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A model to predict modal radiation by finite-sized sources in composite plates with account of caustics

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Abstract

The guided wave field generated by finite size transducers in a composite plate is studied. For isotropic plates, Fraunhofer-like approximations can be found in the literature. Similar approximations fail when the plate is anisotropic. A new calculation method is proposed. Based on a modal decomposition, its principle is to integrate over the transducer surface as seen in energy directions from the calculation point. Special care is taken when dealing with energy paths in direction of caustics. To validate this method, some comparisons are made between our results and those obtained using a full integration over the surface of the source.

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Keywords: guided waves ; anisotropic plate ; modelling ; finite-sized source ; caustic

1. Introduction

Elastic guided waves (GW) are used in various non-destructive testing (NDT) methods to inspect plate-like structures. GW are generated by transducers of finite size, leading to aperture diffraction effects and possibly complex field characteristics. The optimization of transducer positioning for insuring coverage of the structure examined while minimizing the number of transducers used makes it necessary to accurately predict GW field radiated by a transducer. In the literature, Fraunhofer-like approximations have been developed for GW in the case of isotropic and homogeneous plates. They lead to fast computation and accurate prediction of diffraction effects. Applying similar approximations to GW radiated in an anisotropic plate fails at predicting radiated fields even for the less anisotropic modes. To solve this problem, a new transducer integration model is proposed. It is based on the computation of the approximate Green's tensor describing modal propagation from a point source, with account of caustics which are typically seen in anisotropic plates in the directions where we have a strong focusing of GW. The overall principle of the method is to proceed to an angular integration over the transducer surface as seen from the calculation point, based upon the energy paths involved, which are mode-dependent. This computationally efficient method is validated thanks to comparisons of predictions made with the proposed model to those computed by means of a full integration. Examples given concern disk and square shaped transducers.

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2. Formulation of the 3D Green's function

We consider an $2d$ thick infinite plate. The Cartesian coordinate axes are defined with the z axis normal to the plate. The GW field radiated by a finite sized source on this plate can be expressed thanks to the surface convolution of the source $\mathbf{q}(x, y)$ with the 3D Green's function $\mathbf{g}(x, y, z)$ written as a sum over the propagating modes:

$$\mathbf{u}(x, y, z, \mathbf{q}) = \iint_S \sum_m \sum_{n(\varphi)} \mathbf{g}_{m,n}(x - x', y - y', z)\mathbf{q}(x', y')dx'dy'. \quad (1)$$

Modal contributions are computed thanks to the Semi Analytical Finite Element (SAFE) method [1].In some anisotropic cases, the n subscript is necessary when more than one phase contribution appears for certain modes in the observation direction φ. The expression of the Green's functions which is different near and far from caustics, is developed in the literature by Velichko and Wilcox [2] and Karmazin et al. [3] , and can be written as a product of three terms, d which takes in account the distance between the calculation point and the source point, a which is like a modal magnitude and p which is function of the phase:

$$\mathbf{g}_{m,n}(x, y, z) = d(x, y)\mathbf{a}_{m,n}(x, y, z)p_{m,n}(x, y, z). \quad (2)$$

Far from the caustics, we obtain the following equations :

$$d_f(x, y) = (\sqrt{x^2 + y^2})^{-1/2}, \quad (3)$$

$$\mathbf{a}_{fm,n}(x, y, z) = res[\mathbf{G}]_{k=k_{m,n}(\gamma_{m,n})} \frac{|k_{m,n}(\gamma_{m,n})|}{\sqrt{2\pi|\frac{\partial^2\phi_{m,n}(\gamma_{m,n},\varphi)}{\partial\gamma^2}|}}, \quad (4)$$

$$p_{fm,n}(x, y, z) = e^{i\sqrt{x^2+y^2}\phi_{m,n}(\gamma_{m,n},\varphi)} e^{i\frac{\pi}{4}sgn(\frac{\partial^2\phi_{m,n}(\gamma_{m,n},\varphi)}{\partial\gamma^2})} e^{i\frac{\pi}{2}}, \quad (5)$$

where f index stands for far from caustics and with $\mathbf{G}$ being one-dimensional spatial Fourier transform of $\mathbf{g}$,
$\phi_{m,n}(\gamma_{m,n}, \varphi) = k_{m,n}(\gamma_{m,n})cos(\gamma_{m,n} - \varphi)$,
$k_{m,n}$ wave number of the mode,
and $\gamma_{m,n}$ phase direction for a given direction of observation φ given,
whereas it leads to the following relations near the caustics when we can not anymore differentiate two phase contributions (1) and (2):

