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To cite this version:

J. Baccou, J. Zhang, E. Nouy. Towards a systematic approach to input uncertainty quantification methodology. 17th International Topical Meeting on Nuclear Reactor Thermal Hydraulics (NURETH - 2017), Sep 2017, Xi'An, Shaanxi, China. cea-02437064

TOWARDS A SYSTEMATIC APPROACH TO INPUT UNCERTAINTY QUANTIFICATION METHODOLOGY

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ABSTRACT

Taking into account uncertainties is a key step in nuclear applications to ensure that a system cannot move towards unsafe conditions. This topic has already been addressed by several OECD/NEA projects such as UMS or BEMUSE. It appeared that uncertainty analysis methods were mature enough for industrial practices but also that an important effort should be done on input uncertainty quantification. Following this recommendation, the PREMIUM project was a first attempt to tackle this problem by benchmarking several available methods. However, the dispersion of the results prevented from reaching a consensus between participants on estimated input uncertainties. This work is therefore devoted to some recent developments related to the construction of a systematic approach for input uncertainty quantification and validation. It provides an original insight which is not based on a benchmarking of methods and codes but integrates a shared analysis of the different generic steps that should be followed to reduce (or at least understand) the discrepancy in method and process. After recalling the main conclusions from the PREMIUM project that motivated the proposed work, we will introduce the five key elements of the systematic approach. The paper will then describe the scope, methods and expected outcome of the SAPIUM project that is devoted to the full description of this type of approach.

KEYWORDS

Model Input Uncertainty, Quantification, Thermal Hydraulics Code, Validation.

1. INTRODUCTION

Uncertainty assessment associated with Best-Estimate (BE) calculations has become of prime importance in nuclear safety studies. If uncertainty propagation methods are now considered as mature for industrial applications, several open issues need to be tackled when dealing with input uncertainty quantification (IUQ).

In order to progress on this topic, the OECD/NEA PREMIUM [1] ("Post-BEMUSE Reflood Models Input Uncertainty Methods") benchmark (2012-2015) was organized as a first step towards the development and the application of model IUQ methods. However, the analysis of PREMIUM Phases III and IV has shown a large dispersion of participants' results. Moreover, the results were not satisfactory when moving from the FEBA tests to PERICLES tests that were used respectively to quantify and validate input uncertainties. One main reason could be attributed to the lack of common consensus and practices in the followed process and method. A main lesson learned from the project was that a systematic approach devoted to model input uncertainty evaluation (i.e. quantification and validation) should be developed to improve the reliability of the analysis and to ensure the extrapolation of its results to the NPP case. From a methodological point of view, a systematic approach has the advantage of providing a common and generic framework to facilitate both discussions between participants and applications to several industrial problems.

Therefore, a first investigation has led to the identification of five key generic elements that should be considered in the construction of a systematic approach. This five-element structure will be discussed in the frame of the CSNI recently approved SAPIUM project (2017-2019). The proposed work in SAPIUM will exploit the current state of knowledge and lessons learned from the previous benchmarks, as well as the industrial and regulatory practices. The objectives are first to share a common understanding about "good" practices for input uncertainty evaluation, and also to resolve the open issues identified in the PREMIUM benchmark.

This paper is organized as follows: Section 2 is devoted to an overview on the lesson learned from the PREMIUM project. They are used in Section 3 to motivate the construction of a systematic approach for input uncertainty quantification and validation. The five key elements that structure this approach are also introduced. Finally, in Section 4, we describe the new SAPIUM project.

2. LESSON LEARNED FROM THE OECD/NEA PREMIUM BENCHMARK

PREMIUM was addressed to model uncertainties quantification on the basis of so-called "intermediate" tests (ITs) and was focused on the physical models involved in the prediction of the core reflood, a main stage in the loss-of-coolant-accident (LOCA) scenario important for the safety demonstrations of NPP.

2.1. Main Results of the PREMIUM Benchmark

PREMIUM has been organized in 5 consecutive phases.

