Self-acceleration in scalar-bimetric theories

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We describe scalar-bimetric theories where the dynamics of the Universe are governed by two separate metrics, each with an Einstein-Hilbert term. In this setting, the baryonic and dark matter components of the Universe couple to metrics which are constructed as functions of these two gravitational metrics. More precisely, the two metrics coupled to matter are obtained by a linear combination of their vierbeins, with scalar-dependent coefficients. The scalar field, contrary to dark-energy models, does not have a potential of which the role is to mimic a late-time cosmological constant. The late-time acceleration of the expansion of the Universe can be easily obtained at the background level in these models by appropriately choosing the coupling functions appearing in the decomposition of the vierbeins for the baryonic and dark matter metrics. We explicitly show how the concordance model can be retrieved with negligible scalar kinetic energy. This requires the scalar coupling functions to show variations of order unity during the accelerated expansion era. This leads in turn to deviations of order unity for the effective Newton constants and a fifth force that is of the same order as Newtonian gravity, with peculiar features. The baryonic and dark matter self-gravities are amplified although the gravitational force between baryons and dark matter is reduced and even becomes repulsive at low redshift. This slows down the growth of baryonic density perturbations on cosmological scales, while dark matter perturbations are enhanced. These scalar-bimetric theories have a perturbative cutoff scale of the order of 1 AU, which prevents a precise comparison with Solar System data. On the other hand, we can deduce strong requirements on putative UV completions by analyzing the stringent constraints in the Solar System. Hence, in our local environment, the upper bound on the time evolution of Newton's constant requires an efficient screening mechanism that both damps the fifth force on small scales and decouples the local value of Newton constant from its cosmological value. This cannot be achieved by a quasistatic chameleon mechanism and requires going beyond the quasistatic regime and probably using derivative screenings, such as Kmouflage or Vainshtein screening, on small scales.

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I. INTRODUCTION

A very common way of reproducing the late-time acceleration of the expansion of the Universe [1,2] is to add a scalar-field energy density, which would mimic a cosmological constant at small redshifts [3]. Recently, it has been proposed that the acceleration could be an illusion due to the different metrics coupled to either the baryons or dark matter [4]. This was achieved by considering that baryons couple to a metric that can be constructed from both the metric felt by dark matter and the velocity field of the dark matter particles. In the same vein, it has been known for some time that conformally coupled models with a single metric and screening properties, thus evading the local tests of gravity, cannot generate the late-time acceleration of the Universe [5]. In this paper, we generalize the latter approach by introducing two gravitational metrics, with an Einstein-Hilbert term each, and we consider that the baryons and dark matter couple to different dynamical metrics. These metrics are obtained by taking linear combinations of the two gravitational vierbeins, with each of the coefficients dependent on a scalar field. Contrary to dark-energy models (even coupled), we do not require that the scalar field should play any explicit role in generating an effective cosmological constant at late time. Quite the contrary, the scalar is only a free and massless scalar, with positive pressure. The role of the scalar is to provide a time-dependent mapping and transform the deceleration of the two gravitational metrics into an acceleration for the baryonic metric.

Our approach is inspired by the construction of doubly coupled bigravity models [6,7] where the late-time acceleration is due to an explicit cosmological constant, albeit related to the mass of the massive graviton [8,9], and matter couples to a combination (with constant coefficients) of the two dynamical metrics [10]. Here, we remove the potential term of massive gravity and introduce scalar-dependent coupling functions, as our goal is to build self-accelerated solutions. As expected, we find that this leads to major difficulties, as a self-acceleration implies effects of order unity on cosmological scales. This generically gives rise to

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effective Newton constants that evolve on Hubble timescales and a fifth force of the same order as Newtonian gravity.

The models that we construct have no ghost in Minkowski space, where they correspond simply to two copies of General Relativity. When matter is introduced, only the diagonal diffeomorphism invariance is preserved. The order parameter of the symmetry breaking, from two copies of diffeomorphism invariance to the diagonal one, is the Hubble parameter induced by the matter sectors. When performing a Stückelberg analysis of the breaking pattern and introducing the corresponding scalar Goldstone mode, we find that the absence of ghosts associated with an Ostrogradsky instability is guaranteed below the cutoff scale $\Lambda_{\rm cut} = (H^3 M_{\rm Pl})^{1/4}$. This energy scale is smaller than the strong coupling scale $\Lambda_3 = (H^2 M_{\rm Pl})^{1/3}$. Thus, $\Lambda_{\rm cut}^{-1} \sim 1$ AU, whereas $\Lambda_3^{-1} \sim 1000$ km in the late-time Universe, but $\Lambda_{\rm cut}^{-1}$ remains much below cosmological scales. Down to the scale $\Lambda_{\rm cut}^{-1}$ and around compact objects in the weak gravitational regime, the scalar Goldstone mode is decoupled from matter, and no Vainshtein mechanism is at play. For scales below $\Lambda_{\rm cut}^{-1}$, it is very likely that the models should be altered. Notice, too, the analogy with doubly coupled bigravity, where the order parameter of diffeomorphism breaking is given by the graviton mass m as appearing in the potential term of massive gravity and the strong coupling is given by $\Lambda_3 = (m^2 M_{\rm Pl})^{1/3}$. In this case, too, a ghost is known to be present at higher energy, and the models should also be completed.

Our analysis of the presence of ghosts and the existence of a cutoff scale has been performed perturbatively around Friedmann-Lemaitre-Robertson-Walker backgrounds. We have found that at energies higher than the perturbative cutoff scale a ghost is likely to exist due to the mixing between the tensor modes and higher derivatives in the Stückelberg field. It is quite likely that a nonperturbative analysis along the lines of Refs. [7,11,12] would unravel the existence of nonperturbative effects which would lower the cutoff scale and reduce the domain of validity of the scalar-bimetric models. This is left for future work. Here, we only focus on the perturbative cutoff scale and treat the corresponding range as the one where the scalar-bimetric models are well defined.

We mostly focus on the late-time Universe, in the matter and dark-energy eras. However, doubly coupled bigravity theories suffer from instabilities in the radiation era [13–15] for tensor and vector modes. We briefly rederive these behaviors for our models. Tensor modes have a tachyonic regime that implies an anomalous growth in the early Universe. This has some effect on the cosmic microwave background B-modes which may be amplified [16] in models where there is a nonlinear coupling between the metrics, such as in bigravity theories. Similarly, the vector modes that are decoupled from matter suffer from a gradient instability which could pose serious problems for the viability of the models. However, in our case, these instabilities only affect "hidden" modes that are not seen by the matter metrics (at the linear level).

In this paper, we do not perform detailed comparisons with cosmological and astrophysical data, as our goal is only to distinguish which families of solutions offer a realistic framework, which may deserve further investigations. Indeed, imposing a Λ-CDM expansion history for the cosmological background (which ensures consistency with cosmological data at the background level), we find that the tight constraint on the velocity of gravitational waves [17] already provides significant constraints on the model. Moreover, we find that a nonlinear screening mechanism [18] must come into play on small scales, to ensure convergence to General Relativity in the Solar System. This follows from the upper bound on the local time dependence of Newton's constant [19], which would have to be obeyed by any UV completion on scales below the cutoff of order 1 AU. This must go beyond the quasistatic approximation and probably rely on derivatives of the scalar field (as in Kmouflage [20–22] or Vainshtein mechanisms [23]), while quasistatic chameleon screening [24,25] cannot occur. We leave the analysis of this regime for future work.

This article is organized as follows. We first define the bimetric model in Sec. II and next provide the equations of motion in Sec. II C. We describe the cosmological background in Sec. III. We show how to construct solutions that mimic a Λ-CDM expansion and discuss both the simplified cases where all metrics have the same conformal time and the cases where they have different conformal times. We turn to linear perturbations in Sec. IV, for both baryonic and matter density fluctuations. We then describe in Sec. V how linear perturbations behave beyond the quasistatic approximation. We consider the possible presence of ghosts in Sec. VI. We then compare our results to doubly coupled bigravity in Sec. VII. We discuss consistency with small-scale tests of General Relativity in Sec. VIII and conclude in Sec. IX. Several Appendixes are dedicated to more technical details.

II. SCALAR-BIMETRIC MODELS

A. Defining the models

In the following, we focus on models where the dynamics are driven by two independent metrics coupled to a scalar field. We do not add any nontrivial dynamics for the scalar field, which we choose to be massless with a canonical kinetic term. We consider models with the scalar-bimetric action

$$S = S_{\rm grav} + S_{\rm mat}, \tag{1}$$

with

$$S_{\rm grav} = \int d^4x \frac{M_{\rm pl}^2}{2}[\sqrt{-g_1}R_1 + \sqrt{-g_2}R_2], \quad (2)$$

and

$$S_{\rm mat} = \int d^4x[\sqrt{-g_{\rm d}}\mathcal{L}_{\rm d}(\varphi, \psi^i_{\rm dm}; g_{\rm d}) + \sqrt{-g_{\rm b}}\mathcal{L}_{\rm b}(\psi^i_{\rm b}; g_{\rm b})]. \quad (3)$$

The gravitational action $S_{\rm grav}$ contains two Einstein-Hilbert terms for the two gravitational metrics $g_{1\mu\nu}$ and $g_{2\mu\nu}$. The matter action $S_{\rm mat}$ contains the dark sector Lagrangian $\mathcal{L}_{\rm d}$, which includes dark matter fields $\psi^i_{\rm dm}$ and an additional scalar field φ, and the baryonic Lagrangian $\mathcal{L}_{\rm b}$, which includes the ordinary particles of the standard model, both matter and radiation (photons) components. These two matter Lagrangians involve two associated dynamical metrics, $g_{{\rm d}\mu\nu}$ and $g_{{\rm b}\mu\nu}$. In the following, we will usually omit the subscript b, as this is the main sector that is probed by observations and experiments.

We split the dark sector Lagrangian in its scalar-field and dark matter components,

$$\mathcal{L}_{\rm d} = \mathcal{L}_\varphi(\varphi; g_{\rm d}) + \mathcal{L}_{\rm dm}(\psi^i_{\rm dm}; g_{\rm d}), \quad (4)$$

and for simplicity, we only keep the kinetic term in the scalar-field Lagrangian,

$$\mathcal{L}_\varphi(\varphi) = -\frac{1}{2} g_{\rm d}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi, \quad (5)$$

as we wish to recover the late-time acceleration of the expansion of the Universe through a dynamical mechanism, rather than through an effective cosmological constant associated with a nonzero minimum of the scalar-field potential.

The dark and baryonic metrics are functions of the two gravitational metrics $g_{1\mu\nu}$ and $g_{2\mu\nu}$. We write this relationship in terms of the vierbeins of these four metrics. Thus, introducing the vierbeins $e^a_{1\mu}$ and $e^a_{2\mu}$ of the metrics $g_{1\mu\nu}$ and $g_{2\mu\nu}$,

$$g_{1\mu\nu} = e^a_{1\mu}e^b_{1\nu}\eta_{ab}, \qquad g_{2\mu\nu} = e^a_{2\mu}e^b_{2\nu}\eta_{ab}, \quad (6)$$

we define the dark and baryonic metrics as

$$g_{{\rm d}\mu\nu} = e^a_{{\rm d}\mu}e^b_{{\rm d}\nu}\eta_{ab}, \qquad g_{\mu\nu} = e^a_\mu e^b_\nu\eta_{ab}, \quad (7)$$

with

$$e^a_{{\rm d}\mu} = s_{\rm d1}(\varphi)e^a_{1\mu} + s_{\rm d2}(\varphi)e^a_{2\mu},$$
$$e^a_\mu = s_1(\varphi)e^a_{1\mu} + s_2(\varphi)e^a_{2\mu}. \quad (8)$$

Thus, both dynamical matter metrics are a combination of the two gravitational metrics that depend on the scalar field φ. This leads to nonminimal couplings between the matter sectors and the scalar field.

B. Number of components and degrees of freedom

Although we have defined the model in terms of the vierbeins of the two dynamical metrics, we treat the theory as a metric theory, which is a function of the two metrics $g_{\ell\mu\nu}$, $i = 1, 2$. The two Einstein-Hilbert terms are invariant under two copies of the diffeomorphism group. The coupling to matter, which involves the diagonal subgroup, reduces the diffeomorphism invariance to one diagonal copy. The two metrics involve 20 components, which can be reduced to 16 when the diagonal gauge invariance under reparametrization of coordinates has been used. The vierbeins are four-by-four matrices, which involve $2 \times 16 = 32$ components. This is redundant even when the diagonal diffeomorphism invariance has been used, reducing to 28 the number of components. The two vierbeins have two copies of the local Lorentz symmetry group as an invariance group. Again, this is broken to the diagonal Lorentz group by the coupling to matter. This removes six components, bringing it down to 22. This is still more than the 16 components of the metric description. This can be made to coincide by imposing the symmetric condition [26]

$$Y_{\mu\nu} = Y_{\nu\mu}, \quad (9)$$

where we have introduced the tensor

$$Y_{\mu\nu} = \eta_{ab}e^a_{1\mu}e^b_{2\nu} = e^a_{1\mu}e_{2a\nu}. \quad (10)$$

The 4×4 tensor $Y_{\mu\nu}$ can be decomposed into ten symmetric components and six antisymmetric ones which are imposed to be vanishing. This brings the number of vierbein components down to 16, matching the ones for a bimetric theory.

The consequences of the symmetric conditions are well known; let us recall some salient features here. First of all, let us define

$$X^\mu_{2\nu} = g_2^{\mu\lambda}Y_{\lambda\nu} = e^\mu_{2a}e^a_{1\nu}, \qquad X^\mu_{1\nu} = g_1^{\mu\lambda}Y_{\lambda\nu} = e^\mu_{1a}e^a_{2\nu}; \quad (11)$$

then, we have that

$$X^\theta_{2\nu}X^\nu_{2\lambda} = g_2^{\theta\nu}g_{1\nu\lambda}, \qquad X^\theta_{1\nu}X^\nu_{1\lambda} = g_1^{\theta\nu}g_{2\nu\lambda}, \quad (12)$$

which implies that in matrix notation

$$Y = g_2(g_2^{-1}g_1)^{1/2} = g_1(g_1^{-1}g_2)^{1/2}, \quad (13)$$

with an appropriate definition for the square root of a matrix [26]. As a result, $Y_{\mu\nu}$ becomes a function of the two metrics $g_{\ell\mu\nu}$, which implies that the dark and baryonic metrics,

$$g_{\mu\nu} = s_1^2(\varphi)g_{1\mu\nu} + 2s_1(\varphi)s_2(\varphi)Y_{\mu\nu} + s_2^2(\varphi)g_{2\mu\nu},$$
$$g_{\mathrm{d}\mu\nu} = s_{\mathrm{d}1}^2(\varphi)g_{1\mu\nu} + 2s_{\mathrm{d}1}(\varphi)s_{\mathrm{d}2}(\varphi)Y_{\mu\nu} + s_{\mathrm{d}2}^2(\varphi)g_{2\mu\nu}, \quad (14)$$

are simply functions of the two metrics, too.

Not all the components become physical degrees of freedom. For instance, when no matter is present, our models reduce to two copies of General Relativity and as such only carry two copies of massless gravitons, i.e., 2×2 physical degrees of freedom. When matter is present, in particular cosmologically, the Hubble expansion rate of one of the two types of matter becomes the order parameter of the symmetry breaking pattern $(\mathrm{diff}_1 \times \mathrm{diff}_2)/\mathrm{diff}_{\mathrm{diag}}$, where the two copies of diffeomorphism invariance are broken down to the diagonal subgroup. As such, we could expect that four Goldstone bosons ξ^μ could become physical. In fact, we find that out of the divergenceless vector and the two scalars associated with ξ^μ only the two independent components of the vector are dynamical. The validity of the model can be probed by a Stückelberg analysis, where we focus on the scalar $\xi^\mu = \partial^\mu \pi$, and we show in Sec. VI below that no ghost appears below the cutoff scale of order $\Lambda_{\mathrm{cut}} = (H^3 M_{\mathrm{Pl}})^{1/4}$. This is the physical regime we analyze in this paper. In particular, it applies to cosmology as the horizon scale H^{-1} is always much larger than $\Lambda_{\mathrm{cut}}^{-1}$ since the very early Universe. Only in the Solar System, as the cutoff scale is of order 1 AU, shall we be prevented from strong conclusions for want of explicit UV completions.

C. Equations of motion

1. Einstein's equations

We cannot obtain the Einstein equations by requiring the functional derivatives of the action with respect to the vierbeins $e^a_{1\mu}$ and $e^a_{2\mu}$ to vanish. Indeed, because of the symmetry condition (9), which reduces the number of components to those of $g_{1\mu\nu}$ and $g_{2\mu\nu}$, the vierbeins are correlated and constrained by Eq. (9). This means that we must take the variations along the directions that span the subspace defined by the constraint (9). If we vary the metric g_1 while keeping g_2 fixed, hence we vary $e^a_{1\mu}$ at fixed $e^a_{2\mu}$, the symmetric constraint (9) reads as

$$\delta e^a_{1\mu} e_{2a\nu} = \delta e^a_{1\nu} e_{2a\mu} \quad \text{for all } \{\mu,\nu\}. \quad (15)$$

We can check that these constraints are satisfied if the variations $\delta e^a_{1\mu}$ are of the form

$$\delta e^a_{1\mu} = \delta Z_{1\mu\nu} e_2^{a\nu}, \quad (16)$$

where $\delta Z_{1\mu\nu}$ is an arbitrary infinitesimal symmetric matrix, $\delta Z_{1\mu\nu} = \delta Z_{1\nu\mu}$. As expected, the matrix $\delta Z_{1\mu\nu}$ provides the same number of components as the metric $g_{1\mu\nu}$. This also gives $\delta g_{1\mu\nu} = \delta Z_{1\mu\lambda} X^\lambda_{2\nu} + \delta Z_{1\nu\lambda} X^\lambda_{2\mu}$. Then, the Einstein equations follow from the variation of the action with respect to $\delta Z_{1\mu\nu}$. We can write

$$\frac{\delta e^a_{1\lambda}}{\delta Z_{1\mu\nu}} = \Theta_{\mu\nu}(\delta^\mu_\lambda e_2^{a\nu} + \delta^\nu_\lambda e_2^{a\mu}), \quad (17)$$

where $\Theta_{\mu\nu} = 1$ if $\mu < \nu$ and $\Theta_{\mu\nu} = 1/2$ if $\mu = \nu$. Here, we restrict to $\mu \le \nu$ as $\delta Z_{1\mu\nu} = \delta Z_{1\nu\mu}$ so that $\delta Z_{1\mu\nu}$ and $\delta Z_{1\nu\mu}$ are not independent. This gives

$$\text{for all } \{\mu,\nu\}: \ \frac{\delta S}{\delta e^a_{1\mu}} e_2^{a\nu} + \frac{\delta S}{\delta e^a_{1\nu}} e_2^{a\mu} = 0. \quad (18)$$

This provides the expected 16 symmetric Einstein equations (hence, ten equations, before we use diffeomorphism invariance), which read as

$$M^2_{\mathrm{Pl}}\sqrt{-g_1}[G_1^{\mu\sigma}X^\nu_{2\sigma} + G_1^{\nu\sigma}X^\mu_{2\sigma}]$$
$$= s_1\sqrt{-g}[T^{\mu\sigma}(s_1 X^\nu_{2\sigma} + s_2\delta^\nu_\sigma) + T^{\nu\sigma}(s_1 X^\mu_{2\sigma} + s_2\delta^\mu_\sigma)]$$
$$+ s_{\mathrm{d}1}\sqrt{-g_{\mathrm{d}}}[T_{\mathrm{d}}^{\mu\sigma}(s_{\mathrm{d}1}X^\nu_{2\sigma} + s_{\mathrm{d}2}\delta^\nu_\sigma) + T_{\mathrm{d}}^{\nu\sigma}(s_{\mathrm{d}1}X^\mu_{2\sigma} + s_{\mathrm{d}2}\delta^\mu_\sigma)]. \quad (19)$$

The Einstein equations with respect to the second metric g_2 are obtained by exchanging the indices $1 \leftrightarrow 2$ [27]. Here, $T^{\mu\nu}$ and $T_{\mathrm{d}}^{\mu\nu}$ are the baryonic and dark-energy energy-momentum tensors, defined with respect to their associated metrics,

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\mathrm{b}})}{\delta g^{\mu\nu}}, \qquad T_{\mathrm{d}\mu\nu} = \frac{-2}{\sqrt{-g_{\mathrm{d}}}}\frac{\delta(\sqrt{-g_{\mathrm{d}}}\mathcal{L}_{\mathrm{d}})}{\delta g_{\mathrm{d}}^{\mu\nu}}. \quad (20)$$

We recover the standard Einstein equations when the two metrics are identical (case of a single-metric model) with $s_1 + s_2 = 1$ and $s_{\mathrm{d}1} + s_{\mathrm{d}2} = 1$, as it yields $X^\mu_{2\nu} = X^\mu_{1\nu} = \delta^\mu_\nu$.

