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A logical loophole in the derivation of the Bell inequalities

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Abstract. The Bell inequalities are based on a tacit assumption of a *common* probability distribution that precludes their application to the experiments of Aspect et al.

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1 The Bell inequalities and their application to the experiments of Aspect et al.

The subject matter of the Bell inequalities and the experiments of Aspect *et al.* hardly needs any introduction [1]. However, the argument has often been blurred by drawing in unnecessary issues, leading to some confusion. We give here a minimalistic derivation that removes all unnecessary considerations. This will show how elementary the argument is and how very hard it is to question the validity of the inequalities.

We consider 4 variables $a_1 \in S$, $a_2 \in S$, $b_1 \in S$, $b_2 \in S$, where $S = \{0, 1\}$. The idea is that 0 corresponds to absorption in a polarizer, and 1 to transmission. a_j will correspond to polarizer settings in one arm of the set-up, b_k to polarizer settings in the other arm. There are thus 16 possible combinations for the values of (a_1, a_2, b_1, b_2) . By making a table of these 16 combinations it is easy to verify that we always have:

$$\forall (a_1, a_2, b_1, b_2) \in S^4 : Q = a_1 b_1 - a_1 b_2 - a_2 b_1 - a_2 b_2 + a_2 + b_2 \in S. \quad (1)$$

We consider now functions $a_j \in F(V, S)$ and $b_k \in F(V, S)$. The notation means that the domain of the functions is V , while the functions take their values in S . Here V is a set of relevant variables for the experiment. We can call the set V the set of hidden variables, even if some of them may not really be hidden. One can imagine that V could be a subset of a vector space \mathbb{R}^n or of a manifold, e.g. a non-abelian Lie group like $SO(3)$. We have then:

$$\forall \lambda \in V : 0 \leq Q(\lambda) = a_1(\lambda)b_1(\lambda) - a_1(\lambda)b_2(\lambda) - a_2(\lambda)b_1(\lambda) - a_2(\lambda)b_2(\lambda) + a_2(\lambda) + b_2(\lambda) \leq 1. \quad (2)$$

We can now consider a probability density p over V , i.e. $p(\lambda) d\lambda$. The function p belongs then to the set of functions $F(V, [0, \infty[)$ with domain V and values in $[0, \infty[$. We further require that $\int_V p(\lambda) d\lambda = 1$. We can now integrate Eq. 2 with p over V . Introducing the notations:

$$p(\alpha_j \wedge \beta_k) = \int_V a_j(\lambda) b_k(\lambda) p(\lambda) d\lambda, \quad p(\alpha_j) = \int_V a_j(\lambda) p(\lambda) d\lambda, \quad p(\beta_k) = \int_V b_k(\lambda) p(\lambda) d\lambda, \quad (3)$$

we obtain then:

$$0 \leq p(\alpha_1 \wedge \beta_1) - p(\alpha_1 \wedge \beta_2) - p(\alpha_2 \wedge \beta_1) - p(\alpha_2 \wedge \beta_2) + p(\alpha_2) + p(\beta_2) \leq 1. \quad (4)$$

This is the CHSH Bell inequality used in the experiments of Aspect et al. It is a purely mathematical identity and does not depend on any physical considerations. It is also free of any considerations about statistical correlations and statistical independence. These correlations can be anything. They must just be laid down correctly in the definition of $p \in F(V, [0, \infty[)$. In fact, the derivation of Eq. 4 solely relies on the assumption that all data of which we will make statistics in order to define expectation values are just numbers 0 or 1. Eq. 4 can be applied to physical experiments by identifying 1 with the presence and 0 with the absence of a detector signal, as outlined above. The probabilities in Eq. 4 are identified with the mathematical expressions for the outcomes of the photon polarization experiments reported by Aspect *et al.*:

$$p(\alpha_j \wedge \beta_k) = \frac{1}{2} \cos^2(\alpha_j - \beta_k), \quad p(\alpha_j) = \frac{1}{2}, \quad p(\beta_k) = \frac{1}{2}, \quad (5)$$

where α_j and β_k are the angles of the polarizer settings in the two arms of the experiment. According to quantum theory the mathematical expressions are the limits of the measured probabilities when the number of registered events tends to infinity, i.e. when the statistics become perfect. For a function $f \in F(\mathbb{N}, \mathbb{R})$, the limit $n \rightarrow \infty$ is defined by:

$$\lim_{n \rightarrow \infty} f(n) = F \Leftrightarrow (\forall \varepsilon > 0)(\exists N \in \mathbb{N})(n > N \Rightarrow |f(n) - F| < \varepsilon). \quad (6)$$