2.1.1. Phase 1: methodology review

In Phase I the participants presented their methods of model uncertainty quantification. They are fully described in Phase I CSNI report [2]. Two methods were offered to the participants and used by several of them: CIRCÉ [3], developed by CEA, and FFTBM [4], developed by the University of Pisa. GRS, IRSN, KAERI, PSI and Tractebel developed their own approach.

2.1.2. Phase 2: preliminary quantification of uncertainties

During Phase 2, a preliminary quantification of uncertainties and sensitivity calculations on the unblocked tests of the series I of the FEBA experiment [5] allowed to identify the most influential physical models and the associated parameters from the point of view of reflooding.

2.1.3. Phase 3: quantification of uncertainties of the most influential parameters

To perform the quantification of uncertainties, the number of tests exploited by participants varied from only one test to all six tests of the Series I of the FEBA experiment.

Regarding the type of measurement, all participants considered cladding temperatures and almost all of them quench front progression. Some of them took also into account pressure drop measurements. Only a few included measured data of water carried over.

The obtained uncertainties for the model parameters of different TH codes were related, with few exceptions, to wall heat transfer, interfacial heat transfer and interfacial friction. The results exhibited a large variability and discrepancy among participants as shown for interfacial friction in Fig. 1. The legend of the horizontal axis refers to the name of the TH code and the name of the method used for IUQ.

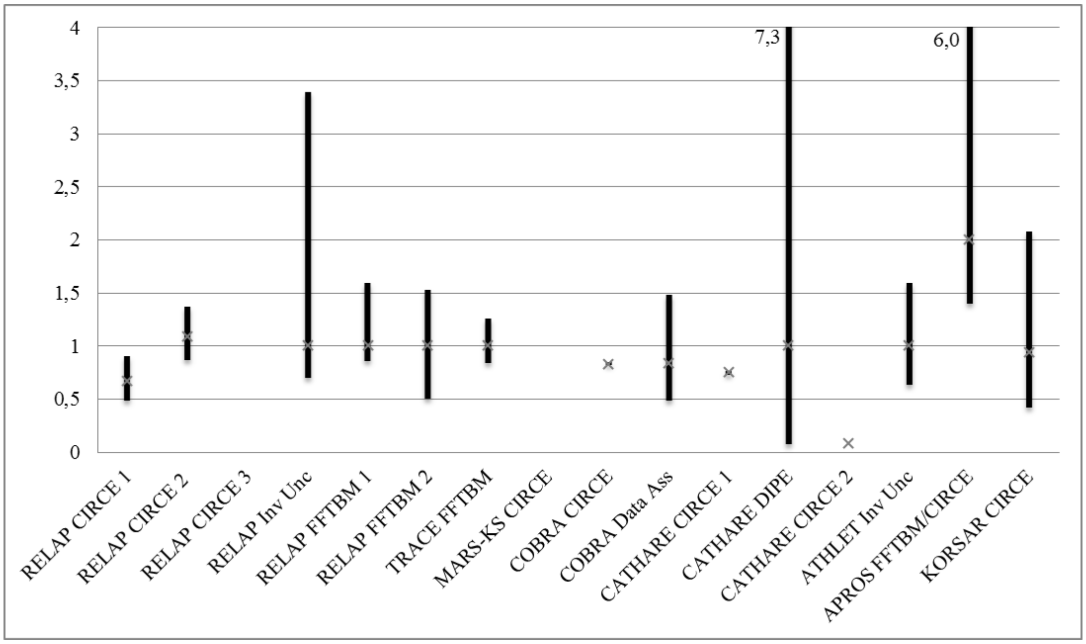

Figure 1. Quantified uncertainty ranges for interfacial friction.

2.1.4. Phase 4: confirmation and validation of the IUQ

To confirm and validate the estimated uncertainties, two experiments were used: the six unblocked FEBA tests already used for IUQ and six 2-D reflood tests from PERICLES facility [6], the main feature of this last step being that is was performed blindly. The PERICLES test facility consists of a test section with a larger number of fuel rod simulators and investigation of the radial power distribution effect in 4 out of the 6 considered PERICLES tests.