When the metrics are diagonal, that is, we have

$$g_{*\mu\nu} \subset \delta_{\mu\nu}, \qquad e^a_{*\mu} \subset \delta^a_\mu, \qquad Y_{\mu\nu} \subset \delta_{\mu\nu}, \qquad X^\mu_{\ell\nu} \subset \delta^\mu_\nu,$$

where $* = \{1, 2, \mathrm{d}, \mathrm{b}\}$ and $g_{*\mu\nu} \subset \delta_{\mu\nu}$ means that $g_{*\mu\nu} = 0$ for $\mu \neq \nu$, the Einstein equations (19) simplify along the diagonal as (no summation over μ)

$$M^2_{\mathrm{Pl}}\sqrt{-g_1}G_1^{\mu\mu}e^a_{1\mu} = s_1\sqrt{-g}T^{\mu\mu}e^a_\mu + s_{\mathrm{d}1}\sqrt{-g_{\mathrm{d}}}T_{\mathrm{d}}^{\mu\mu}e^a_{\mathrm{d}\mu}$$

with $a = \mu$. This coincides with the Einstein equations that would have been obtained by taking derivatives with respect to the vierbeins without taking care of the symmetric constraint (9). However, the off-diagonal Einstein equations remain modified.

2. Scalar-field equation of motion

The dependence of the matter metrics $g_{\mathrm{d}\mu\nu}$ and $g_{\mu\nu}$ on the scalar field φ, through Eq. (8), gives rise to source terms in the Klein-Gordon equation that governs the scalar-field dynamics,

$$\partial_\mu[\sqrt{-g_{\mathrm{d}}}g_{\mathrm{d}}^{\mu\nu}\partial_\nu\varphi] + \sqrt{-g_{\mathrm{d}}}T_{\mathrm{d}}^{\mu\nu}\sum_{\ell=1}^{2}\frac{ds_{\mathrm{d}\ell}}{d\varphi}e_{\ell\mu}^{a}e_{\mathrm{d}\nu}^{b}\eta_{ab} + \sqrt{-g}T^{\mu\nu}\sum_{\ell=1}^{2}\frac{ds_\ell}{d\varphi}e_{\ell\mu}^{a}e_{\nu}^{b}\eta_{ab} = 0. \tag{21}$$

3. Matter equations of motion

The equations of motion of the dark and baryonic matter components take their standard form in their Jordan frames,

$$\nabla_{\mathrm{d}\mu}T_{\mathrm{dm}\nu}^{\mu} = 0, \qquad \nabla_\mu T_\nu^\mu = 0, \tag{22}$$

where $\nabla_{\mathrm{d}\mu}$ and ∇_μ are the covariant derivatives with respect to the metrics $g_{\mathrm{d}\mu\nu}$ and $g_{\mu\nu}$.

III. COSMOLOGICAL BACKGROUND

In this section, we investigate the cosmological backgrounds that can be achieved in these scalar-bimetric scenarios. We show how we recover a standard cosmology at high redshift, when the scalar field is almost constant and plays no role and all metrics follow the same expansion, whereas a self-accelerated expansion without a cosmological constant can be achieved at low redshift thanks to the running of the scalar field, through the interplay between the matter and gravitational metrics.

A. Friedmann's equations

We consider diagonal metrics of the form

$$g_{*\mu\nu}(\tau) = \mathrm{diag}(-b_*^2(\tau), a_*^2(\tau), a_*^2(\tau), a_*^2(\tau)), \tag{23}$$

where $* = \{1, 2, \mathrm{d}, \mathrm{b}\}$, with the vierbeins

$$e_{*\mu}^{a} = \mathrm{diag}(b_*, a_*, a_*, a_*), \tag{24}$$

and we denote by $\mathcal{H}_{b_*} = d\ln b_*/d\tau$, $\mathcal{H}_{a_*} = d\ln a_*/d\tau$, the conformal expansion rates of the time and spatial components. We can choose to define the conformal time τ with respect to the baryonic metric $g_{\mu\nu}$, so that

$$b = a, \qquad g_{\mu\nu}(\tau) = \mathrm{diag}(-a^2, a^2, a^2, a^2), \tag{25}$$

and we use either τ or $\ln(a)$ as the time variable. From the definitions (8), we obtain the constraints

$$b = a = s_1 b_1 + s_2 b_2, \qquad a = s_1 a_1 + s_2 a_2,$$
$$b_{\mathrm{d}} = s_{\mathrm{d}1} b_1 + s_{\mathrm{d}2} b_2, \qquad a_{\mathrm{d}} = s_{\mathrm{d}1} a_1 + s_{\mathrm{d}2} a_2. \tag{26}$$

The (0,0) component of the Einstein equations (19) reads

$$3M_{\mathrm{Pl}}^2 a_\ell^3 b_\ell^{-2}\mathcal{H}_{a_\ell}^2 = s_\ell a^3(\bar\rho + \bar\rho_\gamma) + s_{\mathrm{d}\ell}a_{\mathrm{d}}^3(\bar\rho_{\mathrm{dm}} + \bar\rho_\varphi), \tag{27}$$

while the (i,i) components read

$$M_{\mathrm{Pl}}^2 a_\ell^2 b_\ell^{-1}[2\mathcal{H}'_{a_\ell} + 3\mathcal{H}_{a_\ell}^2 - 2\mathcal{H}_{a_\ell}\mathcal{H}_{b_\ell}] = -s_\ell a^3\frac{\bar\rho_\gamma}{3} - s_{\mathrm{d}\ell}a_{\mathrm{d}}^2 b_{\mathrm{d}}\bar\rho_\varphi. \tag{28}$$

Here, we assumed nonrelativistic matter components, $p_{\mathrm{dm}} = p = 0$, and we used $p_\gamma = \rho_\gamma/3$ and $\bar p_\varphi = \bar\rho_\varphi$ for the radiation and scalar pressure.

B. Conservation equations

The Jordan-frame equations of motion (22) lead to the usual conservation equations; hence,

$$\bar\rho_{\mathrm{dm}} = \frac{\bar\rho_{\mathrm{dm}0}}{a_{\mathrm{d}}^3}, \qquad \bar\rho = \frac{\bar\rho_0}{a^3}, \qquad \bar\rho_\gamma = \frac{\bar\rho_{\gamma0}}{a^4}. \tag{29}$$

We define the cosmological parameters associated with these characteristic densities by

$$\Omega_{\mathrm{b}0} = \frac{\bar\rho_{\mathrm{b}0}}{3M_{\mathrm{Pl}}^2 H_0^2}, \quad \Omega_{\gamma0} = \frac{\bar\rho_{\gamma0}}{3M_{\mathrm{Pl}}^2 H_0^2}, \quad \Omega_{\mathrm{dm}0} = \frac{\bar\rho_{\mathrm{dm}0}}{3M_{\mathrm{Pl}}^2 H_0^2}, \tag{30}$$

where H_0 is the physical expansion rate associated with the baryonic metric today, at $a = 1$. We also define the rescaled scalar-field energy density ξ as the ratio of the scalar field to dark matter energy densities

$$\xi(a) = \frac{a_{\mathrm{d}}^3\bar\rho_\varphi}{3M_{\mathrm{Pl}}^2 H_0^2}, \qquad \frac{\xi(a)}{\Omega_{\mathrm{dm}0}} = \frac{\bar\rho_\varphi(a)}{\bar\rho_{\mathrm{dm}}(a)}. \tag{31}$$

It is convenient to introduce the dimensionless combination

$$\ell = 1,2:\ \omega_\ell = a_\ell^3 b_\ell^{-2}\frac{\mathcal{H}_{a_\ell}^2}{H_0^2}. \tag{32}$$

Then, the Friedmann equations (27) and (28) simplify as

$$\omega_\ell = s_\ell\left(\Omega_{\mathrm{b}0} + \frac{\Omega_{\gamma0}}{a}\right) + s_{\mathrm{d}\ell}(\Omega_{\mathrm{dm}0} + \xi(a)), \tag{33}$$

$$\frac{b_\ell}{a_\ell}\frac{\mathcal{H}}{\mathcal{H}_{a_\ell}}\frac{d\omega_\ell}{d\ln a} = -s_\ell\frac{\Omega_{\gamma0}}{a} - s_{\mathrm{d}\ell}3\frac{b_{\mathrm{d}}}{a_{\mathrm{d}}}\xi. \tag{34}$$

We recover the usual Friedmann equations of General Relativity with $\xi = 0$, $b_* = a_*$, $\mathcal{H} = \mathcal{H}_{a_\ell}$, $s_\ell = 1$, and $s_{\mathrm{d}\ell} = 1$. In this case, we can check that the second Friedmann equation is a consequence of the first Friedmann equation and of the conservation equations, as it is the derivative of Eq. (33) with respect to $\ln a$.

By taking the first derivative of Eq. (33) and combining with Eq. (34), we obtain the useful combinations

$$\frac{ds_\ell}{d\ln a}\left(\Omega_{\mathrm{b}\,0} + \frac{\Omega_{\gamma 0}}{a}\right) + \frac{ds_{\mathrm{d}\ell}}{d\ln a}(\Omega_{\mathrm{dm}\,0} + \xi(a)) = s_\ell \frac{\Omega_{\gamma 0}}{a}\left(1 - \frac{a_\ell}{b_\ell}\frac{\mathcal{H}_{a_\ell}}{\mathcal{H}}\right) - s_{\mathrm{d}\ell}\xi\left(\frac{d\ln\xi}{d\ln a} + 3\frac{b_{\mathrm{d}}a_\ell}{a_{\mathrm{d}}b_\ell}\frac{\mathcal{H}_{a_\ell}}{\mathcal{H}}\right). \tag{35}$$

This shows that the evolutions of the baryonic and dark matter couplings are correlated and related to the running of the scalar field ($\xi > 0$) and the deviations between the different expansion rates $\mathcal{H}_*$. In the absence of the scalar field in the dynamics, the relation (35) reduces to the branch of solutions

$$\frac{a_\ell}{b_\ell}\mathcal{H}_{a_\ell} = \mathcal{H}, \tag{36}$$

which appears in doubly coupled bigravity [15].

C. Scalar-field equation of motion

The scalar-field energy density reads as

$$\bar{\rho}_\varphi = \frac{1}{2b_{\mathrm{d}}^2}\left(\frac{d\bar{\varphi}}{d\tau}\right)^2. \tag{37}$$

Then, we can check that the background Klein-Gordon equation (21) can be written in terms of $\bar{\rho}_\varphi$. Using the rescaled scalar-field density ξ of Eq. (31), this gives

$$\frac{b_{\mathrm{d}}}{a_{\mathrm{d}}^3}\frac{d}{d\ln a}[a_{\mathrm{d}}^3\xi] + (\Omega_{\mathrm{dm}\,0} + \xi)\sum_\ell \frac{ds_{\mathrm{d}\ell}}{d\ln a}b_\ell - 3\frac{b_{\mathrm{d}}}{a_{\mathrm{d}}}\xi\sum_\ell\frac{ds_{\mathrm{d}\ell}}{d\ln a}a_\ell + \left(\Omega_{\mathrm{b}0} + \frac{\Omega_{\gamma 0}}{a}\right)\sum_\ell\frac{ds_\ell}{d\ln a}b_\ell - \frac{\Omega_{\gamma 0}}{a}\sum_\ell\frac{ds_\ell}{d\ln a}a_\ell = 0. \tag{38}$$

From $\xi(a)$, we obtain the evolution of the scalar field $\bar{\varphi}(a)$ by integrating Eq. (37). With the initial condition $\bar{\varphi}(0) = 0$, this gives

$$\bar{\varphi}(a) = M_{\mathrm{Pl}}\int_0^a \frac{da}{a}\frac{b_{\mathrm{d}}H_0}{\mathcal{H}}\sqrt{\frac{6\xi}{a_{\mathrm{d}}^3}}. \tag{39}$$

We can actually check that the Klein-Gordon equation (38) is also a consequence of the Friedmann equations (33) and (34), supplemented with the constraints (26). Therefore, as in General Relativity, the Friedmann equations and the conservation equations are not independent. In General Relativity, it is customary to work with the first Friedmann equation and the conservation equations of the various matter components, leaving aside the second Friedmann equation that is their automatic consequence. In this paper, because we have two symmetric sets of Friedmann equations (33) and (34), for $\ell = 1$, 2, and the Klein-Gordon equation (38) takes a complicated form with its new source terms, we instead work with the four Friedmann equations, and we discard the Klein-Gordon equation (38), which is their automatic consequence.

D. Einstein-de Sitter reference

When the scalar field is a constant, it should not play any role, and we expect to recover a standard cosmology. Because we did not introduce any cosmological constant, this must be an Einstein-de Sitter universe without late-time acceleration (more precisely, a universe with only matter and radiation components). In this reference universe, obtained within General Relativity with only one metric, the Friedmann equations (33) and (34) read as

$$\omega^{(0)} = a\frac{\mathcal{H}^{(0)2}}{H_0^2} = \Omega_{\mathrm{dm}0} + \Omega_{\mathrm{b}\,0} + \frac{\Omega_{\gamma 0}}{a}, \quad \frac{d\omega^{(0)}}{d\ln a} = -\frac{\Omega_{\gamma 0}}{a}. \tag{40}$$

Here and in the following, we denote with the superscript (0) quantities associated with this Einstein-de Sitter reference universe, which follows General Relativity. As noticed above, here, the second Friedmann equation is trivial as it is a mere consequence of the first Friedmann equation and of the conservation equations, which have already been used in the first part of Eq. (40).

We can recover the standard cosmology (40) within the bimetric model (1) by the simple solution

$$a_\ell^{(0)} = s_\ell^{(0)}a, \quad a_{\mathrm{d}}^{(0)} = a, \quad b_*^{(0)} = a_*^{(0)}, \quad \mathcal{H}_*^{(0)} = \mathcal{H}^{(0)}, \quad \omega_\ell^{(0)} = s_\ell^{(0)}\omega^{(0)}, \quad \xi^{(0)} = 0, \quad s_{\mathrm{d}\ell}^{(0)} = s_\ell^{(0)}, \tag{41}$$

where the coefficients $s_\ell^{(0)}$ are constants that obey the condition

$$(s_1^{(0)})^2 + (s_2^{(0)})^2 = 1. \tag{42}$$

The scalar field φ is also constant, as the derivatives in the source terms of Eq. (21) vanish. In this solution, all four metrics are essentially equivalent, as $b_* = a_*$ and all scale factors a_* are proportional. The common expansion rate $\mathcal{H}_*(a)$ follows the standard Einstein-de Sitter reference $\mathcal{H}^{(0)}(a)$ of Eq. (40).

E. Λ-CDM reference

To match observations, the expansion rate $\mathcal{H}(a)$ must deviate from the Einstein-de Sitter reference (40) and remain close to the concordance Λ-CDM cosmology. To ensure that this is the case, in this paper, we constrain the baryonic expansion rate $\mathcal{H}(a)$ to follow exactly a reference Λ-CDM cosmology. Of course, in practice, small deviations from the Λ-CDM limit are allowed by the data, and we could also generalize the solutions that we consider in this paper by adding small deviations. However, by definition, this would not significantly modify the properties of these solutions. Besides, being able to recover a Λ-CDM expansion rate is sufficient to show that the bimetric model can be made consistent with data at the level of the cosmological background.

In the Λ-CDM cosmology, we add a cosmological constant to the components of the Universe. The usual Friedmann equation reads as

$$\frac{\mathcal{H}^2}{H_0^2} = \frac{\Omega_{\rm dm0} + \Omega_{\rm b0}}{a} + \frac{\Omega_{\gamma 0}}{a^2} + \Omega_{\Lambda 0} a^2, \qquad (43)$$

where $\Omega_{\Lambda 0}$ is the cosmological parameter associated with the cosmological constant. In terms of the variable ω, this gives

$$\omega(a) = \omega^{(0)}(a) + \Omega_{\Lambda 0} a^3, \qquad (44)$$

which explicitly shows the deviation from the Einstein-de Sitter reference (40). (Here, the Einstein-de Sitter reference is normalized with $\Omega_{\rm dm0} + \Omega_{\rm b0} + \Omega_{\gamma 0} = 1 - \Omega_{\Lambda 0} \neq 1$, because we normalize the cosmological densities by H_0 instead of $H_0^{(0)}$.)

The bimetric solution with a constant scalar field, which was able to reproduce the Einstein-de Sitter cosmology (40), cannot mimic the Λ-CDM cosmology (44) because of the extra term on the right-hand side of the Friedmann equation [a constant scalar field implies $\xi = 0$ in Eq. (33)]. Therefore, to recover a Λ-CDM expansion rate, we must consider more general solutions with a nonconstant scalar field. In particular, even if the scale factors a_i of the gravitational metrics keep decelerating at late times, the baryonic scale factor $a = s_1 a_1 + s_2 a_2$ can accelerate at late times if s_1 or s_2 grows sufficiently fast. Then, the acceleration experienced by the baryonic metric is a dynamical effect due to the time-dependent relationship between this metric and the two gravitational metrics.

On the other hand, at early times where data show that the dark-energy density is negligible, we converge to the simple Einstein-de Sitter solution (41). This will be the common early-time behavior of all the solutions that we consider in this paper. We can check that the integral in Eq. (39) is indeed finite and goes to zero for $a \to 0$, both in the radiation and dark matter eras, provided

$$\xi \to 0 \quad \text{for } a \to 0. \qquad (45)$$

This also ensures that the dark-energy density is negligible as compared with the dark matter density. As we shall see below, the families of solutions that we build in this paper are parametrized by $\xi(a)$, which is treated as a free function of the model. Therefore, the condition (45) is easily satisfied, by choosing functions $\xi(a)$ that exhibit a fast decay at high redshift.

F. Solutions with common conformal time

To illustrate how we can build bimetric solutions that follow a Λ-CDM expansion rate, we first consider solutions with

$$a_* = b_*; \qquad (46)$$

that is, the conformal time τ is the same for all metrics. Then, at the background level, each metric is defined by a single scale factor a_*, and the two constraints in the first line in Eq. (26) reduce to one, $a = s_1 a_1 + s_2 a_2$. As all metrics are proportional, at the background level, this scenario is similar to a single gravitational metric model, $\tilde{g}_{\mu\nu}$, where the baryonic and the dark matter metrics are given by different conformal rescalings, $g_{\mu\nu} = A^2(\varphi)\tilde{g}_{\mu\nu}$ and $g_{\mu\nu} = A_{\rm d}^2(\varphi)\tilde{g}_{\mu\nu}$.

1. Symmetric solution

We first consider a simple symmetric solution where we split the single constraint $a = s_1 a_1 + s_2 a_2$ into two symmetric constraints:

$$\frac{s_1 a_1}{a} = (s_1^{(0)})^2, \qquad \frac{s_2 a_2}{a} = (s_2^{(0)})^2. \qquad (47)$$

This is consistent with the initial conditions defined by the early-time solution (41)–(42). Then, Eq. (47) gives $s_\ell(a)$ as an explicit function of $\{a, a_\ell(a)\}$, and we solve for the two sets $\{a_\ell(a), \omega_\ell(a), s_{{\rm d}\ell}(a)\}$. Thanks to the splitting (47), these two sets of variables can be solved independently. Then, the three functions $\{a_\ell(a), \omega_\ell(a), s_{{\rm d}\ell}(a)\}$ are determined by the two Friedmann equations (33) and (34) and the definition (32). The definition (32) provides $\mathcal{H}_\ell$ at each time step, hence $d\ln a_\ell / d\ln a$. The second Friedmann equation (34) gives $d\omega_\ell / d\ln a$. The first Friedmann equation (33) provides $s_{{\rm d}\ell}(a)$. The dark sector scale factor $a_{\rm d}$ is given by Eq. (26), $a_{\rm d}(a) = s_{\rm d1} a_1 + s_{\rm d2} a_2$. The scalar-field energy density $\xi(a)$ is an arbitrary function, which is a free function of the bimetric model. It must be positive, and we only request that it vanishes at early times to recover the high-redshift cosmology (41).

This procedure provides a family of solutions that are parametrized by the initial coefficients $s_\ell^{(0)}$ and the scalar energy density $\xi(a)$ and which follow the Λ-CDM expansion history for $\mathcal{H}(a)$. The latter enters the dynamical equations through the factors $\mathcal{H}(a)$ in Eqs. (34) and (32) [when we write $d\ln a_\ell/d\tau = \mathcal{H}(d\ln a_\ell/d\ln a)$]. As the coefficients $s_\ell^{(0)}$ do not appear in these equations, the two metrics are actually equivalent, with

$$\frac{a_1}{a_2} = \frac{s_1}{s_2} = \frac{s_1^{(0)}}{s_2^{(0)}}, \qquad \mathcal{H}_1 = \mathcal{H}_2. \tag{48}$$

We show in Figs. 1 and 2 the evolution with redshift of the main background quantities, in such a solution with $s_1^{(0)} = \sqrt{3}/2$, $s_2^{(0)} = 1/2$. The scalar-field energy density $\xi(z)$ is chosen to vanish at high z and to remain much smaller than the dark matter energy density at all times, $\xi \ll 1$. More specifically, we use the simple form

$$\xi(a) \propto \frac{u^{3/2}}{1+u^{3/2}}, \qquad u = \frac{\Omega_{\Lambda 0} a^4}{\Omega_{\gamma 0} + (\Omega_{\mathrm{dm}\,0} + \Omega_{\mathrm{b}\,0}) a}. \tag{49}$$

From Eq. (43), the quantity $u(a)$ is a natural measure of the deviation of the Λ-CDM cosmological background from the Einstein-de Sitter background. It is also the ratio of the effective dark-energy density to the matter and radiation energy densities, and we have $\omega = \omega^{(0)}(1+u)$. In this paper, we write the free functions of the models in terms of powers of $u(a)$, to ensure that we recover the Einstein-de Sitter reference of Sec. III D at early times. This also means that the effects of the scalar field only appear at low redshifts, where the departure from the Einstein-de Sitter reference is associated with a running of the scalar field.