Here $f(n)$ would be the measured probabilities after n detection events, F the theoretical expression $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$, and ε the statistical accuracy of the experiment required. An experimentalist has to worry about the statistical and instrumental precisions. For practical reasons the experimentalist can only reach a reasonable accuracy ε . But this should be well enough to establish beyond any reasonable doubt if the Bell inequality is satisfied or otherwise. We will adopt a mathematician's viewpoint and assume that the expressions $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ are exact, trusting that at least in principle the experimentalist could prove this to any accuracy ε , by improving the experimental protocol. We introduce thus the assumption (or act of faith) that the algebra of quantum mechanics is exact. This frees us from all imaginable headaches about experimental bias and loopholes. For certain values of $(\alpha_1, \alpha_2, \beta_1, \beta_2) \in [0, 2\pi]^4$, the expressions in Eq. 5 do not satisfy the inequality in Eq. 4. This violation of the Bell inequality shows that the mathematical expressions in Eq. 5 are not compatible with Eq. 4. This seems to invalidate the hidden-variable *Ansatz* and confirm Bohr's thesis that the polarizations cannot exist prior to a measurement and must be created by the measurement. But it is then extremely puzzling that we can obtain a definite correlation $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ because the polarizers can be separated by arbitrarily large distances. It looks like the spooky action at a distance Einstein talked about and which has been called entanglement in the aftermath of the experiments. In the experiments of Aspect this issue is tested by ensuring Einstein separation of the detection events in both arms. The solution of this conundrum is in our opinion summarised in the last paragraph of Subsection 3.3.

2 The tacit assumption

The derivation of the inequality looks unassailable. It is indeed ought to be too simple to possibly hide a logical loophole. But it does! What is not acknowledged is that the identification in Eq. 5 introduces a tacit assumption, which is admittedly hard to discern, namely that all quantities $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ can be obtained from *one single common distribution function* p rather than from *four different distributions* p_{jk} . In other words, it has been assumed that:

$$\exists V, \exists! p \in F(V, [0, \infty[) \parallel \forall (j, k) \in \{1, 2\}^2 : \frac{1}{2} \cos^2(\alpha_j - \beta_k) = \int_V a_j(\lambda) b_k(\lambda) p(\lambda) d\lambda. \quad (7)$$

Here $\exists!$ stands for "there exists a unique". However, it can *a priori* not be excluded that in reality we can only obtain the quantities $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ from four different distributions $p_{jk} \in F(V_{jk}, [0, \infty[)$ according to:

$$\frac{1}{2} \cos^2(\alpha_j - \beta_k) = \int_{V_{jk}} a_j(\lambda) b_k(\lambda) p_{jk}(\lambda) d\lambda \quad (8)$$

In fact, to measure $\frac{1}{2} \cos^2(\alpha_1 - \beta_1)$ we must set the polarizer in one arm to the angle α_1 and the polarizer in the other arm to the angle β_1 . If we want to measure now another quantity $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$, we must change the set-up by switching α_1 to α_j and/or β_1 to β_k . Measuring the four quantities $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ requires this way four different set-ups. In the experiments of Aspect *et al.* the switching is done very quickly and many times in a random way, in order to rule out the possibility that information about the polarizer setting α_j could be present at the position where the other polarizer is set to β_k and *vice versa*. To rule out this possibility we base ourselves on the theory of relativity: The information would have to travel faster than light.