The input uncertainties quantified in Phase 3 were propagated through each TH code following a probabilistic approach: a probability distribution was assigned to each input parameter and the propagation was based on a Monte-Carlo method using a Simple Random Sampling (SRS) strategy. 200 code runs were recommended and the uncertainty band was defined by estimating the 2.5% and 97.5% percentiles thanks to order statistics [7].

A quantitative analysis using the IRSN synthesis method [8] was performed by computing two criteria called informativeness (that measures the precision of the uncertainty band) and calibration (related to the discrepancy between predictions and experimental values).

Their values for FEBA and PERICLES tests are shown in Fig. 2. Each point of the graph corresponds to a participant. It is specified the name of the TH code and of the IUQ method that she/he used. The green (resp. orange) colour stands for CIRCÉ users (resp. users of others IUQ methods). The blue (resp. brown) solid line connects the participants using RELAP (resp. CATHARE).

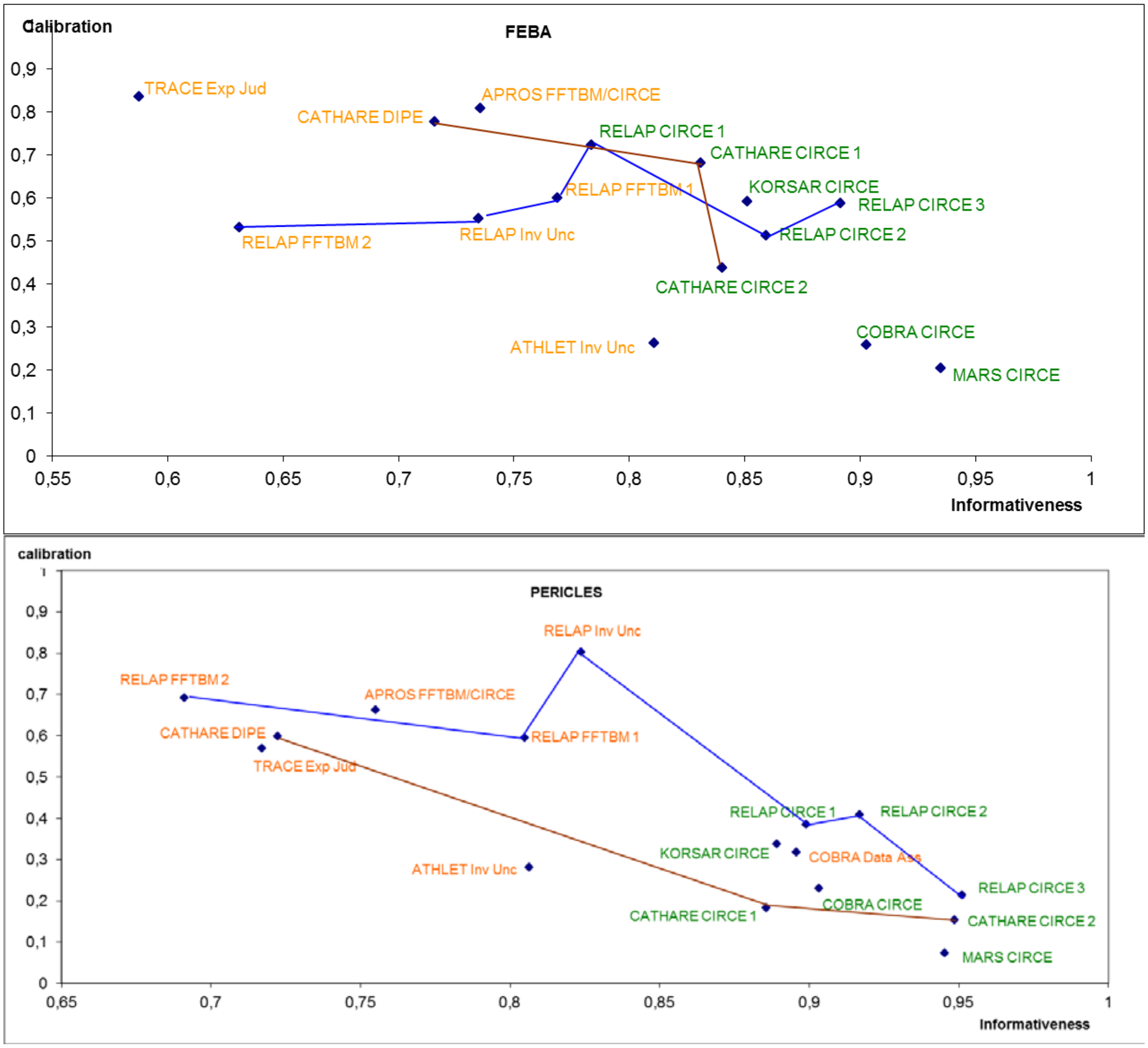

Figure 2. Informativeness and calibration for FEBA and PERICLES.

For FEBA tests, most of the uncertainty bands envelop the experimental values but with varied width.
The results on PERICLES were less satisfactory compared to the FEBA ones. The negative correlation between the two criteria (informativeness and calibration) that is noticeable on the bottom graph of Fig. 2 means that it was unlikely to find a participant with both high informativeness and calibration. In other words, narrow uncertainty bands were not able to encompass most of the PERICLES experimental values. On the contrary, participants who succeeded to envelop the experimental values had wide, even very wide, uncertainty bands.

Finally, the results do not clearly exhibit a code effect (see solid lines in Fig. 2). They appeared to be more method-dependent than code-dependent.

2.2. Phase 5: Main Conclusions and Lessons Learned

All PREMIUM results are detailed in the final report [9]. Among the important conclusions, one can mention that this activity has been a valuable test bed for input uncertainty quantification. However, it was also noticed a large dispersion in the results that prevented from reaching a consensus on the quantified input uncertainties. Moreover, PREMIUM did not demonstrate a high capability for extrapolation from FEBA to PERICLES.

These two last conclusions could be explained by different choices in the quantification process that strongly affected the final results. They concerned:

- The selected outputs of interest used in the quantification,
- The selected input parameters to be quantified,
- The selected experimental database for quantification and validation,