We can see in Fig. 1 that a_1/a and a_2/a decrease at low z while s_1 and s_2 increase. Indeed, because of the absence of a cosmological constant, the scale factors $a_i(\tau)$ of the gravitational metric tend to follow an Einstein-de Sitter expansion rate, which falls below the Λ-CDM expansion rate of $a(\tau)$. The latter manages to mimic the Λ-CDM history thanks to the late-time growth of the factors s_ℓ in Eq. (26). On the other hand, the dark factors $s_{\mathrm{d}\ell}$ decrease at low z, in a fashion that is opposite to the baryonic factors s_ℓ. This follows from the relationship (35), which gives

$$\xi \ll 1, \Omega_{\gamma 0} \ll 1: \ \frac{ds_{\mathrm{d}\ell}}{d\ln a} \simeq -\frac{\Omega_{\mathrm{b}0}}{\Omega_{\mathrm{dm}0}} \frac{ds_\ell}{d\ln a} \quad \text{for } a \sim 1. \tag{50}$$

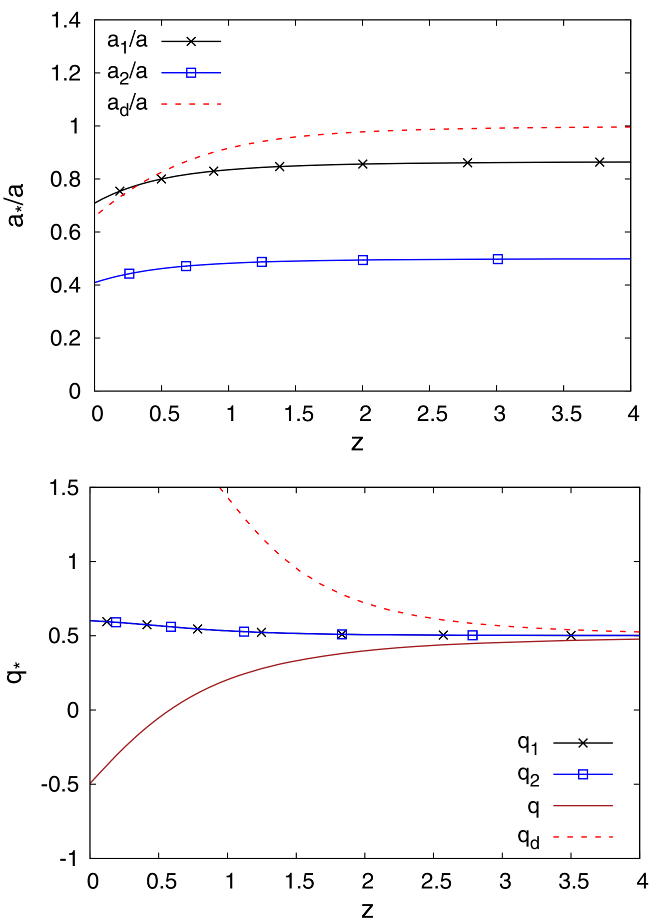

FIG. 1. Background quantities for a symmetric solution of the form (47), as a function of redshift. *Upper left panel:* ratio of the various scale factors a_* to the baryonic scale factor a. *Upper right panel:* the various expansion rates $\mathcal{H}_*$ normalized to H_0. *Lower left panel:* the various deceleration parameters q_*. *Lower right panel:* coefficients s_ℓ and $s_{\mathrm{d}\ell}$ of Eqs. (8) and (26).

Then, from Eq. (26), the dark sector scale factor $a_{\rm d}(\tau)$ grows even more slowly than the gravitational scale factors $a_\ell(\tau)$ at late times, and we have $\mathcal{H}_{a_{\rm d}} < \mathcal{H}_{a_\ell} < \mathcal{H}$.

These different cosmic evolutions are clearly shown by the deceleration factors q_*, defined for each metric with respect to its cosmic time $dt_* = b_* d\tau$ by

$$q_* = -\frac{\ddot{a}_* a_*}{\dot{a}_*^2} = -\frac{d^2 a_*}{dt_*^2} a_* \left(\frac{da_*}{dt_*}\right)^{-2}. \tag{51}$$

Thus, we can see that the gravitational metrics g_1 and g_2 show no acceleration. They keep behaving like an Einstein-de Sitter cosmology, except for a slightly stronger deceleration at low z. Only the baryonic metric shows an accelerated expansion with $q < 0$. Because of the opposite behavior of the dark sector coefficients $s_{\rm d\ell}$, as compared with the baryonic coefficients s_ℓ, the dark sector metric shows instead a stronger deceleration at late times than the Einstein-de Sitter cosmology. This clearly shows that the apparent acceleration of the baryonic metric is not due to a dark-energy component, associated for instance with the scalar field φ, as the "Einstein-frame" metrics g_1 and g_2 do not accelerate. It is only due to the time-dependent mapping (26) between these metrics and the baryonic metric. Therefore, this provides a "self-accelerated model," in the sense that the acceleration is not due to a hidden cosmological constant (e.g., the nonzero minimum of some potential or a dark-energy fluid with negligible kinetic energy).

As we wish to mimic a Λ-CDM cosmology, with $\Omega_{\Lambda 0} \simeq 0.7$, the deviations from the Einstein-de Sitter cosmology are of order unity at low z. This implies that the deviation of the coefficients s_ℓ and $s_{\rm d\ell}$ from their initial value is also of order unity at low z, while from Eq. (39), we have $\bar{\varphi} \sim M_{\rm Pl}\sqrt{\xi}$,

$$z = 0:\; s_\ell - s_\ell^{(0)} \sim 1, s_{\rm d\ell} - s_\ell^{(0)} \sim 1, \frac{\bar{\varphi}}{M_{\rm Pl}} \sim \sqrt{\xi}. \tag{52}$$

As explained below, after Eq. (70), we cannot take ξ too small as this would give rise to a large fifth force. On the other hand, we wish to keep the scalar-field energy density subdominant. We choose for all the solutions that we consider in this paper the same scalar-field energy density, shown in the upper panel in Fig. 2. It is of order $\Omega_{\rm dm0}/10$ at $z = 0$ and decreases at higher z. The $u^{3/2}$ falloff of $\xi(a)$ is fast enough to make the scalar field subdominant and to converge to the Einstein-de Sitter solution (41). It is also slow enough to enforce $ds_*/d\varphi \to 0$, as we have $ds_*/d\varphi = (ds_*/d\ln a)/(d\varphi/d\ln a) \sim u\mathcal{H}\sqrt{a/\xi}$. This yields vierbein coefficients $s_\ell(\varphi)$ that look somewhat more natural than functions with a divergent slope at the origin. We can see in the lower panel that the functions $s_*(\varphi)$ built by this procedure have simple shapes and do not develop

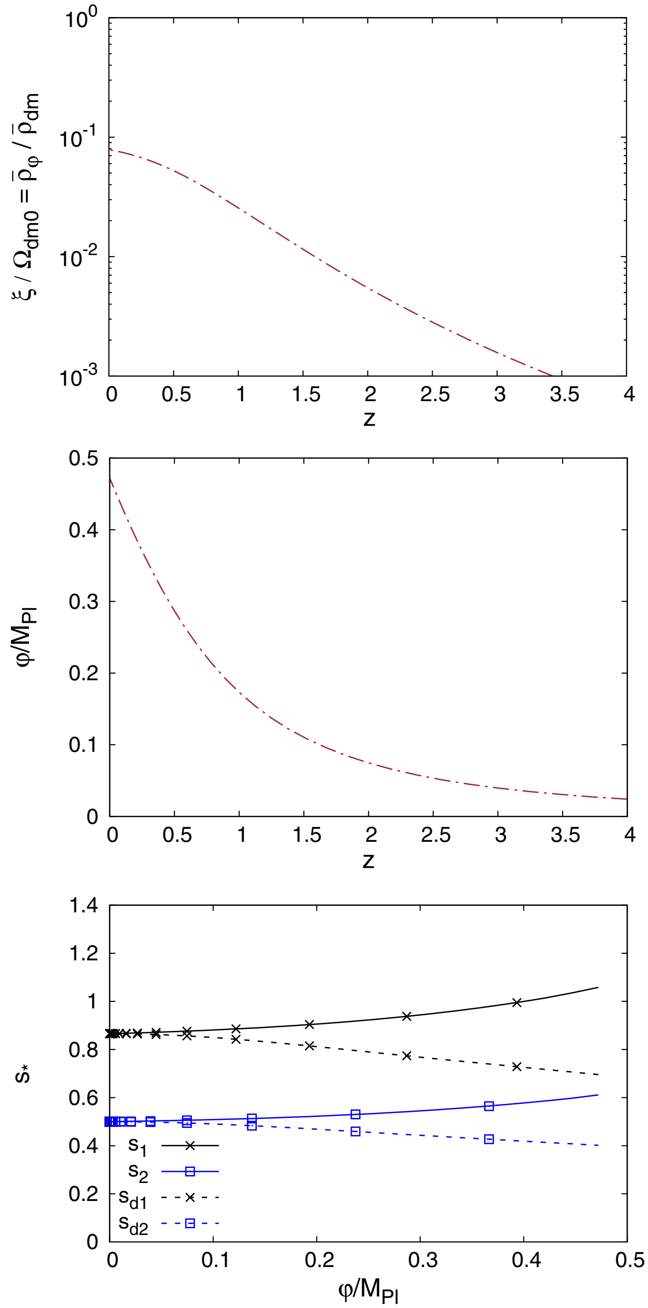

FIG. 2. Background quantities for a symmetric solution of the form (47). *Upper panel:* ratio of the scalar-field energy density to the dark matter energy density. *Middle panel:* value of the scalar field in Planck mass units. *Lower panel:* coefficients s_ℓ and $s_{\rm d\ell}$ as a function of the scalar field.

fine-tuned features. The model chosen for $\xi(a)$ gives scalar-field excursions of about $M_{\rm Pl}/2$ at $z = 0$.

2. Nonsymmetric solution

We can also build nonsymmetric solutions, which do not obey Eq. (47). Instead of splitting the constraint $a = s_1 a_1 + s_2 a_2$ into the two conditions (47), we can add another condition, such as requiring the ratio s_2/s_1 to follow an arbitrary function of time $\kappa(a)$. Then, the function $\kappa(a)$ parametrizes this extended family of solutions. The symmetric solution of Sec. III F 1 corresponds to the particular case $\kappa(a) = s_2^{(0)}/s_1^{(0)}$. Because the effective Newton constant is given by $s_1^2 + s_2^2$, in units of $\mathcal{G}_{\rm N} = 1/8\pi M_{\rm Pl}^2$, as we shall see in Eq. (84) below, we choose instead to parametrize the solutions by the sum $s_1^2 + s_2^2$, as a function of redshift. Thus, we solve the system

$$s_1 a_1 + s_2 a_2 = a, \qquad s_1^2 + s_2^2 = \lambda(a), \tag{53}$$

where $\lambda(a)$ is a new arbitrary function that parametrizes this extended family of solutions. These two equations now provide $\{s_1, s_2\}$ as a function of $\{a, a_1, a_2\}$,

$$s_1 = \frac{a a_1 + \epsilon a_2 \sqrt{\lambda(a_1^2 + a_2^2) - a^2}}{a_1^2 + a_2^2},$$
$$s_2 = \frac{a a_2 - \epsilon a_1 \sqrt{\lambda(a_1^2 + a_2^2) - a^2}}{a_1^2 + a_2^2}, \tag{54}$$

where $\epsilon = \pm 1$. Then, we can again solve for the two sets $\{a_\ell(a), \omega_\ell(a), s_{{\rm d}\ell}(a)\}$ from Eqs. (32), (33), and (34), the only difference being that these two sets of variables are now coupled.

We show in Fig. 3 the evolution of the background quantities for a solution of the form (53), where s_2/s_1 is no longer constant and we impose that $d\lambda/da = 0$ at $z = 0$. Despite this difference, the scale factors and the Hubble expansion rates are very close to those of Fig. 1. This is because, at late times after the radiation-to-matter transition, $a \gg a_{\rm eq}$, and for $\xi \ll 1$, the second Friedmann equation (34) reduces to

$$\frac{d\omega_\ell}{d\ln a} \sim -\frac{\Omega_{\gamma 0}}{a} - \xi, \qquad \text{hence} \left|\frac{d\omega_\ell}{d\ln a}\right| \ll 1. \tag{55}$$

Since the dark-energy era and the running of the scalar field occur much later than the radiation-to-matter transition, we can actually see from the first Friedmann equation (33) that we must have

$$\omega_\ell \simeq s_\ell \Omega_{\rm b\,0} + s_{{\rm d}\ell}\Omega_{\rm dm0} \simeq s_\ell^{(0)}\Omega_{\rm b0} + s_\ell^{(0)}\Omega_{\rm dm0}. \tag{56}$$

Thus, we recover the relationship (50), and we also find that for the general class of solutions with a common conformal time the quantities ω_ℓ are set by the initial conditions and show a negligible dependence on the late-time evolution of the coefficients s_ℓ and $s_{{\rm d}\ell}$ and on the scalar field (as long as it remains subdominant). This explains why we recover almost the same evolution for the scale factors a_* and the Hubble expansion rates $\mathcal{H}_*$, which are determined by the definition (32). Then, the deceleration parameters q_* are also close to those obtained in Fig. 1. The change to the factors s_ℓ associated with different solutions is almost fully compensated by the change to the dark coefficients $s_{{\rm d}\ell}$ that is implied by the constraint of recovering a Λ-CDM expansion rate for the baryonic metric. By the same mechanism, we also find that in these solutions, despite the different behaviors of s_1 and s_2, the two gravitational metrics are mostly equivalent, with again the same expansion rates $\mathcal{H}_1 \simeq \mathcal{H}_2$ up to negligible deviations.

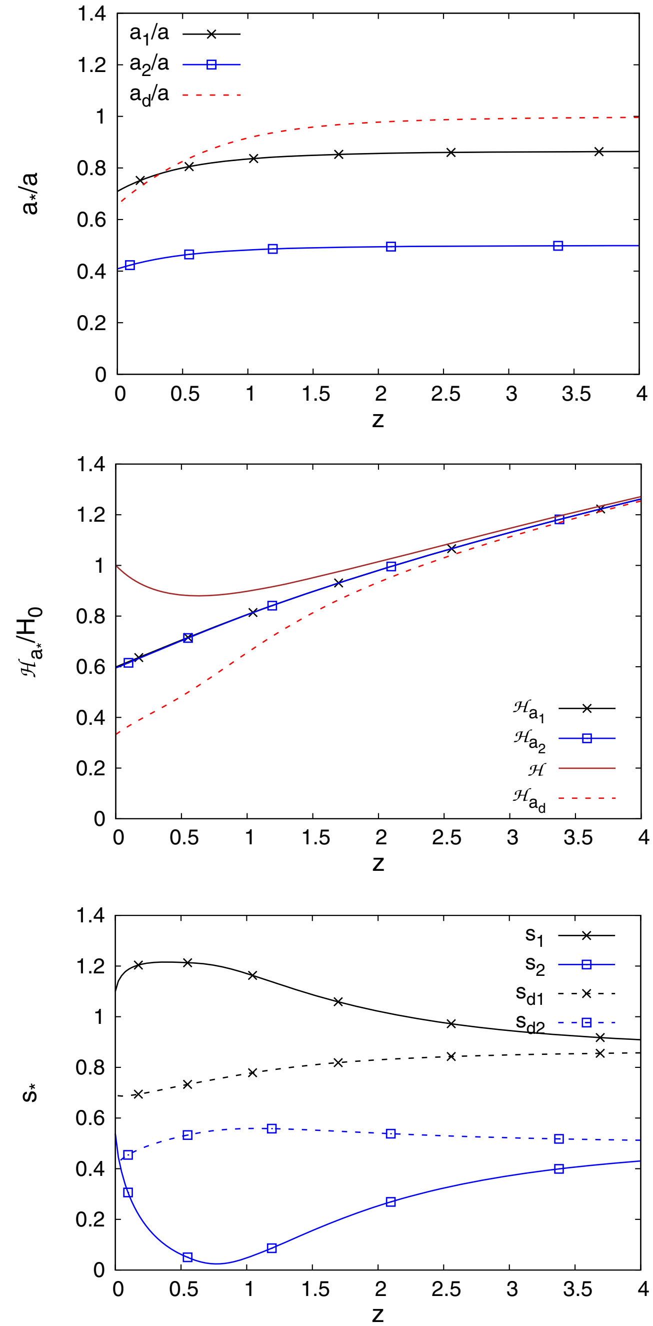

FIG. 3. Background quantities for a solution of the form (53), where s_2/s_1 is not constant.

G. Solutions with different conformal times

We now turn to even more general solutions, which still follow the Λ-CDM expansion rate for the baryonic metric, but where

$$a_\ell \neq b_\ell, \qquad a_{\rm d} \neq b_{\rm d}. \tag{57}$$

Then, the conformal times τ_* of the various metrics are different. As the metrics are not proportional, already at the background level, this scenario is different from models where the baryonic and the dark matter metrics are given by different conformal rescalings of a single Einstein-frame metric $\tilde{g}_{\mu\nu}$.

Defining the scale-factor ratios

$$r_\ell(a) = \frac{b_\ell}{a_\ell}, \qquad r_{\rm d}(a) = \frac{b_{\rm d}}{a_{\rm d}}, \tag{58}$$

the two constraints in the first line of Eq. (26) read as

$$a = s_1 a_1 + s_2 a_2, \qquad a = s_1 r_1 a_1 + s_2 r_2 a_2. \tag{59}$$

These two linear equations provide $\{s_\ell\}$ as a function of $\{a, a_\ell\}$,

$$s_1 = \frac{a(1-r_2)}{a_1(r_1-r_2)}, \qquad s_2 = \frac{a(1-r_1)}{a_2(r_2-r_1)}, \tag{60}$$

when we are given the arbitrary free functions $r_\ell(a)$. As in the previous cases, $\{a_\ell, \omega_\ell, s_{{\rm d}\ell}\}$ are obtained from Eqs. (32) and (33) and (34), while $a_{\rm d}$ and $b_{\rm d}$ are obtained from the second line of Eq. (26).

We require $a_* > 0$, $b_* > 0$, $s_\ell > 0$, $s_{{\rm d}\ell} > 0$, to avoid singularities. This implies $s_1 s_2 > 0$, and Eq. (60) leads to

$$(1-r_1)(1-r_2) < 0, \tag{61}$$

and we can choose for instance

$$r_1 < 1 < r_2. \tag{62}$$

The recent detections of gravitational waves from a binary neutron star merger by the LIGO-VIRGO Collaboration (GW170817) [29], with electromagnetic counterparts in the gamma-ray burst [17] and in UV, optical, and near infrared bands [30], place very stringent limits on the speed of gravitational waves, $|c_g - 1| \leq 3 \times 10^{-15}$ [17]. For the bimetric action (2), we have two gravitons associated with the two Einstein-Hilbert terms R_ℓ. We can obtain their equations of motion from the nonlinear Einstein equations (19), starting at the level of the vierbeins. In the case of a constant scalar field φ, we recover the results obtained from the quadratic action at the level of the metrics in Refs. [13–15]. This gives for the first graviton h_{1ij}

$$M_{\rm Pl}^2 \frac{a_1^2}{b_1}\left[h''_{1ij} + (3\mathcal{H}_{a_1} - \mathcal{H}_{b_1})h'_{1ij} - \frac{b_1^2}{a_1^2}\nabla^2 h_{1ij}\right] - a_2(\bar{p} s_1 s_2 a^2 + \bar{p}_\varphi s_{\rm d1} s_{\rm d2} b_{\rm d} a_{\rm d})(h_{1ij} - h_{2ij}) = 0, \tag{63}$$

and the equation of motion of the second graviton h_{2ij} is given by the permutation $1 \leftrightarrow 2$. Here, we note $\bar{p}$ is the total pressure of the baryonic sector fluids. In the radiation and matter eras, this is simply the radiation pressure, $\bar{p} = \bar{p}_\gamma = \bar{\rho}_\gamma/3$, while during the inflationary era, it is the pressure $\bar{p}_\chi = -\bar{\rho}_\chi$ of the inflaton χ. We can see that the speed of the two gravitons is given by $c_{g\ell} = b_\ell / a_\ell$, which differs from the speed of light when $r_\ell \neq 1$.