If we could make a movie of the experiment in the old-fashioned way of registering it reel-to-reel on celluloid with a non-digital camera, then we could cut the celluloid afterwards with cissors into many pieces, whereby each piece would correspond to the registration of one event. We could then paste these single-event pieces back together in a different order, such that the movie would look now as a succession of four experiments: First we measure all events which define the quantity $\frac{1}{2} \cos^2(\alpha_1 - \beta_1)$, then all events which define the quantity $\frac{1}{2} \cos^2(\alpha_1 - \beta_2)$, etc... This would not change the statistics of the events and the numbers $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ obtained. In reality, we do not assemble the quantities $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ by cutting and pasting films, because this would be too laborious. We do it by electronics, but the experimental results remain the same.

The original movie looks like the narrative of a single experiment. The movie obtained by cutting and pasting reveals however that we are making in reality four experiments to determine the four quantities $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$. The movie obtained by cutting and pasting suggests therefore that we should make the assumption expressed by Eq. 8. It is obvious that if we turn a polarizer, we turn a distribution of molecules. If one believes in hidden variables, then this distribution of molecules must be part of the hidden variables, because $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ changes when we turn one of the polarizers. Hence by turning a polarizer we change the hidden variables, such that *there are indeed four different sets* V_{jk} and four different distributions p_{jk} .¹ These four sets correspond to four different set-ups that are mutually incompatible. We cannot measure $\frac{1}{2} \cos^2(\alpha_1 - \beta_1)$ with the set-up designed to measure $\frac{1}{2} \cos^2(\alpha_1 - \beta_2)$, etc... A viable hidden-variable scenario must thus correspond to a compromise between the viewpoints of Einstein and Bohr. From Einstein we accept the existence of hidden variables, but from Bohr we must accept the rôle played by the interaction of the particles (here the photons) with the measuring device (which consists here of two polarizers). And one does not need only hidden variables to describe the particles, one also needs hidden variables to describe this measuring device! The question who of the two was right, Einstein or Bohr, traps us in a false dilemma. Both of them had parts of it right and parts of it wrong.

The analysis of the experiments of Aspect *et al.* is however based on the assumption expressed by Eq. 7. This is because we believe that it should be easy enough to make up for the difference between the two assumptions. All one would have to do is to enlarge the sets V_{jk} to a set V , where $\forall (j, k) \in \{1, 2\}^2 : V_{jk} \subset V$. We will see that we can indeed achieve this, but that it comes at a cost: When we change the definition domains, the normalization conditions for the probability distributions change as well, from:

$$\text{from } \int_{V_{jk}} p_{jk}(\lambda) d\lambda = 1 \quad \text{to} \quad \int_V p(\lambda) d\lambda = 1, \quad (9)$$

be it only because p (which may coincide with p_{jk} on V_{jk}) is now defined and non-zero on a set larger than V_{jk} . We must thus renormalize the probabilities in the process, and the renormalized probabilities may then no longer violate the Bell inequality. Hence, the problem is not only that we have to construct V . There is also a normalization problem as *the measured quantities* $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ *are beyond any doubt normalized individually according to* $\int_{V_{jk}} p_{jk}(\lambda) d\lambda = 1$ *rather than globally according to* $\int_V p(\lambda) d\lambda = 1$ (if V exists). In fact, the three probability distributions $p_{j'k'}$ for $(j', k') \neq (j, k)$ do not intervene in the measurement of $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$, which stands on its own and is independent of our intentions to consider other experiments with the aim of measuring four quantities in total. This normalization problem may well be the reason for the violation of the Bell inequality, such that we could have here a genuine logical loophole. We will work this out in Section 3 where we will also discuss an approach based on Cartesian products of sets rather than on unions of sets.

3 Would-be simple scenarios for obtaining a common probability distribution that fail

3.1 A logically correct extension based on unions of sets

Let us now explain why enlarging the sets V_{jk} to a set V in such a way that p will engulf all probabilities p_{jk} is all but trivial. In trying to follow our intuition and to define a common distribution function p one will run into all sorts of difficulties, which is normal because they are there to prevent us from deriving a contradiction from the mathematics. But as physicists we have been taught to take our strides with mathematical rigour, such that we are prone to make some booby traps go off. Very often we get away with our lack of rigour, but not this time. This time we have paid very dearly.

