A search for theories enabling analyses of spatial effects in highly coupled sfr cores

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A SEARCH FOR THEORIES ENABLING ANALYSES OF SPATIAL EFFECTS IN HIGHLY COUPLED SFR CORES.

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ABSTRACT

SFR are a solution for the future of nuclear energy. The safety constraints brought by nowadays standards, especially about Sodium void reactivity effect, require core designs which are highly heterogeneous axially and rather flat. These features require an in depth analysis of spatial effects which involves appropriate methodologies. This paper presents a comparison of results obtained with first order perturbation, flux harmonics and coupled-core theories. This last one presents some advantages compared to the other ones for understanding the spatial effects of large SFR cores, and for propagating nuclear data uncertainties.

Key Words: **Flux Harmonics, Perturbation Theory, Coupling Coefficient.**

1. INTRODUCTION AND THE STUDIED GEOMETRY

In nuclear reactors, the in-core power distribution continuously changes due to the control rod movement, the reactivity feedback effects and fuel burn-up. The modal expansion is a very convenient method to characterize these changes. However, recent works show that hundreds of flux harmonics may be needed to properly describe the spatial dependence [1]. Even if the harmonics may give valuable information on the reactor sensitivities, they are insufficient for quantitative purposes. Nevertheless, the *EigenValue Separation* (EVS) is a crucial parameter which strongly influences the responses of the model. The EVS appears naturally when deriving the first-order theory [2]. It is also an indicator of *coupling effects* in a core. To show this relation, the definition of K. Kobayashi [3] is chosen due to its simplicity and physical meaning.

The studied case is a 1D slab model treated with the diffusion assumption. 1D geometry was chosen in a demonstration purpose, but the method may be applied with 3D-transport codes without any major development. Figure 1 presents the model studied, based on an inner core sub-assembly of the CFV (low void effect core) design [4]. This evolutionary core design combines various types of geometrical options, each of them favorable to the sodium void effect reduction like internal blanket zone, upper Na plenum, upper absorbing zone, small core height…

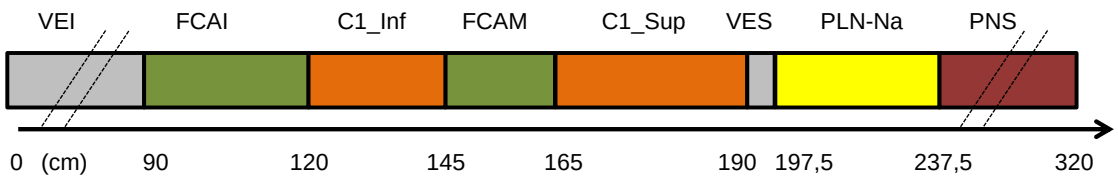

Figure 1. 1D Slab Model representing a inner core sub-assembly of the CFV design.

During the irradiation, the fertile zones become fissile ones, due to the U-238 capture reaction and the Pu weight proportion in each zone changes. Several isotopic concentrations are used to match various operating conditions. This isotopic change directly affects the neutron properties of the system, and therefore its sodium void effect. The sodium void worth is defined by the reactivity change between the sodium voided and nominal states.

In order to check the validity domain of the first-order perturbation theory, we are both interested in total void effect, but also in the sodium density effect. That is why we consider three void intensities. The remaining sodium density is respectively 0%, 50% and 90%. In this work, three voiding scenarios are specified. The 1st one deals with the absorption component by voiding the fissile and fertile regions, and is positive. In the 2nd one, we are interested in the leakage component (negative): all regions above the upper fuel zone are voided. The 3rd scenario is the combination of the two others.

Results from direct computations with modified cross-section depending on the sodium density are given in section 2. They are the reference, and some integral indicators are proposed to compare the voiding scenarios. Section 3 recalls the formalism of the standard perturbation theory based on flux harmonics expansion. Section 4 is devoted to the coupled core theory and its application to match any kind of perturbation. Finally, the performance of both approaches is discussed in the final section, which also gives some possible applications of the new coupled core perturbation theory, especially for uncertainty analysis on the power shape distribution.