- The code modelling and the numerical implementation,
- The quantified models, which, in general depend on the TH code being used,
- The quantification method including different assumptions related to the input uncertainty modelling (interval/pdfs, type of pdfs, calibration or not of the reference calculation).

In the following sections, we summarize the main lessons learned on these topics.

2.2.1. Information formalization for input uncertainty evaluation

The selection of the outputs of interest used for input quantification depends on the user. Usually safety related quantities are chosen as outputs of interest but they must be enough diversified and directly or indirectly connected to the model parameters. The source of some participant’s poor results was clearly identified as their choice of experimental outputs.

The selection of input parameters whose uncertainties should be evaluated needs to be clarified in order to reduce the discrepancy between users (even users of the same TH code). It can be organized in a hierarchical structure, so that an individual model encompasses several sub-models or correlations. The different results obtained by quantifying different sub-models or by quantifying the complete model via a global multiplier should be analyzed.

The set of quantified input parameters should include the most influential ones on the responses; otherwise the resulting uncertainty may be completely misleading. But, it is reasonable to ask if in other applications the set of identified parameters would be the same or different and to wonder if it would be advisable to include in the quantification all potentially important model parameters, not only the most influential ones.

The PERICLES results are in part explained by the test conditions and the geometry for FEBA which are rather different from those of the PERICLES tests. They are less representative of a reactor reflood, due to the smaller size of the bundle and the absence of radial profile.
Therefore, it appears important to quantify the model uncertainties based on well selected large and representative assessment data, in order to better cover the conditions of the reactor application.
The database will define the range of validity of the quantified uncertainties and a compromise must be found between a specific (related to the foreseen application) and a generic (applicable to a wide spectrum of simulations) position.

PREMIUM demonstrates that the code modelling and the numerical implementation affect significantly the IUQ process. The use of a different modelling (1-D/3D) for the two facilities or an important difference in the number of meshes appear to be inappropriate.

Moreover, the quality of the nominal calculation is important to succeed in the envelop calculations especially if the uncertainty bands are narrow.