Let us try to define an extension.² In the following m stands actually for a compound index of the type (jk) above. We note the sets V_{jk} as V_m . We note an element $\lambda \in V_{jk}$ as $x_m \in V_m$, and we note the probability distribution over V_m as $p_m \in F(V_m, [0, \infty[)$. We have thus $\int_{V_m} p_m(x_m) dx_m = 1$. We want to make an extension that includes two sets of the type V_m , which we will call V_1 and V_2 . In reality, we will need four of them because we have four different set-ups. But the example with two sets will show the method works in general. The reader might think that our approach is awkward and object that there is a much more elegant one that does not lead to a normalization problem. But we

¹ One may hold the dogmatic viewpoint that the particles have some properties and that a polarizer just reads the value of such a property, viz. its polarization along a given direction. But as there exists an infinity of possible polarizer angle settings, this viewpoint would imply that the particle has an infinity of properties. An alternative is to consider the experimental result as the outcome of a physical process based on a completely deterministic interaction between the particle and the distribution of molecules, whereby only one property (or perhaps a small finite number of properties) of the particle comes into play. We do not choose here between these two alternatives. We just investigate if the second alternative could lead to a logical loophole.

² Defining in an alternative approach everything in terms of $V = V_{11}$, will keep us in the presence of 4 different distributions p_{jk} .

will see that this elegant approach is logically flawed. The present approach is not flawed. The only problem it has will turn out to be its normalization problem. To obtain a correct definition for $p_m(x)$ when $x \in V_1 \cap V_2$ we define:

$$\bar{V}_m = \{(x_m, m) \mid x_m \in V_m\}, \quad V = \bar{V}_1 \cup \bar{V}_2, \quad (10)$$

We have then:

$$\bar{V}_1 \cap \bar{V}_2 = \emptyset, \quad (11)$$

such that we avoid the problem that V_1 and V_2 can physically spoken not exist simultaneously because two physical objects like polarizers cannot be present at a same place at the same time. Since we have:

$$\forall(x, \ell) \in V : \begin{cases} \ell = 1 \Rightarrow \exists x_1 \in V_1 \mid x = x_1 \\ \ell = 2 \Rightarrow \exists x_2 \in V_2 \mid x = x_2 \end{cases} \quad (12)$$

we can define:

$$\forall(x, \ell) \in V : \begin{cases} \ell = 1 \Rightarrow p(x, \ell) = p_1(x_1) \\ \ell = 2 \Rightarrow p(x, \ell) = p_2(x_2) \end{cases} \quad (13)$$

We can then write:

$$p_m(x_m) = p(x, \ell) \delta_{m\ell} \quad (14)$$

That would permit to write the two probability distributions as one distribution.³ But this leads to a normalization problem, because:

$$\int_V p(x, \ell) dx = \sum_{m=1}^2 \int_{V_m} p_m(x_m) dx_j = 2. \quad (15)$$

We would thus have to allow for a re-normalization. *An extension $V_{jk} \rightarrow V$ changes the normalizations of the probabilities p_{jk} .* In the approach that treats the four set-ups, the quantities $\frac{1}{2} \cos^2(\alpha_j - \beta_k)$ would then have to be re-normalized to $\frac{1}{8} \cos^2(\alpha_j - \beta_k)$ in applying the inequality to the theoretical data. This is completely in agreement with the result given by Khrennikov [3].

3.2 A logically flawed extension based on Cartesian products of sets

Let us explore now the more elegant scenario. Let us write it out in full for the four possible joint polarizer settings. We put $V_{jk} = A_j \times B_k \times P$. Here A_j and B_k are the sets of hidden variables in the two polarizers. P is the set of hidden variables of the photons. Every set-up contains the hidden variables A_j , B_k , and P . To obtain a common description we must use $A_1 \times A_2 \times B_1 \times B_2 \times P$. Let us note the elements of A_j by λ_j , those of B_k by μ_k , and those of P by ζ . The function p_{jk} is defined over $A_j \times B_k \times P$. Suppose that $p_{jk}(\lambda_j, \mu_k, \zeta)$ can be written as a product $f_j(\lambda_j)g_k(\mu_k)h(\zeta)$. In fact, the properties of a polarizer do not depend on the properties of the photons and *vice versa*. The properties of a polarizer do also not depend on the properties of another polarizer. Therefore the distributions over the three sets must be independent, such that we can multiply them. We now also need the characteristic functions of the sets A_j and B_k . Call $\bar{a}_j \in F(A_j, S)$ the characteristic function of A_j defined by: $\forall \lambda_j \in A_j \Rightarrow \bar{a}_j(\lambda_j) = 1$, $\forall \lambda_j \notin A_j \Rightarrow \bar{a}_j(\lambda_j) = 0$, Analogously, we call *mutatis mutandis* $\bar{b}_k \in F(B_k, S)$ the characteristic function of B_k defined by: $\forall \mu_k \in B_k \Rightarrow \bar{b}_k(\mu_k) = 1$, $\forall \mu_k \notin B_k \Rightarrow \bar{b}_k(\mu_k) = 0$.