To explain the multimessenger event GW170817, at least one of these two gravitons must propagate at the speed of light (up to an accuracy of 10^{-15}) in the local and recent Universe, $d \lesssim 40$ Mpc and $z \lesssim 0.01$. In principle, a nonlinear screening mechanism might change the laws of gravity and ensure convergence to General Relativity in the local environment. However, it is unlikely that it would apply over 40 Mpc. Moreover, in most parts of the trajectory, between the host galaxy and the Milky Way, the local density is below or of the order of the cosmological background density. Besides, it would require a fine-tuned cancellation to make the average speed $c_g = 1$ over the full trajectory, inside the two galaxies and the low-density intergalactic medium. Then, at least one of the lapse factors r_ℓ must converge to unity at low z. If both coefficients r_ℓ go to unity, we converge to the solutions studied in Sec. III F. For illustration, we consider in Fig. 4 the case where only one of the coefficients r_ℓ goes to unity at low redshift, for instance r_1 [with again the same initial conditions $\{s_1^{(0)}, s_2^{(0)}\}$ and scalar-field energy density $\xi(a)$ as in Fig. 1]. In this limit, the system effectively reduces again to a single metric for the baryonic sector. Indeed, Eq. (60) implies that $s_2 \to 0$ if $r_1 \to 1$ (and $s_1 \to 0$ if $r_2 \to 1$). Then, the baryonic metric $g_{\mu\nu}$ becomes proportional to the metric $g_{1\mu\nu}$. However, the dark matter metric remains sensitive to both gravitational metrics g_1 and g_2, as $s_{\rm d2}$ remains nonzero, so that the baryon + dark matter system remains different from the common conformal time scenarios of Sec. III F. In particular, the baryonic and dark matter metrics are not proportional, so this scenario remains different from models where the baryonic and the dark matter metrics are given by different conformal rescalings of a single Einstein-frame metric $\tilde{g}_{\mu\nu}$.

We can see that in this scenario the scale factors a_* remain similar to those obtained in Fig. 1 for the symmetric solution (47). However, we can now distinguish the difference between the two expansion rates $\mathcal{H}_{a_1}$ and $\mathcal{H}_{a_2}$ at low z. The main difference with respect to the previous solutions is the behavior of the lapse functions b_ℓ. Thanks to the additional degrees of freedom r_ℓ, the lapses b_ℓ can behave in a significantly different way than the scale factors a_ℓ. In the example shown in Fig. 4, the two lapses even evolve in

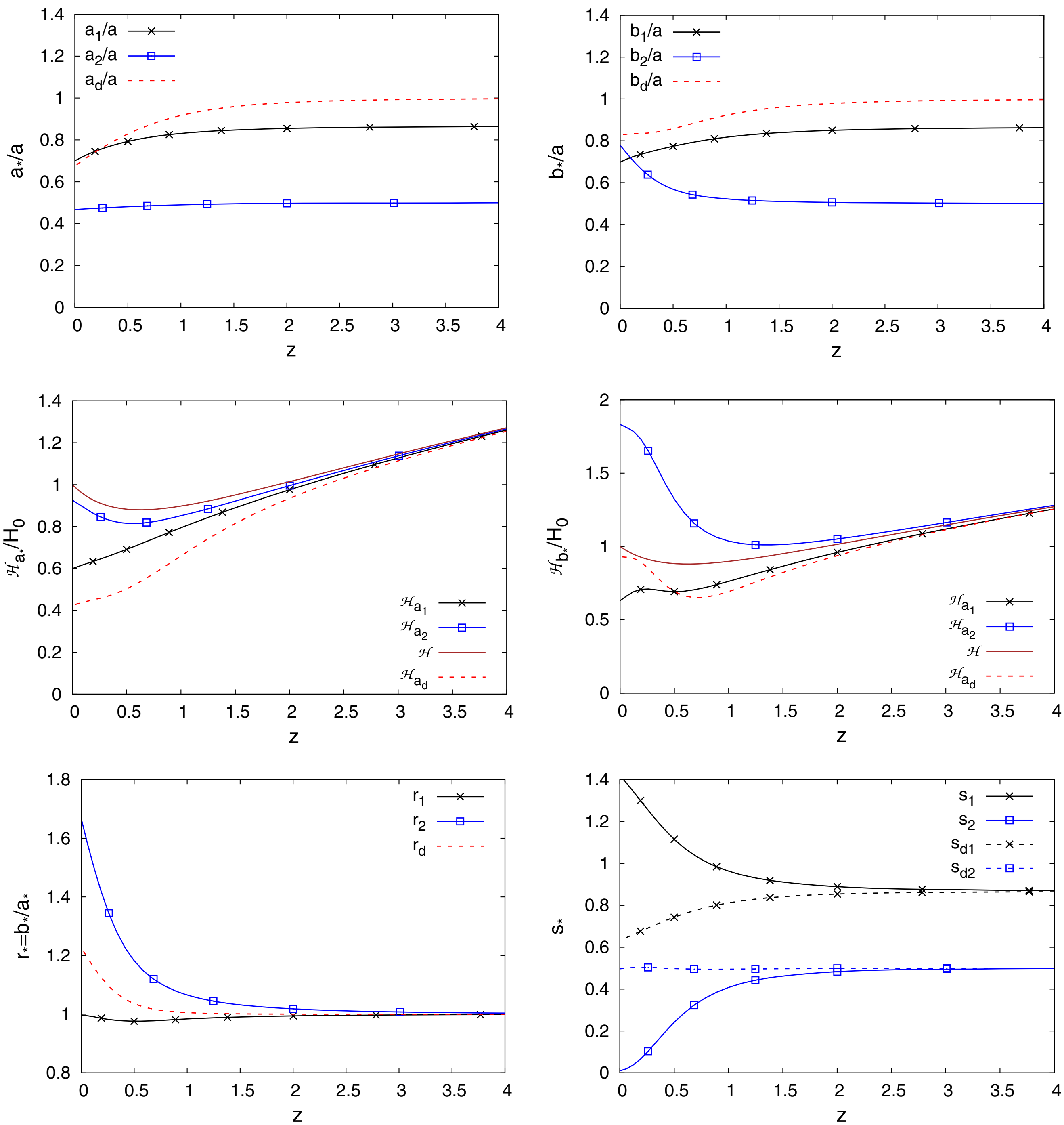

FIG. 4. Background quantities for a solution of the form (62), where the different metrics have different conformal times (i.e., are not proportional), but $r_1 \to 1$ at $z = 0$.

different directions and cross each other at $z \simeq 0.1$. This leads to rates that are significantly different with $\mathcal{H}_{b_2} > \mathcal{H}$. As explained in Sec. III F 1, because of the lack of a cosmological constant, the gravitational expansion rates $\mathcal{H}_{a_\ell}$ are typically smaller than the Λ-CDM expansion rate $\mathcal{H}$. This remains true for the more general solution shown in Fig. 4. But the lapse functions are not so strongly constrained, and it is possible to have one of them growing faster than a. For the choice (62), this corresponds to b_2, with $\mathcal{H}_{b_2} > \mathcal{H}$. This requires a ratio r_2 that significantly departs from unity at low z, as seen in the lower left panel.

The coefficients s_ℓ and $s_{\mathrm{d}\ell}$ follow similar behaviors to those obtained in Figs. 1 and 3, with opposite deviations at low redshift for the baryonic and dark sector coefficients. Because of the constraint $|r_1 - 1| < 3 \times 10^{-15}$ at $z = 0$, the coefficient s_2 almost goes to zero, with $s_2 \lesssim 10^{-15}$ at $z = 0$.

IV. COSMOLOGICAL PERTURBATIONS

We have seen that it is possible to build several families of solutions that follow a Λ-CDM expansion history for the baryonic metric. In the case of metrics that are not proportional, the multimessenger neutron star merger GW170817 also implies that at least one of the two gravitational metrics, g_1 and g_2, becomes proportional to the baryonic metric (i.e., $r_\ell = 1$) at low redshift.

We show below that these models are actually severely constrained by the behavior of perturbations. Here, we focus on the scalar perturbations in the quasistatic approximation, which applies to the formation of large-scale structures. Then, the relevant metric perturbations are set by the four gravitational potentials $\{\phi_\ell, \psi_\ell\}$ as in the usual Newtonian gauge.

A. Scalar-field perturbations

On small scales in the quasistatic approximation, the Klein-Gordon equation (21) becomes

$$\frac{1}{a_{\rm d}^2}\nabla^2\delta\varphi = m^2\delta\varphi + \frac{\beta_{\rm dm}}{M_{\rm Pl}}\delta\rho_{\rm dm} + \frac{\beta}{M_{\rm Pl}}\delta\rho, \tag{64}$$

with $\delta\varphi = \varphi - \bar{\varphi}$, $\delta\rho_{\rm dm} = \rho_{\rm dm} - \bar{\rho}_{\rm dm}$, and $\delta\rho = \rho - \bar{\rho}$. Here, we assumed nonrelativistic matter components, $p_{\rm dm} = p = 0$, and we neglected radiation fluctuations. As $\delta\rho_\varphi = \delta p_\varphi = b_{\rm d}^{-2}\frac{d\bar{\varphi}}{d\tau}\frac{\partial\delta\varphi}{\partial\tau}$, we also neglected the linear fluctuations of the scalar-field density and pressure in the quasistatic limit. The scalar-field mass around the cosmological background is

$$m^2 = (\bar{\rho}_{\rm dm} + \bar{\rho}_\varphi)\sum_\ell \frac{d^2 s_{{\rm d}\ell}}{d\varphi^2}\frac{b_\ell}{b_{\rm d}} - 3\bar{\rho}_\varphi \sum_\ell \frac{d^2 s_{{\rm d}\ell}}{d\varphi^2}\frac{a_\ell}{a_{\rm d}} + (\bar{\rho} + \bar{\rho}_\gamma)\sum_\ell \frac{d^2 s_\ell}{d\varphi^2}\frac{a^3 b_\ell}{a_{\rm d}^3 b_{\rm d}} - \bar{\rho}_\gamma \sum_\ell \frac{d^2 s_\ell}{d\varphi^2}\frac{a^3 a_\ell}{a_{\rm d}^3 b_{\rm d}}. \tag{65}$$

Using the relation (35), it is possible to express the dark sector derivatives $d^2 s_{{\rm d}\ell}/d\varphi^2$ and $ds_{{\rm d}\ell}/d\varphi$ in terms of $d^2 s_\ell/d\varphi^2$ and $ds_\ell/d\varphi$. It is then possible to remove the second derivatives $d^2 s_\ell/d\varphi^2$ thanks to the symmetry in $\ell = 1, 2$, using the relations obtained by taking derivatives with respect to $\ln a$ of the constraints $a = s_1 b_1 + s_2 b_2$ and $a = s_1 a_1 + s_2 a_2$. The couplings to matter are

$$\beta = M_{\rm Pl}\sum_\ell \frac{ds_\ell}{d\varphi}\frac{a^3 b_\ell}{a_{\rm d}^3 b_{\rm d}}, \qquad \beta_{\rm dm} = M_{\rm Pl}\sum_\ell \frac{ds_{{\rm d}\ell}}{d\varphi}\frac{b_\ell}{b_{\rm d}}. \tag{66}$$

In Fourier space, this yields

$$\frac{\delta\varphi}{M_{\rm Pl}} = -\frac{3H_0^2}{m^2 + k^2/a_{\rm d}^2}\left[\frac{\Omega_{\rm dm0}\beta_{\rm dm}\delta_{\rm dm}}{a_{\rm d}^3} + \frac{\Omega_{\rm b0}\beta\delta}{a^3}\right], \tag{67}$$

where $\delta_{\rm dm} = \delta\rho_{\rm dm}/\bar{\rho}_{\rm dm}$, $\delta = \delta\rho/\bar{\rho}$.

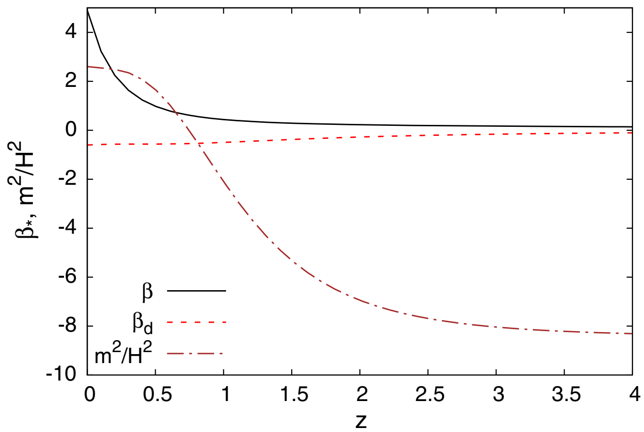

FIG. 5. Scalar-field mass and couplings for the symmetric model of Fig. 1.

It is interesting to consider the scaling in ξ of the scalar-field mass and couplings. From Eq. (52), we have the scalings

$$\frac{ds_*}{d\ln a} \sim 1, \qquad \frac{ds_*}{d\varphi} \sim \frac{1}{M_{\rm Pl}\sqrt{\xi}}, \qquad \frac{d^2 s_*}{d\varphi^2} \sim \frac{1}{M_{\rm Pl}^2\xi}. \tag{68}$$

Then, from Eq. (65), it seems that $m^2 \sim H_0^2/\xi$. However, using the relationship (35), one finds that the terms of order $1/\xi$ cancel out, and we obtain

$$m^2 \sim H_0^2(\Omega_{\gamma 0} + \xi). \tag{69}$$

On the other hand, the couplings scale as

$$\beta \sim \frac{1}{\sqrt{\xi}}, \qquad \beta_{\rm dm} \sim \frac{1}{\sqrt{\xi}}. \tag{70}$$

Therefore, very small values of the scalar-field energy density ξ yield a very large fifth force. This implies that we cannot take ξ too small, which is why we choose $\xi \sim \Omega_{\rm dm0}/10$ at $z = 0$ in the models that we consider in this paper. This feature comes from the fact that we require effects of order 1 from the scalar field onto the background at low redshift, $ds_*/d\ln a \sim 1$, to generate the apparent acceleration of the baryonic metric. This implies $ds_*/d\varphi \propto 1/\bar{\varphi}' \propto 1/\sqrt{\xi}$.

We show in Fig. 5 the scalar-field mass and couplings for the symmetric model of Fig. 1. As expected from the expression (65), the squared mass evolves as $\bar{\rho}/M_{\rm Pl}^2 \sim H^2$, and it is of order H^2. This means that it is negligible on scales much below the horizon, where the quasistatic approximation (64) applies, and does not lead to small-scale instabilities, even when it is negative. The couplings β and $\beta_{\rm d}$ are of order unity and decrease at high z, because $ds_*/d\varphi \to 0$. This is because we choose the high-z decay of the scalar-field energy density, determined by Eq. (49), to be slow enough so that $ds_*/d\varphi \to 0$ at early times. The baryonic and dark matter couplings have opposite signs, with $\beta > 0 > \beta_{\rm dm}$, because we typically have $ds_\ell/d\ln a > 0$ and $ds_{{\rm d}\ell}/d\ln a < 0$, as explained in Sec. III F and in agreement with Eq. (50).

The other solutions considered in Secs. III F and III G give results similar to those found in Fig. 5.

B. Einstein equations

1. Gravitational potentials ϕ_ℓ and ψ_ℓ

We study in details the behavior of linear perturbations in Sec. V below, and we provide in Appendix A explicit expressions of the Einstein equations in the case $a_\ell = b_\ell$. The extra two scalars added to the four Newtonian potentials that cannot be eliminated by gauge freedom (because of the loss of the nondiagonal diffeomorphism invariance) are not dynamical [13], and there is no scalar instability. In this section, we focus on small subhorizon scales, $k \gg \mathcal{H}$, in the quasistatic approximation, where we only keep the higher-order spatial gradients. Then, as in General Relativity, only the four gravitational potentials $\{\phi_\ell, \psi_\ell\}$ remain. The perturbed metrics take the usual form

$$g_{*00} = -b_*^2(1+2\phi_*), \qquad g_{*ii} = a_*^2(1-2\psi_*), \tag{71}$$

while the vierbeins are diagonal with

$$e^0_{*0} = b_*(1+\phi_*), \qquad e^i_{*i} = a_*(1-\psi_*). \tag{72}$$

For nonrelativistic matter components, the (0,0) component of the Einstein equations (19) gives for the metric $g_{1\mu\nu}$

$$\frac{2a_1}{3H_0^2}\nabla^2\psi_1 = s_1\Omega_{\rm b0}\delta + s_{\rm d1}\Omega_{\rm dm0}\delta_{\rm dm} + \Upsilon_{\psi_1}\frac{d\ln a}{d\bar\varphi}\delta\varphi, \tag{73}$$

with

$$\Upsilon_{\psi_1} = \left(\Omega_{\rm b0} + \frac{\Omega_{\gamma 0}}{a}\right)\left[\left(1 + s_1\frac{b_1}{a}\right)\frac{ds_1}{d\ln a} + s_1\frac{b_2}{a}\frac{ds_2}{d\ln a}\right] + (\Omega_{\rm dm0}+\xi)\left[\left(1+s_{\rm d1}\frac{b_1}{b_{\rm d}}\right)\frac{ds_{\rm d1}}{d\ln a} + s_{\rm d1}\frac{b_2}{b_{\rm d}}\frac{ds_{\rm d2}}{d\ln a}\right]. \tag{74}$$

The (i,j) components of the Einstein equations give

$$\frac{b_1}{H_0^2}[-\partial_i\partial_j(\phi_1-\psi_1) + \delta_{ij}\nabla^2(\phi_1-\psi_1)] = \Upsilon_{\phi_1}\frac{d\ln a}{d\bar\varphi}\delta\varphi, \tag{75}$$

with

$$\Upsilon_{\phi_1} = \frac{\Omega_{\gamma 0}}{a}\left[\left(1+s_1\frac{a_1}{a}\right)\frac{ds_1}{d\ln a} + s_1\frac{a_2}{a}\frac{ds_2}{d\ln a}\right] + 3\xi r_{\rm d}\left[\left(1+s_{\rm d1}\frac{a_1}{a_{\rm d}}\right)\frac{ds_{\rm d1}}{d\ln a} + s_{\rm d1}\frac{a_2}{a_{\rm d}}\frac{ds_{\rm d2}}{d\ln a}\right]. \tag{76}$$

We can use the Klein-Gordon equation (67) satisfied by the scalar field to eliminate $\delta\varphi$. In Fourier space, this gives

$$-\frac{2}{3}a_\ell\frac{k^2}{H_0^2}\psi_\ell = (1+\gamma^{\psi_\ell})s_\ell\Omega_{\rm b0}\delta + (1+\gamma^{\psi_\ell}_{\rm dm})s_{{\rm d}\ell}\Omega_{\rm dm0}\delta_{\rm dm} \tag{77}$$

and

$$-a_\ell\frac{k_ik_j-\delta_{ij}k^2}{H_0^2}(\phi_\ell-\psi_\ell) = \gamma^{\phi_\ell}s_\ell\Omega_{\rm b0}\delta + \gamma^{\phi_\ell}_{\rm dm}s_{{\rm d}\ell}\Omega_{\rm dm0}\delta_{\rm dm}. \tag{78}$$

The coefficients γ^*_* arise from the fluctuations of the scalar field φ, which generate fluctuations δs_* of the vierbein coefficients s_* that relate the matter and gravitational metrics. They are given by

$$\gamma^{\psi_\ell} = -\frac{\mathcal{H}}{H_0 r_{\rm d} a^3 s_\ell}\sqrt{\frac{3a_{\rm d}}{2\xi}}\frac{\beta H_0^2}{m^2+k^2/a_{\rm d}^2}\Upsilon_{\psi_\ell},$$
$$\gamma^{\psi_\ell}_{\rm dm} = -\frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}^3 s_{{\rm d}\ell}}\sqrt{\frac{3a_{\rm d}}{2\xi}}\frac{\beta_{\rm dm} H_0^2}{m^2+k^2/a_{\rm d}^2}\Upsilon_{\psi_\ell},$$
$$\gamma^{\phi_\ell} = \frac{\mathcal{H}}{H_0 r_{\rm d} a^3 r_\ell s_\ell}\sqrt{\frac{3a_{\rm d}}{2\xi}}\frac{\beta H_0^2}{m^2+k^2/a_{\rm d}^2}\Upsilon_{\phi_\ell},$$
$$\gamma^{\phi_\ell}_{\rm dm} = \frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}^3 r_\ell s_{{\rm d}\ell}}\sqrt{\frac{3a_{\rm d}}{2\xi}}\frac{\beta_{\rm dm} H_0^2}{m^2+k^2/a_{\rm d}^2}\Upsilon_{\phi_\ell}, \tag{79}$$

where the factors Υ_{ψ_ℓ} and Υ_{ϕ_ℓ} are given in Eqs. (74) and (76). The contribution from the fifth force to the gravitational potentials ψ_ℓ and ϕ_ℓ is negligible if the coefficients γ^*_* are much smaller than unity. Then, we recover Einstein equations for these gravitational potentials that are close to their standard form,

$$|\gamma^*_*| \ll 1: \quad \begin{array}{l} \phi_\ell \simeq \psi_\ell \\ -\frac{2}{3}a_\ell\frac{k^2}{H_0^2}\psi_\ell \simeq s_\ell\Omega_{\rm b0}\delta + s_{{\rm d}\ell}\Omega_{\rm dm0}\delta_{\rm dm}. \end{array} \tag{80}$$

We show in Fig. 6 the coefficients γ^*_* for the symmetric solution of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$. At $z=0$, we expect from Eqs. (79) that $|\gamma^*_*| \simeq (H_0/k)^2$ on small scales. Indeed, we can see in the figure that for $k = 10\mathcal{H}$ we have $|\gamma^*_*| \lesssim 10^{-2}$. Moreover, the amplitude shows a fast decrease at higher z. Therefore, on subhorizon scales, the coefficients γ^*_* are much smaller than unity at all redshifts, and we can always use the approximations (80).

