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Abstract

Unlike finite elements methods, finite volume methods are far from having a clear functional analytic setting allowing the proof of general convergence results. In [4], compactness methods were used to derive convergence results for the Laplace equation on fairly general meshes.
The weak convergence of nonlinear finite volume methods for linear hyperbolic systems was proven in [5] using the Banach-Alaoglu compactness theorem. It allowed the use of general L^2 initial data which is consistent with the continuous theory based on the L^2 Fourier transform [1]. To our knowledge this was the first convergence results applicable to non differentiable initial data. However this weak convergence result seems not optimal with regard of numerical simulations. In this paper we prove that the convergence is indeed strong for a wide class of possibly nonlinear upwinding schemes.
The context of our study being multidimensional, we cannot use the spaces L^1 and BV classically encountered in the study of $1D$ hyperbolic systems [2]. We propose instead the use of generalised p-variation function, initially introduced by Wiener [6] and first studied by Young [7]. These spaces are compactly embedded in L^p (see [9, 8]). They can therefore fit into the L^2 framework imposed by Brenner obstruction result [3]. Using estimates of the quadratic variation of the finite volume approximations we prove the compactness of the sequence of approximations and deduce the strong convergence of the numerical method.
We finally discuss the applicability of this approach to nonlinear hyperbolic systems.

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