2. RESULTS FROM DIRECT COMPUTATIONS

The Sodium void effects are characterized by a reactivity variation and a power distribution change. In order to compare the void worths, we use the concept of associate critical reactor. This concept consists in dividing all production cross-sections by the multiplication coefficient k_0, so that the nominal state becomes critical for all isotopic conditions. Then, to describe the flux changes, we chose an indicator based on the fission source vector. The fission sources are scalar information, integrated over the energy, and the directions (because emission of neutrons by fission is isotropic). For a given flux, this vector provides the number of neutrons emitted by fission in the reactor. The sources are then defined over fissile and fertile regions where fission may occur. We normalize this vector respect to equation (1). Figure 2 gives the shapes of the reference fission sources depending on the isotopic concentrations of the model. For fresh subassemblies, without Pu in the fertile regions, the axial tilt is more significant than for spent sub-assemblies, which leads to higher void effects (see Table 1).

$$S(\vec{r}) = \int_E dE \times \nu\Sigma_f(\vec{r},E) \times \Phi(\vec{r},E) \quad \rightarrow \quad \int_{System} d^3r \times S(\vec{r}) = 1.0 \qquad (1)$$

A perturbation in the Sodium density will change these fission sources. After normalization of this perturbed vector, we define the difference value between modified and nominal states. The integral (or algebraic sum) of this difference vector between two normalized vectors is null. But we can consider two parameters defined by equation (2):

- the $\|\Delta S\|$ norm counts the number of fission chains which are changed in the perturbed case compared to the reference. It is an **integral measure** of the flux distortion due to the perturbation.
- the ΔS_{Norm} normalized vector is the expression of the local changes in the reactor. It is **local information** about the power distribution change in the reactor.

$$\Delta S(\vec{r}) = S_{Pert}(\vec{r}) - S_{\mathrm{Re}\,f}(\vec{r}) \quad \rightarrow \begin{cases} \int_{System} \Delta S(\vec{r}) = 0 \\ \|\Delta S\| = \int_{System} |\Delta S(\vec{r})| \end{cases} \rightarrow \Delta S_{Norm} = \frac{1}{\|\Delta S\|} \times \Delta S \qquad (2)$$

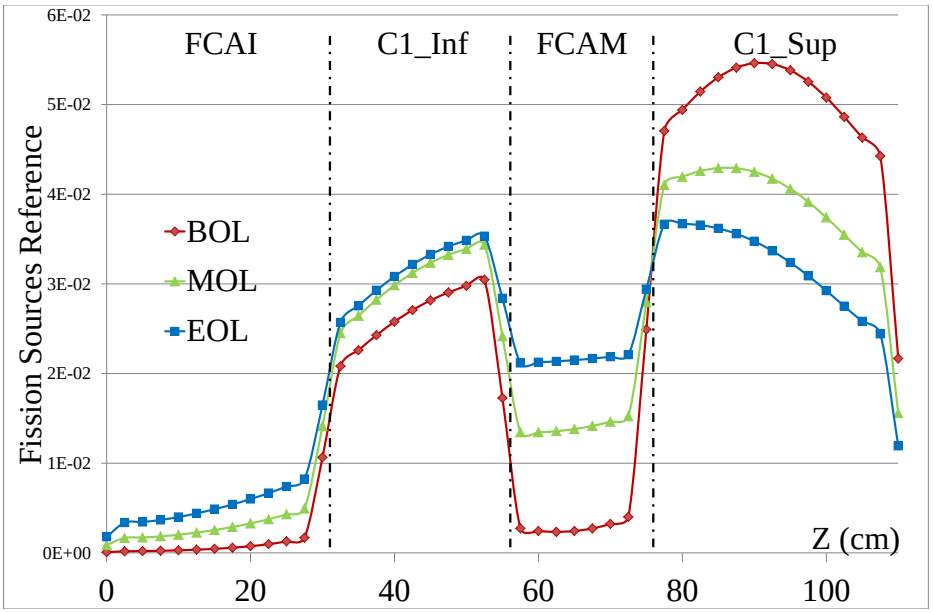

Figure 2. Results of direct reference calculations: the reference normalized fission sources. Plenum is on the right side of all pictures, out of the printed region. BOL : Beginning Of Life ; MOL : Middle Of Life ; EOL : End Of Life.