The other solutions considered in Secs. III F and III G give results similar to those found in Fig. 6.

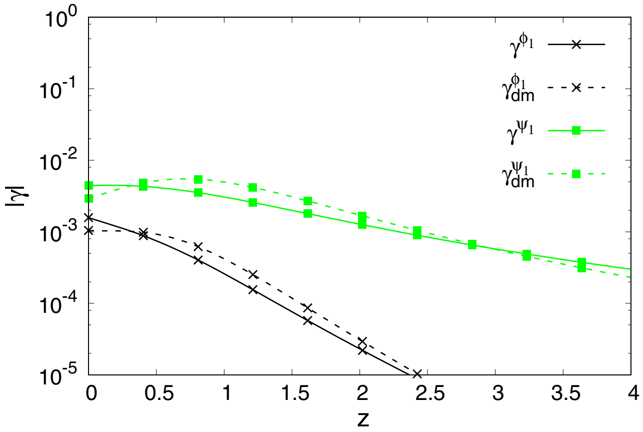

FIG. 6. Absolute value of the coefficients γ^*_* for the symmetric model of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$.

2. Baryonic gravitational potentials ϕ and ψ

In the following, we assume that the properties (80) are satisfied. However, this is not sufficient to remove the fifth force because the dynamics of dark matter and baryons are set by their own metric potentials $\phi_{\rm d}$ and ϕ. Their relationship with the potentials ϕ_ℓ involves the scalar field and will give rise to a fifth force. Indeed, from the vierbeins (72) and their relations (8), we obtain at linear order

$$a\phi = \sum_\ell b_\ell (s_\ell \phi_\ell + \delta s_\ell),$$
$$a\psi = = \sum_\ell a_\ell (s_\ell \psi_\ell - \delta s_\ell). \qquad (81)$$

As for the gravitational potentials ϕ_ℓ and ψ_ℓ, the fluctuations of the coefficients s_ℓ and $s_{{\rm d}\ell}$, due to the perturbations of the scalar field $\delta\varphi$, give rise to nonstandard terms. Using Eq. (80), we obtain

$$-\frac{2}{3} a \frac{k^2}{H_0^2} \phi = \mu^\phi \Omega_{\rm b0} \delta + \mu^\phi_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm},$$
$$-\frac{2}{3} a \frac{k^2}{H_0^2} \psi = \mu^\psi \Omega_{\rm b0} \delta + \mu^\psi_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm}, \qquad (82)$$

with

$$\mu^\phi = \sum_\ell \left[s_\ell^2 r_\ell + \frac{\mathcal{H} a_{\rm d}^2}{H_0 r_{\rm d} a^3} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_\ell}{d\ln a} b_\ell \right]$$
$$\mu^\phi_{\rm dm} = \sum_\ell \left[s_\ell s_{{\rm d}\ell} r_\ell + \frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta_{\rm dm} k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_\ell}{d\ln a} b_\ell \right]$$
$$\mu^\psi = \sum_\ell \left[s_\ell^2 - \frac{\mathcal{H} a_{\rm d}^2}{H_0 r_{\rm d} a^3} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_\ell}{d\ln a} a_\ell \right]$$
$$\mu^\psi_{\rm dm} = \sum_\ell \left[s_\ell s_{{\rm d}\ell} - \frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta_{\rm dm} k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_\ell}{d\ln a} a_\ell \right]. \qquad (83)$$

We recover the standard Poisson equations if $\mu^*_* = 1$.

We can split the coefficients μ^*_* into two parts. The first term, of the form $s_\ell^2 r_\ell$, is similar to a scale-independent renormalized Newton's constant and arises from the coefficients s_ℓ that relate the various metric potentials. The second part, of the form $ds_\ell / d\ln a$, arises from the fluctuations of the scalar field through δs_ℓ and corresponds to a fifth force. It is scale dependent. Thus, we may define the renormalized Newton's constants (in units of the natural Newton's constant, $\mathcal{G}_{\rm N} = 1/8\pi M_{\rm Pl}^2$),

$$\mathcal{G}^\phi = \sum_\ell s_\ell^2 r_\ell, \qquad \mathcal{G}^\phi_{\rm dm} = \sum_\ell s_\ell s_{{\rm d}\ell} r_\ell,$$
$$\mathcal{G}^\psi = \sum_\ell s_\ell^2, \qquad \mathcal{G}^\psi_{\rm dm} = \sum_\ell s_\ell s_{{\rm d}\ell}, \qquad (84)$$

which are all positive.

The two baryonic metric potentials ϕ and ψ are generically different. First, if $r_\ell \neq 1$, the associated effective Newton's constants $\mathcal{G}^\phi$ and $\mathcal{G}^\psi$ are different. Second, the fifth-force contributions that enter ϕ and ψ have the same amplitude but opposite signs.

We show in Fig. 7 the coefficients μ^ϕ_* and μ^ψ_* for the symmetric solution of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$, as well as the effective Newton constants. At early times, when the scalar field has no effect and we converge to the Einstein-de Sitter cosmology, we recover General Relativity with $\mu^*_* \to 1$ and $\mathcal{G}^*_* \to 1$. At late times, these coefficients show deviations of order unity. In this regime, the comparison of the two panels shows that the coefficients μ^*_* are dominated by the fifth-force contributions. This means that the fifth force is greater than Newtonian gravity. Moreover, the coefficients $\mu^\phi_{\rm dm}$ and μ^ψ become negative, which would give rise to very nonstandard behaviors. Thus, the dark matter overdensities repel the baryonic matter at late times.

3. Dark matter gravitational potentials $\phi_{\rm d}$ and $\psi_{\rm d}$

In a similar fashion, the dark sector gravitational potentials $\phi_{\rm d}$ and $\psi_{\rm d}$ obey Poisson equations of the form (82),

$$-\frac{2}{3} a_{\rm d} \frac{k^2}{H_0^2} \phi_{\rm d} = \mu^{\phi_{\rm d}} \Omega_{\rm b0} \delta + \mu^{\phi_{\rm d}}_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm},$$
$$-\frac{2}{3} a_{\rm d} \frac{k^2}{H_0^2} \psi_{\rm d} = \mu^{\psi_{\rm d}} \Omega_{\rm b0} \delta + \mu^{\psi_{\rm d}}_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm}, \qquad (85)$$

with

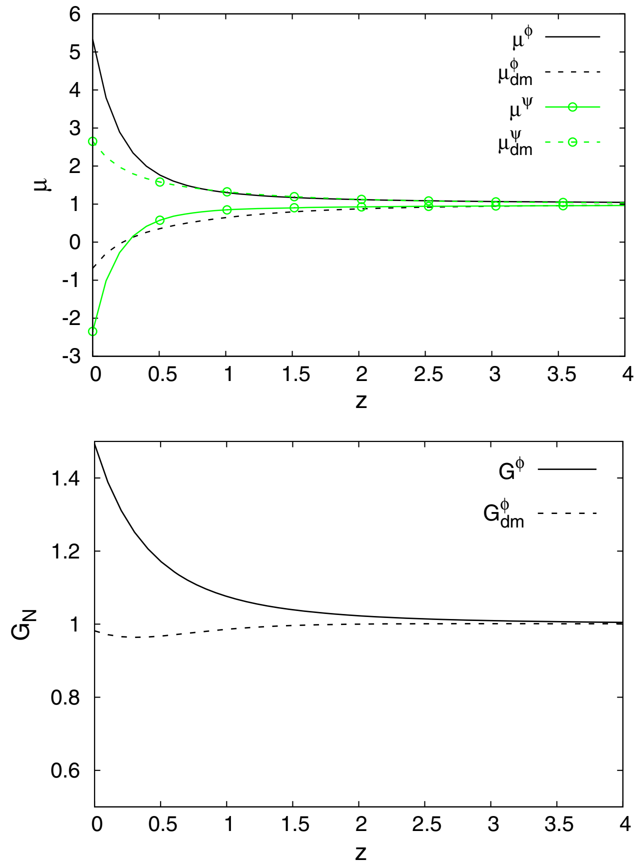

FIG. 7. *Upper panel:* coefficients μ_*^{ϕ} and μ_*^{ψ} for the symmetric model of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$. *Lower panel:* effective Newton constants $\mathcal{G}_*^{\phi}$. For this model, $\mathcal{G}_*^{\phi} = \mathcal{G}_*^{\psi}$.

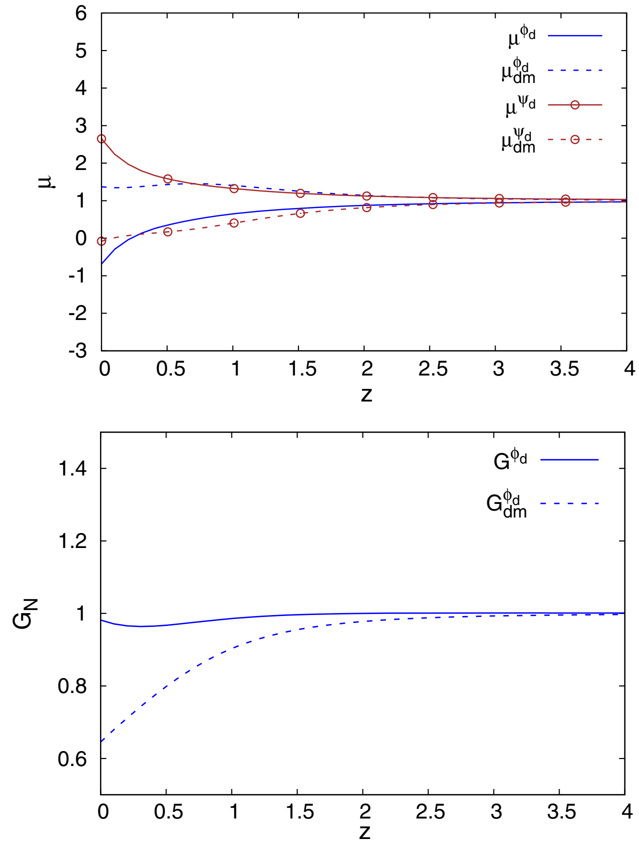

FIG. 8. *Upper panel:* coefficients $\mu_*^{\phi_{\rm d}}$ and $\mu_*^{\psi_{\rm d}}$ for the symmetric model of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$. *Lower panel:* effective Newton constants $\mathcal{G}_*^{\phi_{\rm d}}$ and $\mathcal{G}_*^{\psi_{\rm d}}$.

$$\mu^{\phi_{\rm d}} = \sum_\ell \left[s_{{\rm d}\ell} s_\ell \frac{r_\ell}{r_{\rm d}} + \frac{\mathcal{H} a_{\rm d}^2}{H_0 r_{\rm d}^2 a^3} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_{{\rm d}\ell}}{d\ln a} b_\ell \right]$$
$$\mu^{\phi_{\rm d}}_{\rm dm} = \sum_\ell \left[s_{{\rm d}\ell}^2 \frac{r_\ell}{r_{\rm d}} + \frac{\mathcal{H}}{H_0 r_{\rm d}^2 a_{\rm d}} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta_{\rm dm} k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_{{\rm d}\ell}}{d\ln a} b_\ell \right]$$
$$\mu^{\psi_{\rm d}} = \sum_\ell \left[s_{{\rm d}\ell} s_\ell - \frac{\mathcal{H} a_{\rm d}^2}{H_0 r_{\rm d} a^3} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_{{\rm d}\ell}}{d\ln a} a_\ell \right]$$
$$\mu^{\psi_{\rm d}}_{\rm dm} = \sum_\ell \left[s_{{\rm d}\ell}^2 - \frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta_{\rm dm} k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_{{\rm d}\ell}}{d\ln a} a_\ell \right]. \tag{86}$$

The renormalized Newton's constants are now

$$\mathcal{G}^{\phi_{\rm d}} = \sum_\ell s_{{\rm d}\ell} s_\ell \frac{r_\ell}{r_{\rm d}}, \qquad \mathcal{G}^{\phi_{\rm d}}_{\rm dm} = \sum_\ell s_{{\rm d}\ell}^2 \frac{r_\ell}{r_{\rm d}},$$
$$\mathcal{G}^{\psi_{\rm d}} = \sum_\ell s_{{\rm d}\ell} s_\ell, \qquad \mathcal{G}^{\psi_{\rm d}}_{\rm dm} = \sum_\ell s_{{\rm d}\ell}^2, \tag{87}$$

which are again positive. The comparison with Eq. (84) shows that the cross-terms are related by

$$\mathcal{G}^{\phi}_{\rm dm} = r_{\rm d} \mathcal{G}^{\phi_{\rm d}}, \qquad \mathcal{G}^{\psi}_{\rm dm} = \mathcal{G}^{\psi_{\rm d}}. \tag{88}$$

We show in Fig. 8 the coefficients $\mu_*^{\phi_{\rm d}}$ and $\mu_*^{\psi_{\rm d}}$ for the symmetric solution of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$, as well as the effective Newton constants. We obtain behaviors that are similar to those found in Fig. 7 for the baryonic metric potentials. At late times, the fifth force is again greater than Newtonian gravity and can lead to repulsive effects between baryons and dark matter.

C. Density and velocity fields

In their Jordan frame, associated with the metric $g_{\mu\nu}$, the baryons follow the usual equation of motion $\nabla_\mu T^\mu_\nu = 0$. This gives the standard continuity and Euler equations

$$\frac{\partial \rho}{\partial \tau} + \nabla \cdot (\rho \mathbf{v}) + 3\mathcal{H}\rho = 0,$$
$$\frac{\partial \mathbf{v}}{\partial \tau} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \mathcal{H}\mathbf{v} = -\nabla\phi. \tag{89}$$

Using the Poisson equation (82), we obtain the evolution equation of the linear baryonic matter density contrast,

$$\frac{\partial^2 \delta}{(\partial \ln a)^2} + \left[1 + \frac{d\ln \mathcal{H}}{d\ln a}\right] \frac{\partial \delta}{\partial \ln a} = \frac{3H_0^2}{2a\mathcal{H}^2} [\mu^\phi \Omega_{\rm b0} \delta + \mu^\phi_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm}]. \tag{90}$$

The dark matter also follows its usual equation of motion, $\nabla_{\mathrm{d}\mu} T^\mu_\nu = 0$, where $\nabla_{\mathrm{d}\mu}$ is now the covariant derivative associated with the dark sector metric $g_{\mathrm{d}\mu\nu}$. This gives the continuity and Euler equations

$$\frac{\partial \rho_{\rm dm}}{\partial \tau} + \nabla \cdot (\rho_{\rm dm} \mathbf{v}_{\rm dm}) + 3\mathcal{H}_{a_{\rm dm}} \rho_{\rm dm} = 0,$$

$$\frac{\partial \mathbf{v}_{\rm dm}}{\partial \tau} + (\mathbf{v}_{\rm dm} \cdot \nabla) \mathbf{v}_{\rm dm} + (2\mathcal{H}_{a_{\rm d}} - \mathcal{H}_{b_{\rm d}}) \mathbf{v}_{\rm dm} = -r_{\rm d}^2 \nabla \phi_{\rm d}, \tag{91}$$

where τ is still the conformal time of the baryonic metric. Using the Poisson equation, the evolution equation of the linear dark matter density contrast reads as

$$\frac{\partial^2 \delta_{\rm dm}}{(\partial \ln a)^2} + \left[\frac{2\mathcal{H}_{a_{\rm d}} - \mathcal{H}_{b_{\rm d}}}{\mathcal{H}} + \frac{d\ln \mathcal{H}}{d\ln a}\right] \frac{\partial \delta_{\rm dm}}{\partial \ln a} = \frac{3r_{\rm d}^2 H_0^2}{2a_{\rm d}\mathcal{H}^2} \times [\mu^{\phi_{\rm d}} \Omega_{\rm b0} \delta + \mu^{\phi_{\rm d}}_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm}]. \tag{92}$$

The baryonic and dark matter linear growing modes are coupled and given by the system of Eqs. (90) and (92). We show in Fig. 9 their behavior as a function of redshift for the comoving wave number $k = 0.1h/\mathrm{Mpc}$. At high redshift, they follow the Λ-CDM reference, but at low redshift, the dark matter perturbations grow faster than in the Λ-CDM cosmology, whereas the baryonic perturbations grow more slowly. This is more clearly seen in the lower panel, as the growth rate $f_* = d\ln D_*^+/d\ln a$ amplifies the deviations from the Λ-CDM cosmology because of the time derivative.

The data points in Fig. 9 are only given to compare the magnitude of the deviation of the growth factor with observational error bars but do not provide a meaningful test. Indeed, the Newton constant obtained in this scenario is amplified at $z = 0$, as seen in Fig. 7. This means that to compare with data we would need to run this model again by normalizing Newton's constant to its value at $z = 0$ instead of $z \to \infty$, as we have done so far. We do not go further in this direction in this paper, because this model is already ruled out by the large time derivative $d\ln \mathcal{G}/dt \sim 0.7H_0$ at $z = 0$, as we discuss in the next section.

Nevertheless, it is interesting to note that this model leads to a slower growth for the baryonic density perturbations than in the Λ-CDM cosmology. This is due to the decrease of the gravitational attraction of dark matter onto baryonic matter, shown by the coefficient $\mu^\phi_{\rm dm}$ in Fig. 7, which even turns negative at $z \lesssim 0.1$ (i.e., the fifth force between dark matter and baryons becomes repulsive). This is a distinctive feature of this model, as most modified-gravity scenarios amplify the growth of large-scale structures.

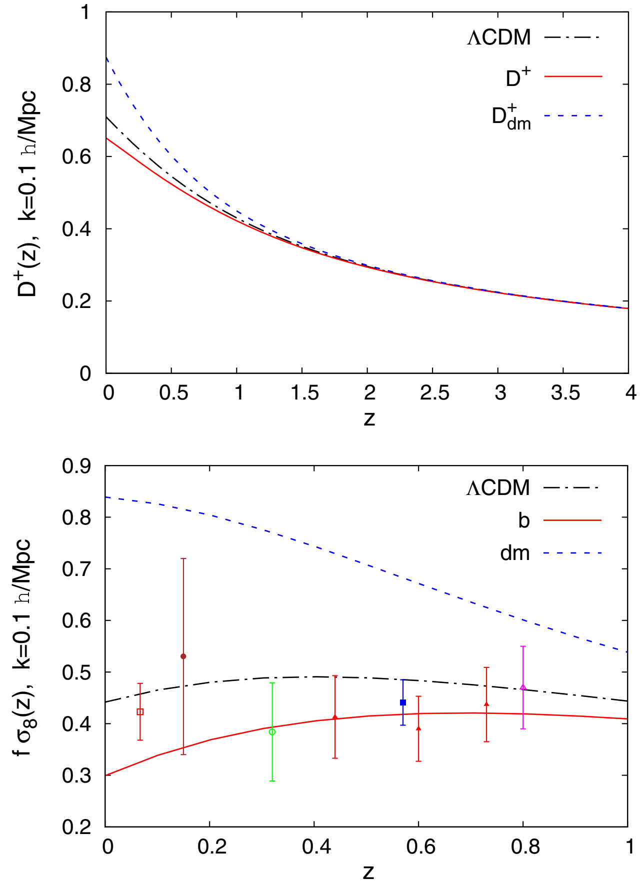

FIG. 9. *Upper panel:* linear growing modes $D^+(k, a)$ and $D^+_{\rm dm}(k, a)$, for the symmetric model of Fig. 1, at comoving wave number $k = 0.1h/\mathrm{Mpc}$. *Lower panel:* growth factors $f\sigma_8$ and $f_{\rm dm}\sigma_{\rm dm\,8}$.

We show in Fig. 10 the growth factors obtained for the case (62) of Fig. 4, where the different metrics have different conformal times. This actually gives similar results for the linear growth of large-scale structures.

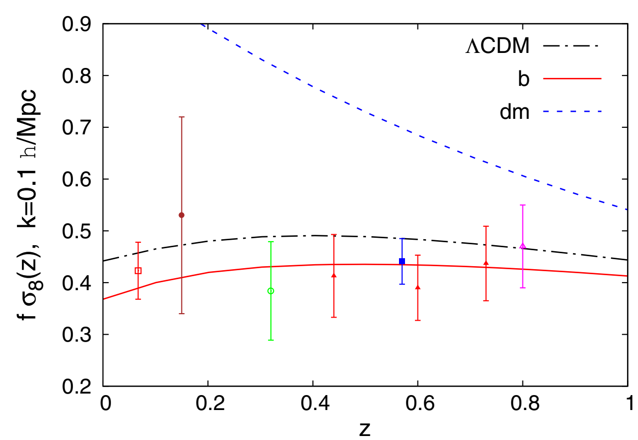

FIG. 10. Growth factors $f\sigma_8$ and $f_{\rm dm}\sigma_{\rm dm8}$, for the model of Fig. 4 with different conformal times.