Let us now write the term that contains $a_1 b_1$ in this formalism. As $a_1 b_1 = a_1 b_1 \bar{a}_2 \bar{b}_2$, because the added factors are equal to 1, we have:

³ Note that it would not be possible to write an equation analogous to Eq. 14 if we used $V_1 \cup V_2$ because we would obtain a probability $p_1(x) + p_2(x)$, $\forall x \in V_1 \cap V_2$, which is wrong. The construction based on Eq. 10 takes thus into account that measurements with different set-ups cannot be carried out simultaneously. We may note that the construction we present here is not general, because it considers only the four set-ups used in the actual experiments. In order to be entirely general, we should allow for an infinity of set-ups with an infinity of angles between the two polarizers. Taking into account the need to renormalize the probabilities we discover in this Subsection, this leads then to zero probabilities as discussed in [2].

$$\begin{aligned}
& \int_{A_1 \times A_2 \times B_1 \times B_2 \times P} a_1(\lambda_1, \zeta) \bar{a}_2(\lambda_2) b_1(\mu_1, \zeta) \bar{b}_2(\mu_2) \cdot f_1(\lambda_1) f_2(\lambda_2) g_1(\mu_1) g_2(\mu_2) h(\zeta) \cdot d\lambda_1 d\lambda_2 d\mu_1 d\mu_2 d\zeta = \\
& \int_{A_1 \times B_1 \times P} a_1(\lambda_1, \zeta) b_1(\mu_1, \zeta) f_1(\lambda_1) g_1(\mu_1) h(\zeta) d\lambda_1 d\mu_1 d\zeta \int_{A_2} \bar{a}_2(\lambda_2) f_2(\lambda_2) d\lambda_2 \int_{B_2} \bar{b}_2(\mu_2) g_2(\mu_2) d\mu_2 = \\
& \int_{A_1 \times B_1 \times P} a_1(\lambda_1, \zeta) b_1(\mu_1, \zeta) f_1(\lambda_1) g_1(\mu_1) h(\zeta) d\lambda_1 d\mu_1 d\zeta \int_{A_2} f_2(\lambda_2) d\lambda_2 \int_{B_2} g_2(\mu_2) d\mu_2 = \\
& \int_{A_1 \times B_1 \times P} a_1(\lambda_1, \zeta) b_1(\mu_1, \zeta) f_1(\lambda_1) g_1(\mu_1) h(\zeta) d\lambda_1 d\mu_1 d\zeta. \quad (16)
\end{aligned}$$

As we can do this for every term in the Bell inequality, this seems to suggest that $f_1 f_2 g_1 g_2 h \in F(A_1 \times A_2 \times B_1 \times B_2 \times P, [0, \infty])$ could be the common distribution desired. This would then be the existence proof. But, once again, this proof is flawed. We stated in the lines just after Eq. 4 that all correlations must be laid down correctly in the definition of $p \in F(V, [0, \infty])$, but this is here not the case. In fact, the photons do not explore the full set $A_1 \times B_1$. They only explore a subset of $A_1 \times B_1$. E.g. if $C_1 \subset \mathbb{R}^3$ is the space occupied by the set of molecules that intervene in A_1 , and $D_1 \subset \mathbb{R}^3$ the space occupied by the set of molecules that intervene in B_1 , then the photons do not explore $C_1 \times D_1$. If $\mathbf{r}_1 \in C_1$ is explored by a photon, then only $-\mathbf{r}_1 \in D_1$ will be explored by its buddy photon, not the full set D_1 . The set of coordinates explored is thus a set of correlated pairs $(\mathbf{r}_1, -\mathbf{r}_1)$, rather than the set $C_1 \times D_1$ of uncorrelated pairs $(\mathbf{r}_1, \mathbf{r}_2)$. Hence, although the probability distributions in the two polarizers are independent, their definition domains are correlated because the photons are correlating them! We can thus not factorize the sets as we have done. We cannot write $V_{jk} = A_j \times B_k \times P$, and therefore there is no common set $A_1 \times A_2 \times B_1 \times B_2 \times P$ on which we could describe simultaneously all different terms that occur in the inequality. Despite the intuitive appeal and prima facie success of the second method, the first method is thus actually logically superior, because we refrain from trying to decompose V_{jk} and this way we avoid making the error that occurs in the second method.