2.2.2. Input uncertainty evaluation: quantification and validation

Experimental databases are made up of separate effects tests (SET), integral effects tests and the so-called intermediate tests (IT). Data from SET are useful for the quantification of “simple” models, e.g. including a single model parameter. In the case of more complex models, effects are difficult to separate, and intermediate tests are more appropriate for the quantification. Taking into account different scales in a database should be also deeply studied.

Almost all PREMIUM participants estimated that experimental uncertainties for FEBA were secondary comparing to the model related uncertainties and decided not to consider them in the quantification process. However, this type of uncertainties should be carefully examined to ensure that they are not influential on the quantification.

Quantification methods may have the option of performing calibration, as well as quantification, of the models. Applying such option means a recalibration of the code. Some participants compared results with and without the recalibration option, concluding that results of the extrapolation were improved when there is no recalibration. This outcome is coherent with the “best estimate” qualification of the TH system codes.
However, quantification methods should not be applied to magnitudes having full physical meaning (initial or boundary conditions, material properties …), unless there is no other source of information about their uncertainty.

The quantified uncertainty obtained for a specific parameter strongly depends of the total set of simultaneously quantified parameters. This means that quantified uncertainties are attributes of the total set of parameters, rather than intrinsic properties of individual parameters. Extrapolation of quantified uncertainties may lead to erroneous results.

Another essential step that was partially addressed during PREMIUM is the validation of input uncertainties that should become a mandatory stage in the IUQ process. It first includes the selection of a set of experiments in the experimental database that are not used for the quantification. It also requires to describe the main properties of a “validated” uncertainty band and to develop technical tools to check them.

3. PROPOSAL FOR A SYSTEMATIC APPROACH

Based on the experience feedback from this previous benchmark, we proposed to investigate in further developments if a systematic approach devoted to model input uncertainty evaluation (i.e. quantification and validation) could improve the reliability of the analysis and ensure the extrapolation of its results to the NPP case.

From a methodological point of view, a systematic approach has already the advantage of providing a common and generic framework to facilitate discussions between participants and applications to several industrial problems.

Such a type of approach is already widely spread in industrial applications where it is used by experts and engineers for the choice and calibration of physical models in computer codes, for V&V procedure of computer codes [10] and also for evaluation of uncertainties associated with code calculations (e.g. CSAU, [11]) . Moreover, Regulatory bodies integrate code and method development and assessment process in their regulatory guides (e.g. EMDAP, [12]). Finally, the development of new procedures for the treatment of model uncertainty remains an active research field of interest ([13], [14]).

The proposed systematic approach is summarized in 5 key elements, as shown by Fig. 3.