D. Gravitational slip

Because the fifth force enters with opposite signs in the ϕ and ψ gravitational potentials, see Eq. (83), the lensing potential $\phi_{\rm lens} = (\phi+\psi)/2$, which deflects light rays, and the dynamical potential ϕ, which determines the trajectory of massive bodies, are different. This means that the lensing mass of clusters of galaxies (deduced from lensing observations) and the dynamical mass (deduced from the galaxy velocity dispersion or the pressure profile of the hot gas in hydrostatic equilibrium) are also different. This is measured by the ratio η, which we define as

$$\eta = \frac{\phi+\psi}{2\phi} = \frac{1}{2} + \frac{\mu^{\psi}\Omega_{\rm b0}\delta + \mu^{\psi}_{\rm dm}\Omega_{\rm dm0}\delta_{\rm dm}}{2[\mu^{\phi}\Omega_{\rm b0}\delta + \mu^{\phi}_{\rm dm}\Omega_{\rm dm0}\delta_{\rm dm}]}. \quad (93)$$

We show in Fig. 11 the gravitational slip η on subhorizon scales, for $\delta = \delta_{\rm dm}$, $\delta_{\rm dm} = 0$ (which corresponds to cases where $\rho \gg \rho_{\rm dm}$), and $\delta = 0$ (for $\rho_{\rm dm} \gg \rho$). In agreement with Fig. 7, the three curves converge to the General Relativity value $\eta = 1$ at high redshift and show deviations of order unity at low z. Because the couplings to baryons and dark matter are different, the gravitational slip η depends on the relative amount of baryons and dark matter in the lens. On cosmological scales down to clusters of galaxies, which are the largest collapsed structures, we expect $\delta \simeq \delta_{\rm dm}$. This gives $\eta > 1$ at low z; hence, the lensing mass would be greater than the dynamical mass. This ratio can reach a factor 3 at $z < 0.1$, but in practice, most cosmological lenses are at redshifts $z \gtrsim 0.5$, as the lensing efficiency goes to zero as the source redshift vanishes. This gives $1 < \eta \lesssim 1.7$. On the other hand, on subgalactic scales where baryons dominate, the gravitational slip is smaller than unity so that the lensing mass is smaller than the dynamical mass by a factor 3 at $z = 0$. In the case where dark matter dominates, η goes to infinity at $z \sim 0.3$ and becomes negative at lower redshift. This is because ϕ goes through zero and changes sign. This follows from $\mu^{\phi}_{\rm dm} < 0$, as seen in Fig. 7. This implies a repulsive fifth force from dark matter onto baryons, which dominates when the lens is mostly made of dark matter. This regime should not be reached in practice, as we have $\delta_{\rm dm} \sim \delta$ on large scales, where the separation of baryons from dark matter due to the fifth force has not yet had time to be efficient, as seen by the small impact on the linear growing modes in Fig. 9, whereas we typically have $\rho \gg \rho_{\rm dm}$ on subgalactic scales because radiative cooling processes make baryons collapse further and eventually form stars.

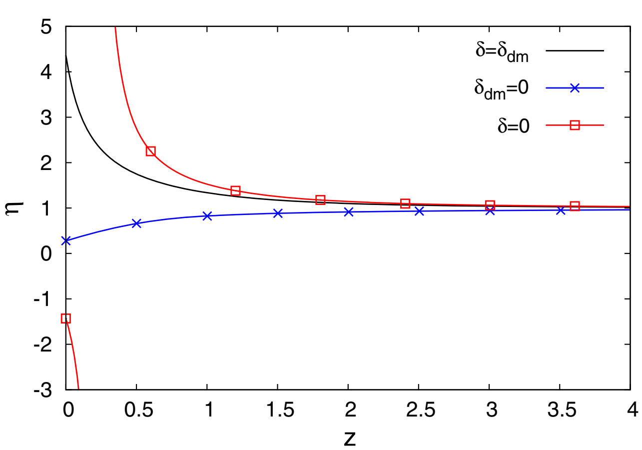

FIG. 11. Gravitational slip η of Eq. (93) for several values of the baryon to dark matter ratio $\delta/\delta_{\rm dm}$.

V. DYNAMICAL DEGREES OF FREEDOM AND LINEAR PERTURBATIONS

In this section, we study the behavior of linear perturbations around the cosmological background for the tensor, vector, and scalar sectors, without using the quasistatic approximation. This allows us to count the number of dynamical degrees of freedom, beyond the simple counting of components described in Sec. II B above. The number of perturbative degrees of freedom in bigravity theories has been discussed in Refs. [13–15]. They obtained the behavior of scalar, vector, and tensor modes by expanding the action up to quadratic order over the fluctuations. We present an alternative derivation, starting directly from the vierbeins as for our derivation of the nonlinear Einstein equations (19). This also allows us to implement explicitly the discussion of Sec. II B and to show how the 32 components of the vierbeins can be reduced to the expected 16 components by successive gauge choices, associated with the diagonal Lorentz and diffeomorphism invariances and with the symmetry constraint (9). Then, constraint equations further reduce the number of dynamical degrees of freedom. We find that there are no ghosts at the level of the quadratic action around the cosmological background.

In Minkowski space-time, i.e., in vacuum, the bimetric action (2) reduces to two independent copies of General Relativity. Therefore, it shows $2\times 2 = 4$ dynamical degrees of freedom (associated with the two massless gravitons of the tensor sector), without ghosts nor dangerous instabilities. In the following, we focus on perturbations around the cosmological background, with nonzero mean density and pressure and with cosmological expansion.

A. Vierbein and metric perturbations, quadratic action

Starting from the vierbeins $\delta e^a_{\ell\mu}$, the metric perturbations $\delta g_{\ell\mu\nu}$ are given from the definition (6) by

$$\delta g_{\ell\mu\nu} = (\delta e^a_{\ell\mu} e^b_{\ell\nu} + e^a_{\ell\mu}\delta e^b_{\ell\nu})\eta_{ab}. \quad (94)$$

For the diagonal background (24), this simplifies to

$$\delta g_{\ell 00} = -2b_\ell \delta e^0_{\ell 0}, \qquad \delta g_{\ell 0i} = \delta g_{\ell i0} = a_\ell \delta e^i_{\ell 0} - b_\ell \delta e^0_{\ell i},$$
$$\delta g_{\ell ij} = a_\ell (\delta e^i_{\ell j} + \delta e^j_{\ell i}). \tag{95}$$

The perturbations of the matrices $X^\mu_{\ell\nu}$ defined in Eq. (11) also simplify as

$$\delta X^0_{20} = -\frac{\delta e^0_{20} b_1}{b_2^2} + \frac{\delta e^0_{10}}{b_2}, \qquad \delta X^i_{20} = -\frac{\delta e^i_{20} b_1}{a_2 b_2} + \frac{\delta e^i_{10}}{a_2},$$
$$\delta X^0_{2i} = -\frac{\delta e^0_{2i} a_1}{b_2 a_2} + \frac{\delta e^0_{1i}}{b_2}, \qquad \delta X^i_{2j} = -\frac{\delta e^i_{2j} a_1}{a_2^2} + \frac{\delta e^i_{1j}}{a_2}. \tag{96}$$

The permutation $1 \leftrightarrow 2$ provides $\delta X^\mu_{1\nu}$.

As in General Relativity, we can split the gravitational perturbations in scalar, vector, and tensor modes. As in Ref. [15], we can do so at the level of the vierbeins, and we can write

$$\delta e^0_{\ell 0} = b_\ell \phi_\ell, \quad \delta e^0_{\ell i} = a_\ell[-\partial_i V_\ell + C_{\ell i}],$$
$$\delta e^i_{\ell 0} = b_\ell[-\partial^i W_\ell + D^i_\ell],$$
$$\delta e^i_{\ell j} = a_\ell[-\psi_\ell \delta^i_j + \partial^i\partial_j U_\ell + \partial_j V^i_\ell + \partial^i W_{\ell j} + h^i_{\ell j}], \tag{97}$$

where the spatial indices are raised and lowered with δ^{ij} and δ_{ij}, so that $\partial^i = \partial_i$ and $V^i_\ell = V_{\ell i}$. The transversality conditions are

$$\partial^i C_{\ell i} = \partial_i D^i_\ell = \partial_i V^i_\ell = \partial^i W_{\ell i} = 0, \qquad \partial_i h^i_{\ell j} = 0,$$

and tracelessness corresponds to

$$h^i_{\ell i} = 0.$$

This provides the perturbations of the gravitational metrics as

$$\delta g_{\ell 00} = -2b_\ell^2 \phi_\ell,$$
$$\delta g_{\ell 0i} = a_\ell b_\ell[\partial_i(V_\ell - W_\ell) + D_{\ell i} - C_{\ell i}],$$
$$\delta g_{\ell ij} = a_\ell^2[-2\psi_\ell \delta_{ij} + 2\partial_i\partial_j U_\ell + \partial_i(V_{\ell j} + W_{\ell j}) + \partial_j(V_{\ell i} + W_{\ell i}) + h_{\ell ij} + h_{\ell ji}]. \tag{98}$$

The baryonic and dark vierbeins and metrics obey the same decompositions, obtained from the combinations (8).

This gives 32 components for the two gravitational metrics: ten scalars $\{\phi_\ell, V_\ell, W_\ell, \psi_\ell, U_\ell\}$, eight vectors $\{C_{\ell i}, D^i_\ell, V^i_\ell, W_{\ell i}\}$, and two nonsymmetric tensors $h^i_{\ell j}$. As explained in Sec. II B, this can be reduced to 16 components when we use the invariance under the diagonal Lorentz transformations and diffeomorphisms and the symmetry constraints (9). It is convenient to handle the Lorentz invariance and the symmetry constraints (9) through the variables $\delta Z_{\ell\mu\nu}$ introduced in Eq. (16).This suppresses the Lorentz degeneracies associated with the vierbeins by the condition $\delta Z_{\ell\mu\nu} = \delta Z_{\ell\nu\mu}$, which implies

$$h_{\ell ij} = h_{\ell ji}, \quad D_{\ell i} = -C_{\ell i}, \quad V_{\ell i} = W_{\ell i}, \quad W_\ell = -V_\ell \tag{99}$$

and removes 2×6 components. The diagonal diffeomorphism invariance still remains.

We study first the dynamics of tensor, vector, and scalar perturbations, in the early-time regime where the scalar field is constant and the background follows the simple solution (41), i.e., all metrics show the same Hubble expansion rate. Then, dark and baryonic matter can be unified in the same matter sector as $s_{d\ell} = s_\ell$. In all three cases, the explicit computation of the Einstein equations (19) at linear order shows that the perturbations separate in two decoupled sectors, S_+ and S_-. The sector S_+ involves the matter perturbations, which act as source terms in the Einstein equations, and the matter metric defined from Eq. (8), which gives

$$h_{\mu\nu} = s_1^2 h_{1\mu\nu} + s_2^2 h_{2\mu\nu}, \tag{100}$$

where $h_{\ell\mu\nu}$ are the linear metric perturbations of the two gravitational metrics, defined by $g_{\ell\mu\nu} = a^2(\eta_{\mu\nu} + 2h_{\ell\mu\nu})$. We find that the Einstein equations of this sector are identical to General Relativity. Therefore, in this regime, there is no deviation from General Relativity in the sector probed by matter and by observations. The hidden sector S_- has no matter source terms and only involves the hidden metric components $h_{-\mu\nu}$, defined by

$$h_{-\mu\nu} = h_{1\mu\nu} - h_{2\mu\nu}. \tag{101}$$

Its equations of motion differ from those of General Relativity by mass terms. [The components $h_{-\mu\nu}$ do not directly define a metric, because if we define the vierbeins $e^a_{-\mu} = s_1^{-1} e^a_{1\mu} - s_2^{-1} e^a_{2\mu}$, which would imply (101), we find that the background vierbeins $\bar{e}^a_{-\mu}$ vanish.]

We can check that the equations of motion of the hidden sector S_- can be derived from the quadratic action defined by the standard expression

$$\delta^2 S = \int d^4x \frac{1}{2} \delta\left(\frac{\delta S}{\delta Z_{\mu\nu}}\right) \delta Z_{\mu\nu}, \tag{102}$$

but where again we work at the level of the vierbeins and use the variables $\delta Z_{\mu\nu}$ of Eq. (16). For instance, using Eq. (17), we obtain for the quadratic part that arises from the first gravitational action $S_1 = \int d^4x (M^2_{\rm Pl}/2)\sqrt{-g_1} R_1$ the expression

$$\delta^2 S_1 = -\frac{M_{\rm Pl}^2}{2}\int d^4x\delta[\sqrt{-g_1}(G_1^{\mu\sigma}X^{\nu}{}_{2\sigma} + G_1^{\nu\sigma}X^{\mu}_{2\sigma})] \times \Theta_{\mu\nu}\delta Z_{1\mu\nu}, \quad (103)$$

where $\Theta_{\mu\nu} = 1$ or $1/2$ was introduced in (17). We recognize the structure of the left-hand side of the Einstein equations (19). The contribution $\delta^2 S_2$ from the second gravitational action S_2 can be obtained from Eq. (103) by the permutation $1 \leftrightarrow 2$. There are also similar contributions from the matter action. For the matter sector S_+, this procedure is more complex because of the coupling to matter. This involves term linear and quadratic in matter perturbations, which enforce the coupling between fluid and metric perturbations and the equations of motion of the fluid. These terms do not contribute to the hidden sector S_-, as can be seen from a direct computation of the Einstein equations from Eq. (19).

B. Tensor modes

In the tensor sector, we consider the evolution of metric perturbations over the background. Because we do not consider matter sources and there are no tensor gauge transformations, the computation from the Einstein equations coincide with the one from the quadratic action (102) where we do not include matter perturbations. Then, the quadratic action separates as

$$\delta^2 S = \delta^2 S_+ + s_1^2 s_2^2 \delta^2 S_- \quad (104)$$

with

$$\delta^2 S_+ = \int d^4x a^2 M_{\rm Pl}^2 [h_{ij}'^2 - (\nabla h_{ij})^2] \quad (105)$$

and

$$\delta^2 S_- = \int d^4x \{a^2 M_{\rm Pl}^2 [h_{-ij}'^2 - (\nabla h_{-ij})^2] + a^4 \bar{p} h_{-ij}^2\}, \quad (106)$$

where the sum is only over the independent components. Thus, at the quadratic order, the action $\delta^2 S_+$ of the matter sector is identical to that of General Relativity, while there exists a second decoupled sector that differs from General Relativity by a new mass term. This leads to $2 \times 2 = 4$ dynamical degrees of freedom in the tensor sector.

We recover the results obtained in Refs. [13–15]. Omitting the indices ij, the two uncoupled gravitons obey the equations of motion

$$h'' + 2\mathcal{H}h' - \nabla^2 h = 0, \quad (107)$$

$$h_-'' + 2\mathcal{H}h_-' - \nabla^2 h_- - \frac{a^2 \bar{p}}{M_{\rm Pl}^2} h_- = 0. \quad (108)$$

The massless graviton h of the baryonic and dark matter metric evolves as in General Relativity. On subhorizon scales, it propagates with the speed of light. On scales greater than the horizon, it contains a constant mode and a decaying mode that evolves as $h' \propto a^{-2}$. This physical mode (in the sense that it is the one seen by the matter metric) is governed by Eq. (107) throughout all cosmological eras and does not mix with the hidden graviton h_-.

The second hidden graviton h_- has a negative squared mass in the radiation era, as $\bar{p} = \bar{p}_\gamma > 0$, which becomes negligible in the matter era. In the radiation era, we have $\mathcal{H} = 1/\tau$ and $a = \sqrt{\Omega_{\gamma 0}} H_0 \tau$. The hidden massive graviton h_- obeys the equation of motion $h_-'' + \frac{2}{\tau} h_-' - \nabla^2 h_- - \frac{1}{\tau^2} h_- = 0$. It oscillates on subhorizon scales. On superhorizon scales, it contains both a decaying mode and a growing mode,

$$k \ll \mathcal{H}\colon\; h_-^- \propto a^{-(1+\sqrt{5})/2},\, h_-^+ \propto a^{(\sqrt{5}-1)/2}, \quad (109)$$

associated with the tachyonic instability. In the matter era, we have $\mathcal{H} = 2/\tau$ and $a \propto \tau^2$. The mass of the second graviton h_- becomes negligible, and it behaves like the massless graviton, with a constant mode and a decaying mode $\propto a^{-3/2}$.

Although h_- is not seen by the matter, it should remain small at all epochs so that the perturbative approach applies. This implies that the initial tensor fluctuations at the onset of the radiation era must be sufficiently small. This is easily satisfied as the squared mass turns positive during the inflation era and the graviton decays [13]. During the inflationary stage, the tensor evolution equation is still given by Eq. (108), where $\bar{p}$ is now the pressure $\bar{p}_\chi = -\bar{\rho}_\chi$ of the inflaton χ. Because we now have $\bar{p}_\chi < 0$, the squared mass becomes positive, and there is no tachyonic instability, and on superhorizon scales, there are only two decaying modes:

$$k \ll \mathcal{H}\colon\; h_-^c \propto a^{-3/2} \cos\left(\frac{\sqrt{3}}{2}\ln a\right),$$
$$h_-^s \propto a^{-3/2} \sin\left(\frac{\sqrt{3}}{2}\ln a\right). \quad (110)$$

Let us consider a mode k that remains above the horizon until the end of the radiation era, $k \le a_{\rm eq} H_{\rm eq}$. It crosses the horizon during the inflationary stage at the time $a_k = k/H_I$, where H_I is the constant Hubble expansion rate of the inflationary de Sitter era. Then, the amplitude of the tensor mode h_- at the end of the radiation era reads as

$$h_-(a_{\rm eq}) = h_-(a_k)\left(\frac{a_f}{a_k}\right)^{-3/2}\left(\frac{a_{\rm eq}}{a_f}\right)^{(\sqrt{5}-1)/2}, \quad (111)$$

where a_f is the scale factor at the end of the inflationary era. For $H_I \sim 10^{-5} M_{\rm Pl} \sim 10^{13}$ GeV, $a_f \sim 10^{-28}$, $a_{\rm eq} \sim 10^{-3}$, we find that all modes with $k \leq a_{\rm eq} H_{\rm eq}$ remain in the perturbative regime, $h_-(a_{\rm eq}) \ll 1$, provided $h_-(a_k) \ll 10^{24}$. As we expect $h_-(a_k) \sim H_I/M_{\rm Pl} \sim 10^{-5}$, if the tensor fluctuations are generated by the quantum fluctuations, all modes remain far in the perturbative regime until the end of the radiation era. This is due to their decay during the inflationary stage on superhorizon scales and to their small initial values associated with quantum fluctuations.

Therefore, the main constraint from the tensor sector is the measurement of the speed of gravitational waves from the binary neutron star merger GW170817 [29], which implies that at least one of the lapse factors r_ℓ is unity at $z = 0$, as discussed in Sec. III G.

C. Vector modes

In the vector sector, the perturbations of the energy-momentum tensor are

$$\delta T^{00} = 0, \qquad \delta T^{0i} = a^{-2}[\bar{\rho} v^i - 2\bar{p} C^i],$$
$$\delta T^{ij} = -2a^{-2}\bar{p}[\partial^i V^j + \partial^j V^i]. \tag{112}$$

As in General Relativity, the equations of motion of matter, $\nabla_\mu T^{\mu\nu} = 0$, decouple from the Einstein equations and read as

$$\frac{\partial}{\partial\tau}[(\bar{\rho}+\bar{p})U^i] + 4\mathcal{H}[(\bar{\rho}+\bar{p})U^i] = 0, \tag{113}$$

where we introduced the usual gauge invariant velocity,

$$U^i = v^i - 2C^i = v^i - 2(s_1^2 C_1^i + s_2^2 C_2^i). \tag{114}$$

The equations of motion follow from the Einstein equations (19). One can check that they also follow from a quadratic action that separates as in (104), with

$$\delta^2 S_+ = \int d^4x a^2 M_{\rm Pl}^2 [\nabla(V'_j + C_j)\cdot\nabla(V'_j + C_j)], \tag{115}$$

and

$$\delta^2 S_- = \int d^4x \left\{ a^2 M_{\rm Pl}^2 [\nabla(V'_{-j} + C_{-j})\cdot\nabla(V'_{-j} + C_{-j})] + a^4\left[\frac{3\bar{\rho}+\bar{p}}{2} C_{-j}^2 + \bar{p}(\nabla V_{-j})^2\right]\right\}. \tag{116}$$

Here, for the matter sector S_+, we focused on the solution $U^i = 0$ of the matter conservation equation (114). As for the tensors, the action $\delta^2 S_+$ of the matter metric is identical to General Relativity, while the second decoupled sector $\delta^2 S_-$ is modified by a new mass term that vanishes in the Minkowski space-time.

Therefore, as in General Relativity, there are no vector dynamical degrees of freedom left in $\delta^2 S_+$, if we set $U^i = 0$. One can see that C_i is not dynamical. Its "equation of motion" reads as $C_i = -V'_i$. Substituting into the action gives $\delta^2 S_+ = 0$, so that V_i is arbitrary. This is due to the diffeomorphism invariance of General Relativity.

In the action $\delta^2 S_-$, C_{-i} is again nondynamical. Its equation of motion reads in Fourier space

$$C_{-i}(\mathbf{k}) = -\frac{2M_{\rm Pl}^2 k^2}{2M_{\rm Pl}^2 k^2 + a^2(3\bar{\rho}+\bar{p})} V'_{-i}(\mathbf{k}), \tag{117}$$

and substituting into the action gives

$$\delta^2 S_- = (2\pi)^3 \int d\mathbf{k}\, d\tau a^4 k^2 \left\{ \frac{M_{\rm Pl}^2(3\bar{\rho}+\bar{p})}{2M_{\rm Pl}^2 k^2 + a^2(3\bar{\rho}+\bar{p})} \times V'_{-j}(\mathbf{k}) V'_{-j}(-\mathbf{k}) + \bar{p} V_{-j}(\mathbf{k}) V_{-j}(-\mathbf{k}) \right\}. \tag{118}$$

The vector V_{-i} is now dynamical when $3\bar{\rho}+\bar{p} \neq 0$. Therefore, we have two dynamical degrees of freedom in the vector sector, associated with the hidden vector V_{-i}.