Table 1. Results of the direct computations (integral parameters). EVS: EigenValue Separation (3).

	Residual Fraction of Na (%)	Voided Fuel		Voided Plenum		Total Void Effect	
		$\Delta\rho$	$\|\Delta S\|$	$\Delta\rho$	$\|\Delta S\|$	$\Delta\rho$	$\|\Delta S\|$
BOL	0	1554	0.04259	-4452	0.16913	-3027	0.20304
k_0 = 1.09028	50	687	0.02078	-1588	0.05701	-914	0.07514
EVS = 4.75	90	131	0.00414	-265	0.00939	-134	0.01308
MOL	0	2021	0.01696	-2774	0.14261	-949	0.15210
k_0 =1.05730	50	923	0.00829	-1002	0.05026	-102	0.05654
EVS = 4.44	90	178	0.00166	-168	0.00838	10	0.00982
EOL	0	2287	0.01072	-1866	0.11190	210	0.11234
k_0 =1.03569	50	1054	0.00501	-666	0.0400	361	0.04046
EVS = 3.99	90	204	0.00096	-111	0.00671	92	0.00688

Table 1 gives the results of direct computations (with modified cross-sections). It shows that for a small reactivity change, there can be a rather significant change of the power distribution (see MOL, 50% Na void). The flux is reduced in some regions and increased in other ones with equivalent importance, so that the reactivity of the system is almost unchanged. Sodium void effect is typical of such behavior due to the compensation between the positive absorption component and the negative leakage component. In table 1, we notice that the fuel region void leads to small $\|\Delta S\|$ values, compared to the plenum void perturbation. In other words, the absorption component is a **spectral perturbation** while the leakage component has higher **spatial effect**.

The shapes of the normalized vectors ΔS_{Norm} were compared. It is remarkable that, for a given configuration, these vectors are almost identical for a small void perturbation and the full void one. Of course, the perturbed flux is different, due to the weight $\|\Delta S\|$ to be given on that normalized form. Finally, based on the results of direct calculations, it may be noticed that an "equivalent" perturbation has significantly different consequences on the system. It appears here that the EOL configuration is the less deformable core. The question here is: can we predict the response of the model in terms of reactivity variation and power distribution change?

3. THE MODAL ANALYSIS APPROACH

Modal analysis approach aims at expanding any perturbed flux over base reference functions (the flux harmonics). These functions are the eigen-modes of the neutron transport equation, where the eigenvalue *k* is added to balance production and disappearing terms. The transport equation has an adjoint formulation, and forward and adjoint harmonics share bi-orthogonality properties. First-order perturbation theory takes benefit from this characteristic, and provides both reactivity variation and expression for the expansion coefficient due to a given perturbation $(\delta A, \delta F)$ of the leakage (and absorption) and production operators.

$$EVS = \frac{k_0 \times k_1}{k_0 - k_1} \;;\; \Phi_{Pert} = \Phi_0 + \sum_{i=1}^{N} a_i \times \Phi_i \;;\; a_i = -\frac{k_0 \times k_i}{k_0 - k_i} \times \frac{\left\langle \Phi_i^+, \left(\delta A - \frac{\delta F}{k_0} \right) \Phi_0 \right\rangle}{\left\langle \Phi_i^+, F\Phi_i \right\rangle} \qquad (3)$$

The EVS defined by equation (3) appears naturally in this theory. Equation (3) shows that the EVS can be physically interpreted as an indicator of the stability of the system. The highest will be the EVS, the highest will be the deformation of the flux for "a same amount of perturbation", as seen in Table 1. This result is well known in the spatial effects studies: A. Sargeni shows the influence of the EVS on the power distribution sensitivity based on the first-order formulation [2]. Another work is G. Rimpault's one, which links the power tilt change per unit of reactivity change when a control rod is introduced in an optimized RZ cylindrical model [5].