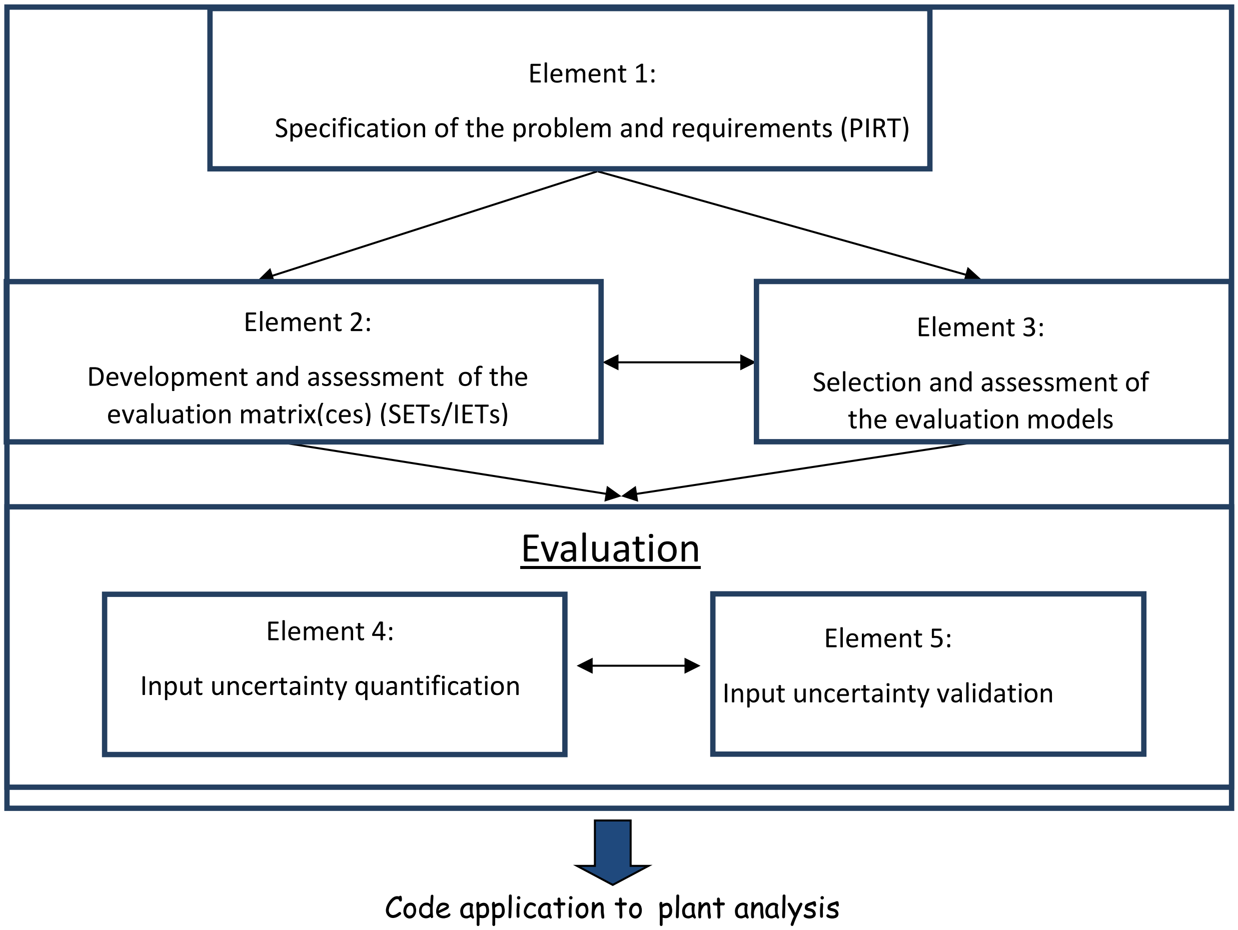

Figure 3. The 5 key elements of the systematic approach.

Element 1 allows to share/check a common understanding between participants on the problem to analyse. It includes the definition of the objectives of the evaluation (e.g. quantify and validate the uncertainties of the reflooding heat transfer models for application to plant analysis), the selection of a NPP and a scenario as well as the code outputs of interest and the important physical phenomena thanks to a PIRT.

Element 2 is related to the construction of the experimental database for input uncertainty quantification and validation that will control the capability of the method to extrapolate its results to real situations. It should be based on available SETs and IETs but can also require extra experiments if necessary. It includes the assessment of adequacy of an experiment and of completeness of an experimental database. At the end of this step, a ranking between experiments within the database could be performed using mathematical tools for data analysis ([15], [16]).

One important issue of this element concerns the question of dependency of the experimental database with respect to the reactor transient. More precisely, if the database is dependent on the reactor transient, this transient is divided into several « parts » within which the uncertainty of a given model should be evaluated. These parts are defined according to several possibilities that may be combined:

- Different components of the reactor (e.g. vessel, downcomer, cold leg, etc.).
- Different period of times (e.g. depressurization, refilling, reflood, Safety injection, … for a LB-LOCA).
- Several macro-phenomena (e.g. reflood, condensation in the cold leg, break flow, etc.) or
- thermalhydraulic conditions (power, pressure, flowrate, quality, …).

Uncertainty quantification should be performed for each “part” and may be validated both parts by parts and globally on IET

If the database is designed independently from the reactor transient, the uncertainty of any physical model depends only on the thermal hydraulic conditions and one solution could be to quantify the uncertainties using all SETs and to validate them on every transient and every reactor (all IETs).

A consensus on the type of strategy to follow is an important step of the systematic approach.

