 6.  $\beta_{\text{eff}}$  C-E comparisons for BERENICE R2 cores

Configuration	R2 clean core	R2 experimental core
<b><math>\beta_{\text{eff}}</math> with ERANOS</b>	740.7	740.1
<b><math>\beta_{\text{eff}}</math> with Nauchi method</b>	721 ± 3	720 ± 3
<b><math>\beta_{\text{eff}}</math> with improved k-ratio method</b>	722 ± 3	721 ± 3
<b><math>\beta_{\text{eff}}</math> with TRIPOLI4 with IFP method</b>	742 ± 6	741 ± 8
<b><math>\beta_{\text{eff}}</math> experiment</b>	744.8 ± 13.5	755.6 ± 9.5

## 6. CONCLUSIONS

The use of the Iterated Fission Probability method in the Monte Carlo code TRIPOLI4 gives credit to deterministic codes such as ERANOS for calculating  $\beta_{\text{eff}}$ . The use of the Monte Carlo code TRIPOLI4 enables a better representation of experimental cores, especially the R2 experimental core which exhibit more heterogeneities for hosting experimental devices. Its use for evaluating the calculated parts of the  $\beta_{\text{eff}}$  has been found essential.

The nuclear data uncertainty propagation has been leading to a 3.5% uncertainty with 2.5% due to direct terms and 2.7% due to indirect terms linked to the ratio fission U238 over fission U235 which drives the contribution of these 2 isotopes.

This 3.5% uncertainty is confirmed by the  $\beta_{\text{eff}}$  C-E bias for BERENICE R2 cores.

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