Thanks to (3), we are able to reconstruct any kind of perturbed flux, provided that many harmonics are known. If too few eigen-functions are used in the expansion, oscillations may arise (even with

13 harmonics used, see Figure 3). To reduce these oscillations **inherent to the method**, the only way is to increase the number of available flux harmonics. But the traditional technique based on the power iteration method and a filtering technique is not efficient to compute a large number of harmonics. Moreover, to study the leakage component of the sodium void effect in the CFV concept, axial harmonics are required. They usually are higher-order than the radial ones, which complicates considerably their computation. In practice, getting by the filtering technique several axial harmonics is impracticable in 3D full model. Some other tools might do better, such as the one based on coupled-core theory presented here after.

4. THEORETICAL BACKGROUND OF THE COUPLED-CORE THEORY

4.1. The Formalism of K. Kobayashi

Here are recalled the main results of the theory derived by K. Kobayashi [3]. For a given partition of the fissile regions, $V = \sum_{m=1}^{N} V_m$, the multi-point equation (4) links the partial sources together. It uses region-wise importance functions $G_m(\vec{r}, E, \vec{\Omega})$. In (5), $\delta_m(\vec{r})$ is the characteristic function of volume V_m. These importance functions defined everywhere in the reactor count the number of neutrons produced at first generation of fission in a particular region *m* due to a primary neutron born at a given position. k_{mn} expresses the number of neutrons produced in the region V_m due to a fission neutron produced in a region V_n.

$$\vec{S} : \left(S_J = \int_{V_J} d^3 r \times S(\vec{r}) \right)_{J=1..N} \quad ; \quad S_m = \sum_{n=1}^{N} \frac{k_{mn}}{k_0} \times S_n \quad \Leftrightarrow \quad \vec{S} = \frac{K}{k_0} \vec{S} \tag{4}$$

$$k_{mn} = \frac{\langle G_m, F\Phi_0 \rangle_{V_n}}{S_n} \quad ; \quad A^{+} G_m(\vec{r}, E, \vec{\Omega}) = \nu\Sigma_f \times \delta_m(\vec{r}) \tag{5}$$

We must notice that in opposition to the flux harmonics, the computation of G_m does not present any convergence difficulty. Moreover, the medium is assumed not multiplicative, so that no outer iteration is required (provided that down-scattering and up-scattering are done in specific loops). Finally, all regions may be solved individually on parallel processors.

4.2. Relation with the Eigenvalues of the Flux Harmonics

Equation (4) allows tracking the population of neutrons produced by fission generation after generation. Between two successive fission generations, if the shape of the fission source is unchanged, but its amplitude is multiplied by the scalar k_i, it means that an eigenfunction is reached. Finally, looking for the eigenvalues of the matrix *K* is similar than solving the neutron transport eigenvalue problem. This technique allows the computation of N harmonics (where N is the size of the fissile partition). An example is provided in [6].

The fundamental mode is used to weight the coefficient k_{mn} in (5). That is why k_0 is always an eigenvalue of the **coupling matrix** *K*. The high-order eigenvalues may be quite different than with the power iteration method (filtering technique), because of this weighting function. Nevertheless, for accurate partitions (small regions), a relatively good estimation of the first harmonic is provided. When the coupling matrix is defined on the same spatial mesh as those used for the power iteration method, the coefficient k_{mn} is no longer dependent on the fundamental mode. In this case, K is named the **fission matrix**. The eigenvalues are then equal to the ones of the traditional filtering technique. Finally, **coupling coefficients give similar information than the EVS.**

5. PERTURBATION ANALYSIS TROUGH THE COUPLING THEORY

5.1. How it Helps for Perturbation Analysis

A perturbation modifies the fission source: specific fission chains are added (reactivity insertion) or removed (reactivity loss) compared to the reference case. Equation (6) gives the definition of modified fission chains at "first generation" (vector $\vec{P}_0$). It can be seen that this vector may be obtained using the traditional tools of neutronic codes systems such as ERANOS [7], with the first-order perturbation theory assumption. These fission chains will then spread in the system until equilibrium. The scalar value w_0 is the norm of the final vector. The sequence of vectors $(\vec{U}_n)$ converges to 0.