Element 3 is related to the code. It consists in assessing the applicability of the code for modeling the identified important phenomena as well as for modeling the considered SETs/IETs. It requires to consider nodalization strategy and model option selection that should be consistent between the experimental facility and similar components in the nuclear power plant. A special attention should be also devoted to the construction of error metrics (to evaluate the accuracy code/experiment) and the definition of a scale of accuracy. Finally, uncertain model input parameters have to be identified in this element.

Element 4 consists in inferring from the experimental knowledge, the information related to input uncertainties. The experimental knowledge is here associated with a subset of the database constructed in Element 2 (the remaining subset will be used for input uncertainty validation). It then requires to select a set of differences between code calculation and experimental value. Finally, the inference can be performed. Besides the choice of the input uncertainty quantification method (FFT, Monte Carlo, Bayesian, …), an appropriate uncertainty modeling for each uncertain input (interval, pdf, possibility,…) should be done by taking into account the real state of knowledge (nature of uncertainty and available information) and by reducing as much as possible extra assumptions. Key questions of this element are also related to the strategy to follow in presence of several experiments (quantification per experiment or a unique quantification for all experiments considered together?) as well as in case of several quantifications (how to combine input uncertainties, keeping in mind that several options exist?).

Element 5 is based on the propagation of the input uncertainties obtained in Element 4 through the computer code. It can be included in an iterative process with Element 4. It exploits the remaining subset of the experimental database identified in Element 2 and not used in Element 3. The propagation first implies the selection of an uncertainty model for each uncertain input (interval, pdf, possibility,…) that can be different from the uncertainty modelling associated with Element 4. Moreover, the input sampling procedure should be specified as well as the quantities of interest derived for the output sample that will be used for validation (e.g. percentiles in the probabilistic framework). Finally, a key point of this step is the definition and computation of validation metrics. It requires to reach a consensus on the definition of "validated uncertainty bands" (i.e. which important properties an uncertainty band has to satisfy to be accepted) and to introduce relevant criteria that mathematically translate this definition.

It should be noted that Elements 1-3 are common to any BEPU methodology based on CSAU or EMDAP, focusing on the application of a quantified (fully verified and validated, with model input uncertainties quantified and validated) code for accident analysis. The good practices from those industrial development and applications will be taken in this framework.

The PREMIUM benchmark has been devoted to Elements 4 and 5. The sources of discrepancy between participants recalled in Section 2.2 are included in Elements 1,2,3 and 4 respectively. Therefore, a "top-down" systematic approach can be seen as a way to share a common understanding about "good" practices for input uncertainty evaluation.

4. THE OECD/NEA SAPIUM project

This approach will be fully described in the frame of the new OECD/NEA SAPIUM (Systematic APproach for Input Uncertainty quantification Methodology) project. The main objective is to progress on the issue of quantification of the input uncertainty of physical models in computer codes. In general this activity addresses thermal-hydraulic safety codes, e.g.: uncertainty of physical models in thermal-hydraulics system or sub-channel codes in multiphysics calculations.

This project can be considered as a follow-up of PREMIUM. However, there are two main differences between the two activities. As mentioned in Section 2, PREMIUM was focused on the applications of methods for model uncertainties quantification (Element 4 in Figure 3) whereas SAPIUM is dedicated to the whole process of model input uncertainties quantification. Moreover, contrarily to PREMIUM that was a comparison of methods on the basis of "intermediate" experiments (FEBA and PERICLES), the outcome of SAPIUM is a methodological document: simple additional studies can be performed by interested participants but with the objective to get reliable insights into methodological key issues. More precisely, the SAPIUM methodology is a top-down approach that will cope with the analyses of all elements of its construction (Fig. 3) including:

- Identification of model uncertainties;
- Preparation of experimental basis for quantification and validation of analysed uncertainties;
- Application of technical tools for models uncertainties quantification and validation.

It will exploit the available state of knowledge coming from previous OECD/NEA projects as well as current practices in research and industries.