Its equation of motion reads

$$-\frac{\partial}{\partial\tau}\left[\frac{a^4 M_{\rm Pl}^2(3\bar{\rho}+\bar{p})}{2M_{\rm Pl}^2 k^2 + a^2(3\bar{\rho}+\bar{p})} V'_{-j}(\mathbf{k})\right] + a^4 \bar{p} V_{-j}(\mathbf{k}) = 0. \tag{119}$$

Thus, the mode V_{-i} shows a gradient instability on subhorizon scales in the radiation and matter eras, where $\bar{p} = \bar{p}_\gamma > 0$, and we recover the results obtained in Refs. [13–15].

Let us consider in turns the inflationary, radiation, and matter eras. In the inflationary era, Eq. (119) gives on subhorizon scales $V''_- + 4\mathcal{H}V'_- + k^2 V_- = 0$ (where we omit the index i), so that the vector mode V_- oscillates with frequency $\omega = \pm k$. On superhorizon scales, we obtain $V''_- - \frac{2}{\tau}V'_- + \frac{3}{\tau^2}V_- = 0$. This is the same evolution equation as for the tensor modes, and we obtain the same two decaying solutions as in Eq. (110).

In the radiation era, on subhorizon scales, we obtain $V''_- - \frac{k^2}{5}V_- = 0$. This gradient instability leads to the two exponential modes

$$k \gg \mathcal{H}:\ V_-^\pm \propto e^{\pm k\tau/\sqrt{5}}. \tag{120}$$

On superhorizon scales, we again recover the same behavior as for tensors, $V''_- + \frac{2}{\tau}V'_- - \frac{1}{\tau^2}V_- = 0$, with the power-law growing and decaying modes (109).

In the matter era, we obtain on subhorizon scales $V''_- + \frac{2}{\tau}V'_- - \frac{8k^2}{9\mathcal{H}_{\rm eq}^2\tau^2}V_- = 0$, which gives the power-law growing and decaying modes

$$k \gg \mathcal{H}:\ V_-^{\pm} \propto \tau^{\lambda_\pm} \quad \text{with} \quad \lambda_\pm = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1+\frac{32k^2}{9\mathcal{H}_{\rm eq}^2}}. \tag{121}$$

On superhorizon scales, we have $V_-'' + \frac{4}{\tau}V_-' - \frac{4\tau_{\rm eq}^2}{\tau^4}V_- = 0$. Long after the radiation-matter equality, $\tau \gg \tau_{\rm eq}$, this gives a constant mode and a decaying mode $V_- \propto \tau^{-3}$.

Let us estimate the magnitude of the unstable vector mode V_- at $z=0$, for a wave number k that goes beyond the horizon at a_k during the inflationary stage and goes below the horizon at a_k' during the radiation era. Collecting the results above, we obtain

$$kV_- \sim kV_-(a_k)\left(\frac{H_I}{H_{\rm eq}}\right)^{-3/2}\left(\frac{a_{\rm eq}}{a_f}\right)^{(\sqrt{5}+2)/2} \times \left(\frac{k}{a_{\rm eq}H_{\rm eq}}\right)^{(4-\sqrt{5})/2} e^{\frac{k}{a_{\rm eq}H_{\rm eq}}\left(\frac{1}{\sqrt{5}}-\frac{\sqrt{2}}{3}\ln a_{\rm eq}\right)}. \tag{122}$$

After the horizon exit during the inflationary era, this mode first decays as $a^{-3/2}$ until the end of the inflationary era at a_f. Next, it grows as $a^{(\sqrt{5}-1)/2}$ during the radiation era, until it enters the horizon. Then, its subhorizon behavior deviates from the one of the tensor mode h_- as it shows the exponential growth (120) until the matter era starts where it shows the power-law growth (121). These last two stages give the exponential factor in Eq. (122), which is actually dominated by the matter era growth factor. If we assume that at horizon exit during the inflationary stage, we have $C_- \sim V_-' \sim kV_- \sim H_I/M_{\rm Pl}$, and we obtain for $H_I \sim 10^{-5}M_{\rm Pl}$ that $kV_- \ll 1$ at $z=0$ for $k \ll 0.3h\mathrm{Mpc}^{-1}$. Therefore, on weakly nonlinear scales and below, the growth of the hidden vector jeopardizes the perturbativity of the model, and the gravitational metrics g_1 and g_2 become nonlinear in this regime. This implies that the initial vector seeds at the horizon exit during the inflationary era should be suppressed or that the scenario must be supplemented by additional mechanisms that damp the growth of this vector mode on small scales at high redshift.

D. Scalar modes

The same decoupling as for tensors and vectors occurs for scalars. The matter sector S_+ is again identical to General Relativity. The hidden sector S_- does not couple to matter and differs from General Relativity by mass terms. Its quadratic action reads as

$$\begin{aligned}\delta^2 S_- = \int d^4x a^2 M_{\rm Pl}^2 \{&\phi_-[-3\mathcal{H}^2\phi_- - 9\mathcal{H}^2\psi_- + 3\mathcal{H}^2\nabla^2 U_- \\ &- 6\mathcal{H}\psi_-' - 4\mathcal{H}\nabla^2 V_- + 2\mathcal{H}\nabla^2 U_-' + 2\nabla^2\psi_-] \\ &+ \psi_-[3(\mathcal{H}'-\mathcal{H}^2)\psi_- - 2(2\mathcal{H}'+\mathcal{H}^2)\nabla^2 U_- + 8\mathcal{H}\nabla^2 V_- \\ &- 4\mathcal{H}\nabla^2 U_-' + 3\psi_-'' - \nabla^2\psi_- + 4\nabla^2 V_-' - 2\nabla^2 U_-''] \\ &+ (4\mathcal{H}^2 - \mathcal{H}')(\nabla V_-)^2\}.\end{aligned} \tag{123}$$

The equations of motion in this scalar sector can be easily carried out and result in no dynamical degree of freedom. Indeed, as the scalar U only enters linearly, it is nondynamical, and it provides a constraint equation that allows us to substitute for ψ''. The scalar V is also nondynamical, and its equation of motion allows us to substitute for ψ'. Then, ϕ only enters linearly; hence, it is also nondynamical and provides another constraint equation, when ψ is also nondynamical. Thus, there are no new dynamical degrees of freedom in the scalar sector.

E. Dynamical degrees of freedom in Einstein-de Sitter space-time

To summarize the results from the previous sections, in Minkowski space-time, we have two copies of General Relativity and 4 dynamical degrees of freedom, associated with the two massless gravitons.

Around the cosmological background, chosen to be the early Universe Einstein-de Sitter solution (41) of the equations of motion, the quadratic action separates as a part $\delta^2 S_+$ that describes the metric seen by matter and a part $\delta^2 S_-$ that describes a second hidden metric. The first part $\delta^2 S_+$ remains identical to General Relativity, with 2 dynamical degrees of freedom associated with the massless graviton. The second part $\delta^2 S_-$ contains new mass terms. It generates 4 degrees of freedom, associated with a massive graviton and a transverse vector that shows a gradient instability. At this level, there are no new scalar dynamical degrees of freedom and no ghosts.

We study the linear perturbations in the general case in Appendix A, when we no longer assume $s_{{\rm dl}\ell}$ and s_ℓ to be equal and the various metrics can have different Hubble expansion rates. As the baryonic and dark matter metrics are different, the quadratic action no longer separates in a sector S_+, which contains all matter and remains identical to General Relativity, and a hidden sector S_- that differs from General Relativity by mass terms and is decoupled from matter. However, from the Einstein equations, we find that linear perturbations behave in the same fashion as in the simpler case presented above. In the tensor sector, we have two massive gravitons, which at high frequency and wave number have a negligible mass and behave as in General Relativity. In the vector sector, we can still separate $\{C_i, V_i\}$ and $\{C_{-i}, V_{-i}\}$. Again, there are only 2 propagating degrees of freedom, associated with V_{-i}, and they still show the gradient instability (120) in the radiation era. In the scalar sector, no new dynamical degrees of freedom or ghosts appear.

In the next section, we will analyze the existence of ghosts and the cutoff of the theory by performing a Stückelberg analysis.

VI. ANALYSIS OF GHOSTS BY THE STÜCKELBERG METHOD

As shown by the explicit computation of linear perturbations around the Einstein-de Sitter cosmological background, in that case, the system decouples in the two sectors S_+ and S_-. The sector S_+ contains the matter metric perturbations $\delta g_{\mu\nu}$ and the matter fluid perturbations, such as $\delta\rho$, and it coincides with General Relativity. It is the sector relevant for observations (at this linear order). The hidden sector S_- contains the other metric components, $\delta g_{-\mu\nu}$, and is not sourced by matter. This shows that around the cosmological background it is more convenient to decompose the metric degrees of freedom in these two metrics, rather than the two gravitational metrics $\delta g_{\ell\mu\nu}$. In particular, it means that the two sets $\delta g_{\ell\mu\nu}$ are strongly coupled and that one cannot study the fluctuations of $g_{1\mu\nu}$ while neglecting its coupling to $g_{2\mu\nu}$.

In contrast, in vacuum, we only have two independent Einstein-Hilbert terms, giving rise to two independent copies of General Relativity. Therefore, around the Minkowski background, the relevant decomposition is over the two gravitational metrics $\delta g_{\ell\mu\nu}$. This shows that the physics is quite different over these two backgrounds, and different treatments are appropriate.

A. Explicit quadratic action around Einstein-de Sitter background

We now check with the Stückelberg method that there is no Boulware-Deser ghost at the linear order of perturbations around the cosmological background. In massive gravity or bigravity theories, a Boulware-Deser ghost [31] can appear in the scalar sector because of the new degrees of freedom, associated with the additional metric or the loss of gauge invariance. In General Relativity, there are no scalar dynamical degrees of freedom around Minkowski or Einstein-de Sitter backgrounds because the gauge invariance removes 2 scalar degrees of freedom (among the four scalar components, two are nondynamical fields or Lagrange multipliers, and the other two are pure gauges). In a bimetric theory like the one we consider in this paper, we have two metrics, but only the diagonal gauge invariance is left. Therefore, as compared to two independent copies of General Relativity, we have additional degrees of freedom, as one gauge invariance is missing in order to remove a few of them. Then, some of these new degrees of freedom may turn out to be ghosts.

The sector S_+ being identical to General Relativity, it is healthy and it makes full use of the diagonal gauge invariance. We will try to restore full diffeomorphism invariance by performing a Stückelberg analysis on the decoupled sector. Because S_- is decoupled (at linear order), we can study the quadratic action (123) alone. Around the cosmological background, a change of coordinates $x^\mu \to x^\mu + \xi^\mu$ corresponds at linear order to a change of the metric

$$\delta g_{\mu\nu} \to \delta g_{\mu\nu} - \frac{\partial \bar{g}_{\mu\nu}}{\partial x^\sigma}\xi^\sigma - \bar{g}_{\sigma\nu}\frac{\partial \xi^\sigma}{\partial x^\mu} - \bar{g}_{\mu\sigma}\frac{\partial \xi^\sigma}{\partial x^\nu}. \tag{124}$$

Because we have lost gauge invariance, the action $\delta^2 S_-$ is not invariant when $\delta g_{-\mu\nu}$ transforms as in (124). Following the Stückelberg formalism, we can introduce an additional field ζ^μ to restore the gauge invariance, by writing [32]

$$\delta g_{-\mu\nu} = \hat{\delta g}_{-\mu\nu} + \frac{\partial \bar{g}_{-\mu\nu}}{\partial x^\sigma}\zeta^\sigma + \bar{g}_{-\sigma\nu}\frac{\partial \zeta^\sigma}{\partial x^\mu} + \bar{g}_{-\mu\sigma}\frac{\partial \zeta^\sigma}{\partial x^\nu}. \tag{125}$$

Then, the action $\delta^2 S_-(\hat{\delta g}_{-\mu\nu}, \zeta^\mu)$ is invariant under the combined gauge transformation where $\hat{\delta g}_{-\mu\nu}$ transforms as in (124), while ζ transforms as $\zeta^\mu \to \zeta^\mu + \xi^\mu$. By choosing another gauge condition than $\zeta^\mu = 0$, one can often read on the Lagrangian terms involving ζ^μ the behavior of dangerous modes. Focusing on the scalar sector, with $\zeta^\mu = \eta^{\mu\nu}\partial_\nu \pi$, this gives for the scalar perturbations of the hidden metric $\delta g_{-\mu\nu}$

$$\begin{aligned} &\phi_- = \hat{\phi}_- - \mathcal{H}\pi' - \pi'', \qquad \psi_- = \hat{\psi}_- + \mathcal{H}\pi' \\ &V_- = \hat{V}_- + \pi', \qquad U_- = \hat{U}_- + \pi. \end{aligned} \tag{126}$$

Substituting into the quadratic action (123), one finds that the Stückelberg field π does not cancel out because the action $\delta^2 S_-$ is not gauge invariant. We could expect quadratic terms with up to four derivatives from (126), which would be the usual signature of the Boulware-Deser ghost. However, the explicit computation from Eq. (123) shows that all third and fourth-order time derivatives cancel out and the action can be written in terms of first-order time derivatives. This means that there is no Ostrogradsky ghost, associated with higher derivative terms in the Lagrangian, at linear order around the cosmological background.

For completeness, the explicit expression of the action is given by $\delta^2 S_- = \delta^2 S_-^{(0)} + \delta^2 S_-^{(1)} + \delta^2 S_-^{(2)}$, where $\delta^2 S_-^{(0)}$ is given by Eq. (123) where we add a hat to the metric variables, $\delta^2 S_-^{(1)}$ is the linear part over π, and reads as

$$\begin{aligned} \delta^2 S_-^{(1)} = \int d^4x a^2 M_{\rm Pl}^2 \{ &\hat{\phi}_- 3\mathcal{H}[\mathcal{H}\nabla^2\pi - (\mathcal{H}^2 + 2\mathcal{H}')\pi'] \\ &+ \hat{\psi}_-[(\mathcal{H}^2 + 2\mathcal{H}')(3\pi'' - 2\nabla^2\pi) \\ &+ 3(\mathcal{H}^3 + 4\mathcal{H}\mathcal{H}' + 2\mathcal{H}'')\pi'] \\ &- 2(2\mathcal{H}^2 + \mathcal{H}')(\nabla^2\hat{V}_-)\pi' - (\nabla^2\hat{U}_-)[(\mathcal{H}^2 + 2\mathcal{H}')\pi'' \\ &+ (\mathcal{H}^3 + 4\mathcal{H}\mathcal{H}' + 2\mathcal{H}'')\pi']\}, \end{aligned} \tag{127}$$

which only involves first-order time derivatives if we integrate π'' by parts, and $\delta^2 S_-^{(2)}$ is the quadratic part over π and reads as

$$\begin{aligned} \delta^2 S_-^{(2)} = \int d^4x \frac{a^2 M_{\rm Pl}^2}{2} \{ &-3(\mathcal{H}^2 + 2\mathcal{H}')\mathcal{H}'\pi'^2 \\ &+ (7\mathcal{H}^2\mathcal{H}' + 2\mathcal{H}'^2 + 2\mathcal{H}(\mathcal{H}^3 + \mathcal{H}''))(\nabla\pi)^2\}, \end{aligned} \tag{128}$$

which only contains first-order time derivatives.

As was the case for the original action (123), we can check from the action $\delta^2 S_- = \delta^2 S_-^{(0)} + \delta^2 S_-^{(1)} + \delta^2 S_-^{(2)}$ that there are no propagating modes and π is not dynamical. This is not apparent from the quadratic part (128), but π is coupled to the other metric components through (127). Then, for instance, U_- again enters linearly into the action and provides a constraint that removes another degree of freedom. After successive simplifications, one finds that there are no physical dynamical modes left.

We obtain the same result in Appendix A for the more general case where the different metrics follow different Hubble expansion rates.

B. Goldstone bosons

We now study how ghosts may appear beyond the linear perturbation theory investigated in the previous section and beyond the Einstein-de Sitter case, when the baryonic and dark matter metrics are different. We again follow the Stückelberg formalism, and we first show that we do not need to explicitly compute the action to recover the previous results at linear order, in the regime of short time and length scales as compared to the horizon and the age of the Universe. Next, we discuss the nonlinear terms. Notice that our analysis remains perturbative around Friedmann-Lemaître-Robertson-Walker (FLRW) backgrounds throughout and that a full investigation of the presence of ghosts should be carried out non-perturbatively.

As noticed above, in the absence of matter, our bimetric theory reduces to two copies of General Relativity, and it is therefore ghost free. This corresponds to the Minkowski background, and one would like to extend this result to the case of FLRW spaces, where the coupling of the two metrics is present through the matter actions and might reintroduce a Boulware-Deser ghost. As in (125), this can be investigated by introducing four Goldstone fields ζ^μ of which the role is to restore the full diffeomorphism invariance of the theory, which is broken by the presence of the matter actions. The order parameter of the breaking of the two copies of diffeomorphism invariance to the diagonal subgroup is the Hubble parameter of the Universe. We will see that it plays the same role as the mass term for gravitons in massive bigravity [10].

In the following, we consider the case where $a_\ell = b_\ell$ (i.e., all metrics have a common conformal time), so that the background vierbeins are diagonal with

$$\bar{e}^a_{\ell\mu} = a_\ell \delta^a_\mu, \tag{129}$$

and we focus on short times compared to the age of the Universe and short distances compared to the horizon,

$$\partial \ln h_{\mu\nu} \gg \mathcal{H}. \tag{130}$$

Here, $h_{\mu\nu}$ stands for the metric perturbations, and $\mathcal{H}$ stands for the conformal Hubble expansion rates, which we take to be of the same order for the different metrics. In contrast with Sec. VI A, we do not restrict to the early-time regime (41). Hence, the baryonic and dark matters follow different metrics $g_{\mu\nu}$ and $g_{\mathrm{d}\mu\nu}$ with different expansion rates, and $s_{\mathrm{d}\ell}$ are different from s_ℓ.

The matter actions break the two copies of diffeomorphism invariance associated with the two Einstein-Hilbert actions. However, in the approximation (130), we can reintroduce the broken symmetry invariance by introducing Stückelberg fields ϕ^μ_ℓ and defining the composite object

$$g_{\ell\mu\nu} = \hat{g}_{\alpha\beta} \frac{\partial \phi^\alpha_\ell}{\partial x^\mu} \frac{\partial \phi^\beta_\ell}{\partial x^\nu}. \tag{131}$$

The metric $g_{\ell\mu\nu}$ is now invariant under the combined transformations

$$\hat{g}_{\ell\mu\nu} \to \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x^\rho}{\partial x'^\nu} \hat{g}_{\ell\lambda\rho}, \qquad \frac{\partial \phi^\alpha_\ell}{\partial x^\mu} \to \frac{\partial x'^\alpha}{\partial x^\lambda} \frac{\partial \phi^\lambda_\ell}{\partial x'^\mu}. \tag{132}$$

We recover the initial action by the gauge choice $\phi^\mu_\ell = x^\mu$. This is the nonlinear extension of (125), with $\phi^\mu = x^\mu + \zeta^\mu$, where we neglect derivatives of the cosmological background $\partial \bar{g}_{\mu\nu}$ thanks to the approximation (130). The definition (131) can also be written at the level of the vierbeins as

$$e^a_{\ell\mu} = \hat{e}^a_{\ell\nu} \frac{\partial \phi^\nu_\ell}{\partial x^\mu}. \tag{133}$$

It is convenient to separate the diffeomorphisms into the diagonal ones, which are not broken by the presence of matter, and the broken ones in the complementary directions which belong to the group quotient $(\mathrm{diff}_1 \times \mathrm{diff}_2)/\mathrm{diff}_{\mathrm{diag}}$

$$\phi^\mu_\ell = x^\mu + \xi^\mu + \gamma_\ell \pi^\mu, \qquad \gamma_1 \neq \gamma_2. \tag{134}$$

Here, ξ^μ is the diagonal diffeomorphism, while π^μ is an arbitrary complementary direction, set by the constant coefficients γ_ℓ. Then, the vierbeins (133) read

$$e^a_{\ell\mu} = \hat{e}^a_{\ell\mu} + \hat{e}^a_{\ell\nu} \left(\frac{\partial \xi^\nu}{\partial x^\mu} + \gamma_\ell \frac{\partial \pi^\nu}{\partial x^\mu} \right). \tag{135}$$

The total action becomes $S(e^a_{\ell\mu}) \to S(\hat{e}^a_{\ell\mu}, \xi^\mu, \pi^\mu)$, which is independent of ξ^μ as the diagonal diffeomorphism invariance is not broken. The field π^μ cannot be gauged away, as if annulled in $g_{2\mu\nu}$ by a diagonal change of coordinates it would reappear in the $g_{1\mu\nu}$ metric and vice versa. Hence, the π^μ fields parametrize orthogonal directions to diagonal gauge transformations.