$$d_c(x, y) = (\sqrt{x^2 + y^2})^{-1/3} \quad (6)$$

$$\begin{aligned}\mathbf{a}_{cm,(1)+(2)}(x, y, z) = {} & res\mathbf{G}G]_{k=k_{m,1}(\gamma_{m,1})}|k_{m,1}(\gamma_{m,1})| \left| \sqrt{\frac{-2\sqrt{S(\varphi)}}{\frac{\partial^2\phi_{m,1}(\gamma_{m,1},\varphi)}{\partial\gamma^2}}} \right| \\ & + res[\mathbf{G}]_{k=k_{m,2}(\gamma_{m,2})}|k_{m,2}(\gamma_{m,2})| \left| \sqrt{\frac{2\sqrt{S(\varphi)}}{\frac{\partial^2\phi_{m,2}(\gamma_{m,2},\varphi)}{\partial\gamma^2}}} \right|\end{aligned} \quad (7)$$

$$p_{cm,(1)+(2)}(x, y, z) = e^{i\sqrt{x^2+y^2}L(\varphi)} Ai(\kappa) \frac{e^{i\frac{\pi}{2}}}{2}, \quad (8)$$

where c index stands for close to caustics and with

$L(\varphi) = \frac{1}{2}(\phi_{m,1}(\gamma_{m,1}, \varphi) + \phi_{m,2}(\gamma_{m,2}, \varphi))$, $\kappa = -(\sqrt{x^2 + y^2})^{\frac{2}{3}} S(\varphi)$,

$S(\varphi) = \left[\frac{3}{4}(\phi_{m,2}(\gamma_{m,2}, \varphi) - \phi_{m,1}(\gamma_{m,1}, \varphi))\right]^{\frac{2}{3}}$,

and Ai the Airy's function.
It is important to notice that, in practice, solution at or close to caustics is straightforwardly computed and continuously tends to that obtained out of caustics when this latter applies.

3. Integration along energy directions

In the case of isotropic plates Fraunhofer-like approximation has been shown to work very well. The cases of rectangular and circular uniform sources of normal stress as treated by Raghavan and Cesnik [4] can be solved analytically. In preliminary studies, we applied the same approximation to the case of GW radiation in an anisotropic plate. Even for the less anisotropic modes, this approximation fails in some directions. These results motivated the finding of a new and less approximate solution to overcome this difficulty.
The proposed solution relies on a change of variable in the general radiation integral. The displacement field is expressed in a coordinate system centered at the computation point. In what follows, r' is the distance between a running source point and the calculation point and θ denotes the angle formed by the direction between these two points and that joining the calculation point to the center of the source. In this new formulation $d(x, y) = d(r)$.

$$\begin{aligned}\mathbf{u}(e_x = 0, e_y = 0, z,\mathbf{q}) = \sum_m \sum_{n(\varphi)} \int_{\theta_{min}}^{\theta_{max}} d\left(\frac{r_1(\theta) + r_2(\theta)}{2}\right) \\ \times \int_{r_1(\theta)}^{r_2(\theta)} \mathbf{a}_{m,n}(\theta, z) p_{m,n}(r', \theta, z)\mathbf{q}(r', \theta) r' dr' d\theta,\end{aligned} \tag{9}$$

$[\theta_{min}, \theta_{max}]$ and $[r_1(\theta), r_2(\theta)]$ are the intervals of integration.
We consider that the source is uniform over the transducer surface. An approximation is made for the distance term d. The main difference with Fraunhofer-like approximation consists in considering variations with θ of the amplitude term.

$$\begin{aligned}I_{m,n}(e_x = 0, e_y = 0) &= \int_{\theta_{min}}^{\theta_{max}} d\left(\frac{r_1(\theta) + r_2(\theta)}{2}\right) \int_{r_1(\theta)}^{r_2(\theta)} \mathbf{a}_{m,n}(\theta, z) p_{m,n}(r', \theta, z) r' dr' d\theta \\ &= \sum_{t=1}^{n_\theta} \frac{\theta_{max} - \theta_{min}}{n_\theta} d\left(\frac{r_1(\theta_t) + r_2(\theta_t)}{2}\right) \int_{r_1(\theta_t)}^{r_2(\theta_t)} \mathbf{a}_{m,n}(\theta_t, z) p_{m,n}(r', \theta_t, z) r' dr',\end{aligned} \tag{10}$$

with $\theta_t = \theta_{min} + (t - \frac{1}{2})\frac{\theta_{max} - \theta_{min}}{n_\theta}$ and n_θ denotes the finite number of directions used in the angular integration.