$$P_0(V_m) = -\frac{\left\langle G_m, \left(\delta A - \frac{\delta F}{k_0} \right) \Phi_0 \right\rangle}{\left\langle F\Phi_0 \right\rangle} \quad ; \quad \vec{P}_0 \xrightarrow{K/k_0} \ldots \xrightarrow{K/k_0} w_0 \times \vec{S}_0 \quad \Rightarrow \begin{cases} \vec{P}_0 = \vec{U}_0 + w_0 \times \vec{S}_0 \\ \vec{U}_{e+1} = \dfrac{K}{k_0}\vec{U}_e \xrightarrow{e\to\infty} \vec{0} \end{cases} \tag{6}$$

This initial vector $\vec{P}_0$ and its descendants change the reference source. We assume that the perturbation fulfills the following criteria. Firstly, the perturbation has only effect on the projection of the fission source over the fundamental mode. Secondly, the perturbation has no effect on the coupling matrix which is unchanged. Under these assumptions, can we find a coupled $\left(k_{Pert}\,;\vec{S}_{Pert}\right)$ in such a way that from one generation to the next one : $\vec{S}_{Pert} \rightarrow k_{Pert} \times \vec{S}_{Pert}$?

We propose the relation :

$$\begin{cases} k_{Pert} = k_0 \times (1 + w_0) \\ \vec{S}_{Pert} = \vec{S}_0 + \dfrac{1}{1+w_0} \times \displaystyle\sum_{n=0}^{\infty} \frac{\vec{U}_n}{(1+w_0)^n} \end{cases} \tag{7}$$

The demonstration of this expression (under the previous assumptions) is provided in appendix A. It

proves also that the sum converges provided that condition (8) is satisfied. The perturbed multiplication coefficient must stay higher than the first harmonic of the reference system. Otherwise, the power iteration method, which converges towards the highest eigenvalue, could fail.

$$\forall i > 0 \, , \hat{k}_i < k_0 \times (1 + w_0) = k_{Pert} \tag{8}$$

5.2. Results provided by the coupled-core theory

Two fissile partitions are studied. The first one has 13 regions, corresponding to the axial mesh used for burnup computations in the 3D geometry. In this case, the coupling matrix is a 13x13 square matrix, and 13 importance functions G_m are computed and saved (to define the initial vector $\vec{P}_0$). The second one is identical to the spatial mesh used in the power iteration method. It means that we are here able to compute all harmonics of the discretized model.

Table 2. Results of the coupled core theory for the BOL configuration (45 coupling regions).

		Voided Fuel			Voided Plenum		
Residual Na (%)		0	50	90	0	50	90
$\Delta\rho$	Coupled-Core	1365	653	130	-2849	-1310	-256
	Bias (pcm)	-189	-34	-1	-1603	-278	-9
$\|\Delta S\|$	Coupled-Core	0.03931	0.02008	0.00411	0.10930	0.04815	0.00911
	Bias (%)	-7.7	-3.4	-0.7	-35.4	-14.9	-3.0