3.3 Caveats

Consider a first person who rebuts the first repair scenario. He might be unaware of the second repair scenario and think then that he has proved conclusively that the Bell inequalities are wrong, while he might be considered as a fool by a second person who believes in the second repair scenario and is not aware of its errors and of the first repair scenario. He will believe that the first person is stupid and completely has missed the point. But the situation is completely symmetrical. The first person may consider the second person as foolish. In reality both persons are trying to contribute to science in a very serious constructive way. They are not at all ridiculous, but they are both falling prey to unfair scorn because the incompleteness of the proof of the Bell inequality turns the context into a minefield. Clauser *et al.* should not have introduced a tacit assumption without proof. They should have given a complete proof without any possible loopholes. Instead of that people are now kept guessing if a complete proof could exist and on what kind of approach it could be based. A correct proof must only leave it to people to check if it is true, and not leave it to people to fill some gaps. The proof that has been given is wrong because it is at the very best incomplete. It forces people then to imagine a plethora of possible approaches that could be attempted in order complete it, because they would like to prove that the inequality is wrong, after figuring out that the proof is wrong.

In fact, as long as we have not proved that the inequality is wrong, we permanently risk that people may come up with alternative scenarios whereby a subtle point has been missed. They will then unwittingly revert the charge of proof, as it becomes then the responsibility of others to figure out in great pains where the unsuspected error occurs. We mentioned one alternative and its error in Footnote 2. It is for this reason that it cannot possibly be our task to prove that Eq. 7 is wrong. This reversal of the charge of proof has already been lingering on for several decades now. In the two examples given above, the error was hiding in a little corner. In both cases I went through immense frustration because for a long time I just could not spot the error that would come to my rescue. That was also the case with the tacit *Ansatz* of a common probability distribution in the first place. In the situation as it stands, the inequality has just not been proved.

Kupczynski [4] and Nieuwenhuizen [5] have also pointed out how the tacit assumption of a common probability distribution thwarts the application of the Bell inequalities to the experiments of Aspect *et al.*, actually a long time before me. I was not aware of this work and figured this out independently. I hope that at least my work may strengthen the case, because entanglement would have been a very serious and puzzling conceptual issue. It is important to point out that in our approach the angle $\alpha_j - \beta_k$ is *non-locally defined*, without any need for signalling and without any violation of relativity, such that this non-locality is not an issue!

3.4 Epilogue

We may note that we could have stopped our rebuttal of the Bell inequalities at the end of Section 2. All the additions are just forced upon us by physicists, who are inert to requests for extreme rigour, such that we are forced to anticipate that they could pooh-pooh the objection in Section 2 despite the fact that it is a completely pertinent, terrible blow to the derivation of the Bell inequalities. We do not have the charge of proof to show that Eq. 7 is wrong. It is mathematically clear enough that the necessity to provide an existence proof is absolutely compelling. Asking more from me amounts to reverting the charge of proof. It cannot be anybody's task to imagine all possible scenarios to define a common probability distribution and prove them wrong. We only have to express reasonable doubt about the hidden assumption, making the reader wonder if it really goes without saying that we can introduce it without proof, under the pretext that it would be self-evident. Nevertheless, we have developed some further considerations in a previous version of this paper [6]. We have removed them because they cannot be assessed without a prior knowledge of the author's work on spinors.

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