From now on, straightforward calculations allow us to analytically calculate the integral over r'. Far from caustics, we have:

$$\begin{aligned}&d\left(\frac{r_1(\theta_t) + r_2(\theta_t)}{2}\right) \int_{r_1(\theta_t)}^{r_2(\theta_t)} \mathbf{a}_{m,n}(\theta_t, z) p_{m,n}(r', \theta_t, z) r' dr' \\ &= d_f\left(\frac{r_1(\theta_t) + r_2(\theta_t)}{2}\right) e^{i\frac{\pi}{4} sgn(\frac{\partial^2 \phi_{m,n}(\gamma_{m,n}, \theta_t)}{\partial \gamma^2})} e^{i\frac{\pi}{2}} \mathbf{a}_{fm,n}(\theta_t, z) \\ &\times \left(e^{ir_2(\theta_t)\phi_{m,n}(\gamma_{m,n},\theta_t)} \frac{ir_2(\theta_t)\phi_{m,n}(\gamma_{m,n}, \theta_t) - 1}{(i\phi_{m,n}(\gamma_{m,n}, \theta_t))^2} - e^{ir_1(\theta_t)\phi_{m,n}(\gamma_{m,n},\theta_t)} \frac{ir_1(\theta_t)\phi_{m,n}(\gamma_{m,n}, \theta_t) - 1}{(i\phi_{m,n}(\gamma_{m,n}, \theta_t))^2}\right).\end{aligned} \tag{11}$$

Close to caustics, we use the same calculation method taking $\kappa(\theta_t) = -\left(\frac{r_1(\theta_t)+r_2(\theta_t)}{2}\right)^{\frac{2}{3}} S(\theta_t)$.

4. Simulations

The plate considered in our simulations is 1mm thick, made of $[0/90]S$ T700GC/M21 cross-ply composite fiber-reinforced polymer(CFRP) [5]. Material parameters are listed in Table 1. Three propagating modes, A0, S0 and SH0 exist at our work frequency of 300 kHz which is under the first cut-off frequency.

Table 1. Plate characteristics.

C11 (GPa)	C22=C33 (GPa)	C12=C13 (GPa)	C23 (GPa)	C44 (GPa)	C55=C66 (GPa)	Mass density (kg/m3)	Ply thickness (mm)
123.4	11.5	5.6	6.4	2.6	4.5	$6x10^3$	0.25

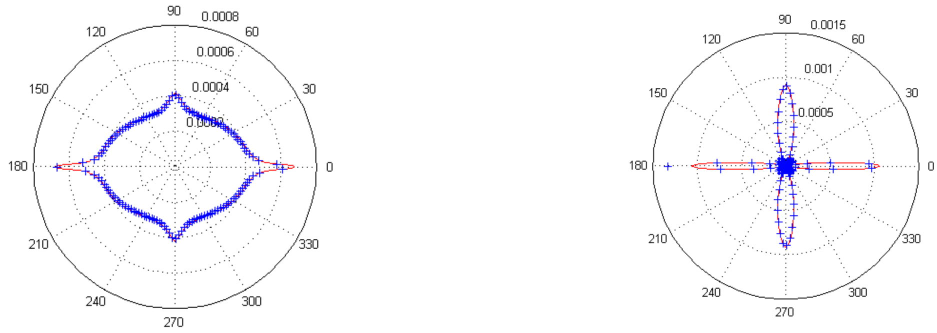

Fig. 1. Out of plane displacement of the A0 mode at a distance of 100mm at the frequency of 300 kHz of an excitation by a circular source (left) or by a square source (right) calculate by a full integration (red line) and an integration along energy direction (blue cross) .

To validate our integration scheme, we compare field predictions computed by means of a full surface integration as defined by Eq. (1) to those computed by our model. For the sake of conciseness, only two results are given which concern the radiation of A0 mode by two transducers of different geometry. Plots show the modulus of the out-of-plane displacement at points located at a distance of 100 mm from the source center, computed by the two methods. Both a disk transducer of radius of 5 mm, and a square transducer of side L=9mm are considered.

In both cases, results computed by our method are in perfect agreement with those computed by the full integration.

5. Conclusion

The combination of SAFE results for GW modal solutions in arbitrary anisotropic plate with Greens functions[2, 3] constitutes a model of general applicability to predict the GW field radiated by a finite-sized source thanks to the further computation of a convolution integral over the source surface. The overall computation is however computer intensive. The surface integral over the source can be avoided under Fraunhofer-like approximation which allows fast and accurate prediction of GW fields in an isotropic plate but cannot be used in the anisotropic case. A specific method of integration has been developed in the latter case based upon a change of integration variables in relation to energy path from the source as seen from the calculation point. By doing so, a single angular integration is involved, far faster to compute.

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