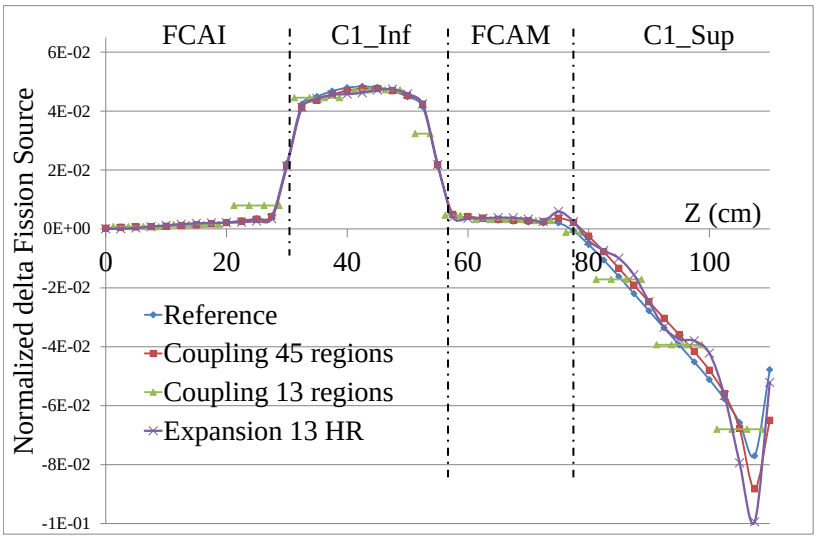

Figure 3. Results for the coupled-core approach for the ΔS_{Norm} vector. BOL configuration, total void perturbation. For the 13x13 coupling matrix, the information is only partial but matches well the reference computation, without oscillation phenomena observed for the expansion method.

The results for the reactivity variations $\Delta\rho$ and the flux distortion measures $\|\Delta S\|$ are very similar to those obtained by the first-order approach (Table 2). It can be demonstrated that the reactivity

variation inferred from equation (7) equals the first-order perturbation one, provided that the fissile partition is identical to the spatial mesh used to compute the fundamental mode. For large perturbation, affecting significantly the power shape (leakage component for example), the first-order theory is insufficient for quantitative purposes, as shown in Table 2. However, the shape of the ΔS_{Norm} vector, representing the power redistribution in the reactor, is well reproduced (see figure 3).

5.3. How it helps for Uncertainties Analysis

Since the coupled-core theory allows propagating a given perturbation through the fission generations, we use this approach for uncertainty analysis. We call a parameter *p* the variation of a given cross-section for a given isotope on a given energetic group. Thanks to (6), an initial $\vec{P}_0$ is defined. Then the perturbed normalized fission source are computed with (7), which allows the computation of power shape sensitivities for this given parameter *p*. Then we use the usual tools to propagate the nuclear data uncertainties on the power shape distribution [8]. Figure 4 gives the relative uncertainties (in %) for the ratio of the local fission rate normalized to the full fission rate.

For the BOL case, the major component to the total uncertainty is the inelastic U-238 reaction, which may lead to a tilt in the power shape. Moreover, the BOL configuration is "more uncertain" that the EOL one, as expected respect to the relative EVS. This profile of uncertainty was verified using the usual generalized perturbation theory. Results are obtained using the COMAC V1 variance covariance matrix [9]. Uncertainties presented here are those of the indirect term of the generalized perturbation theory.

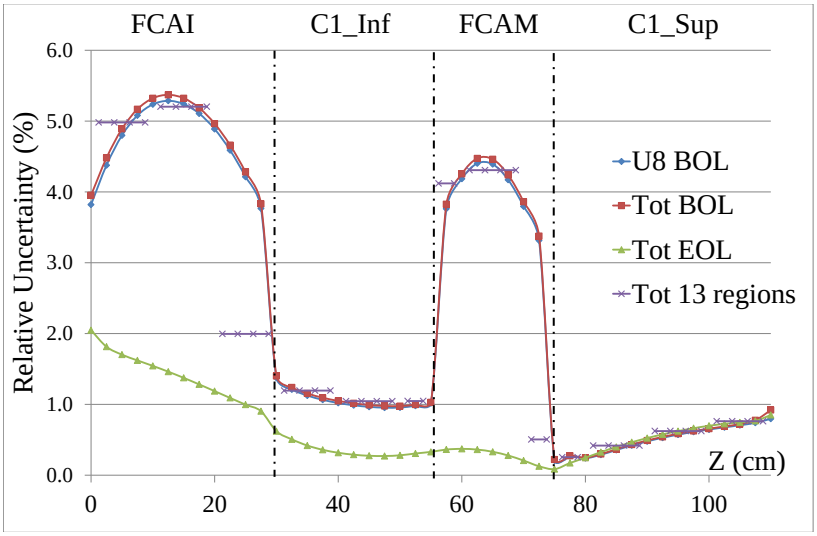

Figure 4. Profiles of total relative uncertainty (in %) due to nuclear data for the normalized fission source distribution, and major component for the BOL configuration

6. CONCLUSION: COMMENTS AND PERSPECTIVES

In the paper, we chose two parameters to study a given perturbation. Firstly, the **reactivity variation** may be computed using the fundamental mode (forward and adjoint flux) and the first-order perturbation theory. Bias may arise for large perturbations, or at least for the ones changing significantly the shape of the flux. High-order formulations could do better, but is out the scope of this document. We are more interested in quantifying **the power distribution deformability.** Are we able to predict any change of the flux shape based on the variations of the cross-sections?