In order to refine the structure of the project, a questionnaire was sent to potentially interested organizations. Its objective was to obtain a better overview about existing knowledge and experiences in the field of model uncertainties quantification and validation. According to the answers, it appeared that most of the efforts should be done in the developments of Elements 2 to 5. Among important key issues that should be addressed to fully describe them, one can mention for example

- the construction and assessment of experimental matrices as basis for model uncertainties quantification and validation (Element 2),
- the construction of error metrics to evaluate the accuracy code/experiment and the definition of a scale of accuracy (Element 3),
- the aggregation of the information coming from different experiments to be used in the quantification (Element 4),
- the construction and computation of criteria to evaluate the information on input uncertainties in the validation process (Element 5).

The first point has already been mentioned during the PREMIUM benchmark. The doubts about representativeness of FEBA tests, used in PREMIUM for models uncertainties quantification, for PERICLES experiment was recognized as an important reason of problems with benchmarking of the methods of quantification.

To summarize, the SAPIUM project will lead to the development of a systematic approach that clearly compiles the different practices and offers a shared understanding about "good" practices for input uncertainty quantification in order to improve the reliability of the analysis and to progress on the validity of extrapolation of its results to the NPP case. The generic structure of the developed methodology will provide a common and generic framework to facilitate discussions between engineers with applications to several industrial problems. The final report will be a first “good practices” document for input uncertainty quantification and validation. It can be used for safety study in order to reduce user effect and to increase the agreement among experts on appropriate practices as well as on remaining open issues for further developments. End users are research institutes and universities, manufacturers, utilities and safety authorities. In other words, they are the developers and the users of BEPU approaches, as well as the organizations in charge of evaluating these approaches.

The SAPIUM project is organized as a writing group including, up to now, IRSN (France), Tractebel (Belgium), CEA (France), NINE (Italy), GRS (Germany), UPC/CSN (Spain), EDF (France), JAEA (Japan) and KINS (Republic of Korea). Participants are expected to provide their own experience on input uncertainty quantification and validation through contributions to the writing of document sections and to the review of the final report. Moreover, if interested, they can perform short applications to facilitate the discussions and reach a consensus on the SAPIUM process.

The SAPIUM project will last two years (January 2017 – January 2019). The kick-off meeting held in Paris, France, on 26-27 January 2017 allowed establishing a first table of contents of the methodological document with the list of contributors. The next meeting is planned in Paris on 29-30 May 2017 and should lead to a refined table of contents with an exhaustive list of key issues and open questions to be addressed. The objective is to prepare the good practices document by mid of 2018.

5. CONCLUSIONS

Some recent developments for input uncertainty quantification and validation have been introduced in this paper. Based on the lesson learned from the OECD/NEA PREMIUM project that showed a lack of common method and process between participants, we have proposed a new systematic approach that includes five steps to handle in order to derive reliable input uncertainties. The advantage of such an approach is to define a common framework for practitioners that will reduce the user-effect noticed during the PREMIUM activity.

A refined description of the five elements of the systematic approach will be performed in the frame of the new OECD/NEA SAPIUM project. The final objective is to write a first “good practices” document to drive input uncertainty studies by exploiting the available state of knowledge coming from previous OECD projects as well as current practices in research and industries.

NOMENCLATURE

BE	Best-Estimate
CIRCÉ	Calcul des Incertitudes Relatives aux Corrélations Élementaires (it can be translated into English by: « Calculation of the Uncertainties Related to the Elementary Correlations »)
CSAU	Code Scaling, Applicability and Uncertainty
EMDAP	Evaluation Model Development and Assessment Process
FFTBM	Fast Fourier Transformation Based Method
IT	Intermediate Tests
IUQ	Input Uncertainty Quantification
LOCA	Loss Of Coolant Accident
NPP	Nuclear Power Plant
PREMIUM	Post-BEMUSE Reflood Models Input Uncertainty Methods
SAPIUM	Systematic APproach for Input Uncertainty quantification Methodology
TH	Thermal-Hydraulic
V & V	Validation and Verification

ACKNOWLEDGMENTS

The authors wish to acknowledge all the participants of the PREMIUM and SAPIUM benchmarks for their inspiring contributions and discussions. They are also grateful to their respective organizations (IRSN, ENGIE and CEA) for their support for participation, and to OECD/NEA for organization of the benchmarks.

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