In that purpose, the limits of the standard perturbation theory based on the expansion method on the flux harmonics to describe properly flux shapes changes have been illustrated. The eigen-values may give valuable qualitative information on the reactor stability: a high EVS comes along with high sensitivities. But many flux harmonics are required to match a perturbed power distribution. Oscillations inherent to the expansion approach occur if too few eigen-functions are known and available for the expansion. However, the traditional power iteration method is inefficient to compute a large number of harmonics.

Another approach based on the coupled-core theory explains the discrepancies between the perturbed fission source and the reference one, by the descendants of specific fission chains. This method presents many advantages for the computation point of view, since most of the operations may be parallelized, and done simultaneously. Moreover, no convergence difficulty is to be treated. The performances of this method are similar to those obtained by the first-order theory for the reactivity variation. Significant biases arise for large perturbations out of the validity domain of the first-order formulation. Nevertheless, the deformability of the power distribution is well-reproduced. Moreover, this information is identical for small sodium densities effect, and for full void perturbation. It opens the door to experimental validation. Another application of this theory, in the first-order domain, is about the propagation of nuclear data uncertainties on the power distribution. For a tiny perturbation in nuclear cross-sections, we are able to provide the reactivity variations (first-order formulation) and the flux changes (on a given fissile partition), which allows uncertainties analysis of the power shape distribution.

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APPENDIX A. DEMONSTRATION OF THE PERTURBED SOURCE EXPRESSION

We remember in equations the notations. The application of the perturbed coupling matrix from one generation to the next one respects the following rules:

- $K\vec{U}_n = k_0 \times \vec{U}_{n+1}$; $\left(\frac{K}{k_0}\right)^{\infty} \vec{U}_0 = \left(\frac{K}{k_0}\right)^{\infty} \vec{U}_n = \vec{0}$ (definition)
- $\delta K \vec{U}_n = \vec{0}$, which is our **main assumption**
- $\delta K\ \vec{S}_0 = k_0 \times \vec{P}_0 = k_0 \times \left(w_0 \times \vec{S}_0 + \vec{U}_0\right)$ (definition)
- $K\ \vec{S}_0 = k_0 \times \vec{S}_0$, definition of the coupling matrix (unchanged ; **other assumption**)

Finally:

$$\left(K+\delta K\right)\vec{S}^{(e)}_{Pert} = \left(K+\delta K\right)\vec{S}_0 + \frac{1}{1+w_0} \times \sum_{n=0}^{\infty} \frac{\left(K+\delta K\right)\vec{U}_n}{\left(1+w_0\right)^n} = k_0 \times \left(\vec{S}_0 + \vec{P}_0\right) + \frac{1}{1+w_0} \times \sum_{n=0}^{\infty} \frac{K\,\vec{U}_n}{\left(1+w_0\right)^n}$$

$$= k_0 \times \left(1+w_0\right) \times \vec{S}_0 + k_0 \times \vec{U}_0 + k_0 \times \sum_{n=0}^{\infty} \frac{\vec{U}_{n+1}}{\left(1+w_0\right)^{n+1}} = k_0 \times \left(1+w_0\right) \times \vec{S}_0 + k_0 \times \sum_{n=0}^{\infty} \frac{\vec{U}_n}{\left(1+w_0\right)^n}$$

$$= k_0 \times \left(1+w_0\right) \times \left[\vec{S}_0 + \frac{1}{1+w_0} \times \sum_{n=0}^{\infty} \frac{\vec{U}_n}{\left(1+w_0\right)^n}\right]$$

To prove that this summation converges, we use the diagonal form of the coupling matrix *K*. Using this form, and recalling that the highest eigenvalue equals to k_0 and that the series $(\vec{U}_n)$ converge towards 0, the demonstration is straight forward.