To investigate the Boulware-Deser ghosts, we can focus on the fields π^μ, which are the Goldstone bosons of the broken symmetry, and consider the scalar mode

$$\pi^\mu = \partial^\mu \pi, \tag{136}$$

for a scalar π. Because of the approximation (130), it does not matter whether ∂^μ is defined with respect to $\eta^{\mu\nu}$ or any of the metrics $\bar{g}_\ell^{\mu\nu}$.

From the definition (8), the baryonic matter and dark matter vierbeins read as

$$e^a_{\star\mu} = s_{\star1}\hat{e}^a_{1\mu} + s_{\star2}\hat{e}^a_{2\mu} + (s_{\star1}\gamma_1\hat{e}^a_{1\nu} + s_{\star2}\gamma_2\hat{e}^a_{2\nu})\partial^\nu\partial_\mu\pi, \tag{137}$$

where the subscript $\star$ stands for b or d (i.e., baryonic or dark matter). The field π could only be removed from the matter action by a change of coordinate if $\gamma_1 = \gamma_2$, associated with a diagonal diffeomorphism. On the other hand, if we choose $\gamma_1 = 1/s^2_{\star1}$ and $\gamma_2 = -1/s^2_{\star2}$, the field π disappears from Eq. (137) at the linear level. This corresponds to the choice used in Sec. VI A, where the scalar π in (126) lived in the sector S_- and was not coupled to matter. Indeed, we can check that with this choice of coefficients γ_ℓ, π contributes to the sector S_- as defined by Eq. (101) and does not contribute to the sector S_+ defined by Eq. (100). This is only possible in the early-Universe regime, where the baryonic and dark matter metrics are identical, with $s_\ell = s_{d\ell}$. In this section, we go beyond this regime, and we do not assume $s_\ell = s_{d\ell}$. Then, it is not possible to find coefficients γ_ℓ that remove the field π from both the baryonic and dark matter actions.

Let us now focus on the scalar π alone, setting the other metric modes to zero, that is, $\hat{e}^a_{\ell\mu} = \bar{e}^a_{\ell\mu}$. The matter vierbeins (137) contain second derivatives $\partial^2\pi$. Therefore, the equations of motion for π coming from the matter actions may contain up to four derivatives and may lead to the propagation of extra ghostlike modes [32]. Specifically, the Euler-Lagrange terms in the equations of motion for π coming from the matter action take the form

$$E_{\star1} \propto \partial^\nu\partial_\mu\left(\frac{\delta S_\star}{\delta e^a_{1\mu}}\bar{e}^a_{1\nu}\right) \propto \partial^\nu\partial_\mu(\sqrt{-g_\star}T^{\mu\sigma}_\star e_{\star\nu\sigma}), \tag{138}$$

where we used the approximation (130) to neglect background derivatives. Using the equation of motion of the matter, $\nabla_{\star\mu}T^{\mu\nu}_\star = 0$, which gives

$$\partial_\mu(\sqrt{-g_\star}T^{\mu\nu}_\star) = -\sqrt{-g_\star}\Gamma^\nu_{\star\mu\lambda}T^{\mu\lambda}_\star, \tag{139}$$

and the property

$$\partial_\mu e_{\star a\lambda} = e_{\star a\nu}\Gamma^\nu_{\star\lambda\mu} - e_{\star b\lambda}\omega^b_{\star a\mu}, \tag{140}$$

where $\omega^{\star ab}_\mu$ is the spin connection defined by

$$\omega^{ab}_{\star\mu} = \frac{1}{2}e^{a\nu}_\star(\partial_\mu e^b_{\star\nu} - \partial_\nu e^b_{\star\mu}) - \frac{1}{2}e^{b\nu}_\star(\partial_\mu e^a_{\star\nu} - \partial_\nu e^a_{\star\mu}) - \frac{1}{2}e^{a\rho}_\star e^{b\sigma}_\star(\partial_\rho e_{\star c\sigma} - \partial_\sigma e_{\star c\rho})e^c_{\star\mu}, \tag{141}$$

we can write Eq. (138) as

$$E_{\star1} \propto \partial^\nu(\sqrt{-g_\star}T^{\mu\sigma}_\star e_{\star b\sigma}\omega^b_{\star a\mu}). \tag{142}$$

The matter vierbeins (137) take the form $e^a_{\star\mu} = \bar{e}^a_{\star\mu} + \bar{A}^{a\nu}_\star\partial_\nu\partial_\mu\pi$, for a given matrix $\bar{A}^{a\nu}_\star$, and substituting into the definition (141), we find $\omega^{ab}_{\star\mu} = 0$, within the approximation (130). As a result, well inside the horizon and on timescales much shorter than the age of the Universe, we find that the contributions to the equations of motion for π coming from the matter terms do not involve higher-order derivatives and therefore do not give rise to ghosts. This is similar to what happens in massive bigravity [10].

This result can be understood in a simpler way that also applies to the two Einstein-Hilbert terms. Within the approximation (130), the matter vierbeins (137) take the form

$$e^a_{\star\mu} = \bar{e}^a_{\star\mu} + \bar{e}^a_{\star\nu}\partial^\nu\partial_\mu\left(\frac{s_{\star1}\gamma_1 a_1 + s_{\star2}\gamma_2 a_2}{a_\star}\pi\right), \tag{143}$$

where we used Eq. (129) for the background vierbeins. This corresponds to a diffeomorphism $x^\mu \to x^\mu + \partial^\mu[(s_{\star1}\gamma_1 a_1 + s_{\star2}\gamma_2 a_2)\pi/a_\star]$, so that the matter action reads as $\sqrt{-g_\star}\mathcal{L}_\star(g_{\star\mu\nu}) = \sqrt{-g_\star}\mathcal{L}_\star(\bar{g}_{\star\mu\nu})$. The gravitational vierbeins $e^a_{\ell\mu}$ also take the form (143), where the fraction is replaced by a simple factor γ_ℓ. Again, the invariance of the Ricci scalar under changes of coordinates implies that the Einstein-Hilbert terms read as $\sqrt{-g_\ell}R(g_{\ell\mu\nu}) = \sqrt{-g_\ell}R(\bar{g}_{\ell\mu\nu})$. Therefore, the scalar π only appears in the two Einstein-Hilbert actions and the two matter actions through the determinants $\sqrt{-g}$. This gives factors of the form

$$\sqrt{-g} = a^4 \det\left(\frac{\partial\phi^\mu}{\partial x^\nu}\right), \tag{144}$$

with $\phi^\mu = x^\mu + \bar{A}\partial_\mu\pi$. Thus, the action is a sum of four terms of the form

$$S \propto \int d^4x \frac{\bar{S}}{4!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}\frac{\partial\phi^{\mu_1}}{\partial x^{\nu_1}}\frac{\partial\phi^{\mu_2}}{\partial x^{\nu_2}}\frac{\partial\phi^{\mu_3}}{\partial x^{\nu_3}}\frac{\partial\phi^{\mu_4}}{\partial x^{\nu_4}} \tag{145}$$

for coefficients $\bar{S}$ related to the Ricci scalars of the two metrics and the matter contents in baryons and CDM, which vanish thanks to the antisymmetry of the Levi-Cività tensor [33].

Thus, we have found that, at leading order in the approximation (130), and setting the other metric modes $h_{\ell\mu\nu}$ to zero, the action does not contain higher-order derivatives such as $(\partial^2\pi)^2$. This agrees with the explicit expression (128) for the quadratic action, obtained without the approximation (130). There, we can see that the leading terms $M_{\rm Pl}^2\mathcal{H}^2(\partial^2\pi)^2$ cancel out and the action only includes the subleading contributions $M_{\rm Pl}^2\mathcal{H}^4(\partial\pi)^2$, with an extra factor $\mathcal{H}^2$ and two fewer derivatives on π. Thus, there is no Boulware-Deser ghost around the cosmological background, at all orders over π but in the small-scale and short-time approximation (130) when we neglect the other metric modes $h_{\ell\mu\nu}$.

We analyze the terms $h\partial^2\pi$ in Appendix B. We find that, even when the baryonic and dark matter metrics are different, the Stückelberg field π only couples to the metric combination h_- as defined in Eq. (101), as in the case $s_{\rm d\ell} = s_\ell$ that was explicitly considered in Sec. VI A. Besides, such terms $h\partial^2\pi$ can be written in terms of first-order time derivatives, after integrating by parts over π''; hence, they do not give rise to ghosts.

C. Cutoff scale

We now investigate at which scale the terms we have neglected above may introduce a ghost. As can be seen from the explicit action (128) and the terms in $M_{\rm Pl}^2\mathcal{H}^4(\partial\pi)^2$, the canonically normalized Stückelberg field $\tilde\pi$ is given by

$$\tilde\pi = \Lambda_3^3\pi \quad \text{with} \quad \Lambda_3 = (M_{\rm Pl}H^2)^{1/3}, \tag{146}$$

up to a numerical factor of order unity. Introducing the canonically normalized gravitons $\tilde h_{\mu\nu} = M_{\rm Pl}h_{\mu\nu}$, the terms that we have neglected above correspond to couplings between $\tilde\pi$ and $\tilde h$ and derivatives of the background. They take the form

$$M_{\rm Pl}^2H^2\frac{\tilde h^n H^{2m-p}\partial^p\tilde\pi^m}{M_{\rm Pl}^n\Lambda_3^{3m}} = \frac{\tilde h^n\partial^p\tilde\pi^m}{\Lambda^{n+p+m-4}}, \tag{147}$$

and they are suppressed by a scale Λ with

$$\Lambda = \Lambda_3\left(\frac{\Lambda_3}{H}\right)^{(2n+2m-p-2)/(n+m+p-4)}. \tag{148}$$

We have $n\geq 0$, $2m-p\geq 0$, and $\Lambda_3 \gg H$. Therefore, $\Lambda \geq \Lambda_3$, except in the case $n=0$ and $2m-p=1$. This corresponds to the combination $H\partial^{2m-1}\tilde\pi^m$, where one partial derivative on $\tilde\pi$ is replaced by a background derivative H. We have already found that there is no ghost in the quadratic action; therefore, such a term can only give rise to ghosts if $m\geq 3$. This yields for the lowest cutoff scale

$$\Lambda_{\rm cut} = \Lambda_3\left(\frac{H}{\Lambda_3}\right)^{1/4} = (M_{\rm Pl}H^3)^{1/4}, \tag{149}$$

which corresponds to $\Lambda_{\rm cut} \sim 1$ AU $\sim 10^{-6}$ pc. Therefore, at energies below $\Lambda_{\rm cut}$, there is no ghost in the model, but the theory cannot be trusted on scales smaller than 1 AU, and new contributions must be added to the action to ensure that there are no ghosts. On the other hand, it can be used as an effective theory on all larger scales, which are relevant for cosmology. The cutoff scale that we have deduced may be modified by nonperturbative effects which are not investigated here.

The fact that the cutoff scale is of order 1 AU prevents our analysis from being applicable in most parts of the Solar System. However, close to compact objects, or in the Solar System, on scales greater than 1 AU and in the weak gravitational field regime, we can use the quadratic theory described in Se. VI A if we can neglect the dark matter. We can then separate the action in the sectors S_+ and S_-, with the dangerous mode π living in the sector S_- at this order. Therefore, the field π does not couple to matter and never enters the nonlinear regime due to matter overdensities. At the classical level, $\pi=0$ is a solution of the equations of motion (with all $h_{-\mu\nu}=0$), even when there are baryonic matter fluctuations. Then, there is no need for a Vainshtein mechanism, down to the scale $\Lambda_{\rm cut}^{-1}$.

VII. LINKS WITH DOUBLY COUPLED BIGRAVITY

The models that we have constructed have similarities with doubly coupled bigravity [13–15]. In doubly coupled bigravity, there is no scalar field, and hence the Jordan-frame vierbein couplings s_* are constant, such as

$$s_\ell = s_{\rm d\ell} = s_\ell^{(0)}, \tag{150}$$

with a universal coupling to all types of matter, i.e., baryons, cold dark matter, and radiation. In both the matter and radiation eras, the scale factors are in the symmetric case

$$a_\ell = b_\ell, \qquad \mathcal{H}_\ell = \mathcal{H}, \tag{151}$$

implying that the two metrics are proportional. The late-time acceleration of the expansion of the Universe is obtained by adding a potential term,

$$S_V = \Lambda^4\int d^4x\sum_{ijk\ell} m^{ijk\ell}\epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}e^a_{i\mu}e^b_{j\nu}e^c_{k\rho}e^d_{\ell\sigma}, \tag{152}$$

comprising one scale and a completely symmetric tensor $m^{ijk\ell}$ which, up to rescaling, is associated to four coupling constants. This term is responsible for the late-time acceleration where Λ^4 plays the role of the vacuum energy.

Moreover, the potential term gives rise to a mass matrix for the gravitons of which the order of magnitude corresponds to $\Lambda^4/M_{\rm Pl}^2 \sim H_0^2$, i.e., very light gravitons.

At the background level, and as long as the scalar field is negligible, the bimetric models considered here coincide with the bigravity theories. They differ when it comes to the phase of acceleration. In bigravity, this is simply realized as Λ^4 plays the role of dark energy. In scalar-bimetric models, there is no vacuum energy, and the acceleration is simply due to the rapid variation of the scalar factors $s_*(\varphi)$, which imply that the baryonic and dark matter metrics do not mimic the ones of the Einstein-de Sitter space-time. In the acceleration phase in doubly coupled bigravity,

$$r_2 \neq r_1; \tag{153}$$

that is, the two gravitational metrics do not have the same conformal time. For scalar-bimetric models, we have seen that natural models obey $r_1 = r_2 = 1$ even at late times. In a similar fashion, in bigravity, the consistency of the Friedmann equations gives a constraint equation that admits two branches of solutions [13–15], the interesting one for cosmology being $\mathcal{H}_{a_1}/r_1 = \mathcal{H}_{a_2}/r_2$ as noticed in Eq. (36). In our case, the scalar field provides an additional degree of freedom, and there is no such constraint. As in General Relativity, the Friedmann equations and the equations of motion of the various fluids are automatically consistent. This follows from the fact that Eq. (35) is no longer a constraint equation, because of the scalar-field dynamics. As we checked in Sec. III C, the equation of motion of the scalar field is not independent of the Friedmann equations and of the equations of motion of the other fluids, as it can be derived from the latter.

When it comes to the scalar perturbations, bigravity in the doubly coupled case and scalar-bimetric models differ more drastically as the cosmological perturbations of the scalar field imply the existence of a scale, related to its effective mass, such that for large enough wave numbers gravity is modified. This leads to a fifth force that is of order of the Newtonian force on cosmological scales at $z = 0$. Moreover, as the scalar field evolves in the late-time Universe, the effective Newton constants (they are not unique anymore but depend on the species) drift with time. This has also an effect on cosmological perturbations.

Vector and tensor perturbations in the radiation era have similar behaviors in doubly coupled bigravity and scalar-bimetric models, with both tensor and vector instabilities. In the matter era, the nontrivial mass matrix for the two gravitons in doubly coupled bigravity implies that the two gravitons oscillate, leading to birefringence [34]. Moreover, in doubly coupled bigravity, the speed of the gravitational waves differs from unity in the late-time Universe as the ratio between the two lapse functions of the two metrics is not equal to 1 anymore. This is severely constrained by the LIGO/VIRGO observations. In contrast, in scalar-bimetric models, we have shown that symmetric solutions where $a_\ell = b_\ell$ can be obtained even during the acceleration phase. In this case, the speed of the gravitational waves is always unity. Moreover, at the linear level, there is no mixing between the tensor and vector instabilities that affects the hidden modes and the matter metrics.

Finally, let us note another analogy between the bimetric models presented here and doubly coupled bigravity. The breaking of the full diffeomorphism invariance to the diagonal subgroup is parametrized by the mass of the gravity m in the latter and the Hubble expansion rate H in the former. In both cases, the strong coupling scale is given by $\Lambda_3 = (v^2 M_{\rm Pl})^{1/3}$, where $v = H$, m is the order parameter of each case. At energies larger than this scale, ghosts are present, and a completion of the models is required. Notice that in the scalar-bimetric models ghosts may actually appear at the lower scale $(H^3 M_{\rm Pl})^{1/4}$. In both theories, around compact objects in the weak gravitational regime for distances larger than their respective cutoff scales, the scalar Goldstone mode decouples without the need for the Vainshtein mechanism.

On the other hand, as we are now going to analyze, the time variation of the scalar field in scalar-bimetric models poses new problems which are late-time issues, i.e., not only restricted to the radiation era, contrary to what happens in bigravity [15].

VIII. RECOVERING GENERAL RELATIVITY ON SMALL SCALES?

As shown in Fig. 7, the scenarios obtained so far are not consistent with small-scale tests of General Relativity. First, the fifth force is too large, being about twice stronger than Newtonian gravity at $z = 0$, as measured by the ratio $\mu^\phi/\mathcal{G}^\phi - 1$. Second, the time derivative of the effective Newton constant is too high at $z = 0$, with $d\ln\mathcal{G}/dt \sim 0.7H_0$ whereas the Lunar Laser Ranging (LLR) experiment gives the upper bound $0.02H_0$ ($d\ln\mathcal{G}/dt < 1.3\times 10^{-12}$ yr^{-1}) [19]. Strictly speaking, this constraint lies beyond the realm of validity of the models as coming from scales below 1 AU. On the other hand, less stringent constraints on the planetary orbits exist [35] at the 10^{-11} level and should be fulfilled. Hence, we will use the LLR bound as a template for any UV completion of scalar-bimetric models. Third, the change of the Newton constant from its large-redshift value to its current value is too large. Indeed, we obtain an increase of $\mathcal{G}$ of about 50% from its high-z asymptote to its value at $z = 0$. Here, we normalize the Planck mass at $z = \infty$ to its measured value in the Solar System today and define the cosmological parameters in terms of the same Planck mass in Eq. (30). Instead, we should normalize both the Newton constant at $z = 0$ and the cosmological parameters (i.e., the matter densities) to the measured value of $\mathcal{G}_{\rm N0}$. However, we would face the same problem. Because we have no dark energy, to recover the Λ-CDM expansion at high z with the same background densities, we need the effective Newton constant at high z to be the same as in the Λ-CDM scenario, which is also the measured value today. Thus, we need $\mathcal{G}_{\rm N}$ at $z=0$ to be equal to $\mathcal{G}_{\rm N}$ at $z \gg 1$, unless we modify the dark matter and radiation densities by a similar amount (with respect to the Λ-CDM reference). However, it is not possible to change the background densities by 50% while keeping a good agreement with the cosmic microwave background and big bang nucleosynthesis constraints.

These three problems are not necessarily connected. In modified-gravity models, the fifth force is assumed to be damped in the local environment by nonlinear screening mechanisms (which use the fact that the Solar System length scale is much smaller than cosmological distances and/or the local density is much higher than the cosmological background densities). However, it is usually assumed that the time dependence of the Newton constant, and often its value, remain set by the cosmological background, which acts as a boundary condition. In particular, derivative screening such as the Vainshtein screening, where the nonlinear terms are invariant under $\varphi \to \varphi + \alpha t$ with arbitrary α, does not seem to prevent a slow drift of Newton constant. Then, unless the local Newton constant can be significantly decoupled from the cosmological background solution (e.g., through a more efficient screening that remains to be devised), we need to modify the background solution itself to decrease both $d\ln\mathcal{G}/dt(z=0)$ and $\Delta\mathcal{G} = \mathcal{G}(z=0) - \mathcal{G}(z=\infty)$.

A. Reducing $d\ln\mathcal{G}/dt$

1. Constant $\mathcal{G}$?

The most elegant way to reduce $d\ln\mathcal{G}/dt$ below the Hubble timescale would be to keep it (almost) constant, so that one would not need any tuning to decrease the time derivative precisely at $z=0$. Moreover, this would ensure that $\mathcal{G}$ would be about the same at $z=0$ and $z \gg 1$.

Scenarios with common conformal time.—Let us first consider the case of the scenarios with $r_\ell = 1$, described in Sec. III F. Then, from Eq. (84), a constant $\mathcal{G}$ corresponds to a constant λ in Eq. (53). Unfortunately, the solution (54) does not exist for any $\lambda(a)$, as the argument of the square root needs to remain positive. Numerically, we found that it is not possible to keep a constant $\lambda(a) = 1$, at all times. This can be understood from the behavior of the scales factors a_ℓ. As noticed in Fig. 3 and explained below Eq. (56), the behavior of the scale factors a_* and Hubble expansion rates $\mathcal{H}_*$ is almost independent of the evolution of the coefficients s_ℓ, because we impose a Λ-CDM-like expansion for the baryonic metric. This implies that the ratios a_ℓ/a decrease with time, as the gravitational metrics $g_{\ell\mu\nu}$ follow an expansion close to the Einstein-de Sitter prediction (because we do not put any cosmological constant or dark-energy component that would play the same role). Then, to keep the square root real in Eq. (54), $\lambda(a)$ must typically increase with time. In any case, its value at $z=0$ must be greater than unity. From the values of a_ℓ/a read in Fig. 3, we find $\lambda(z=0) \gtrsim 1.5$. This means that Newton's constant $\mathcal{G}$ at $z=0$ must be about 50% greater than its value at high redshift.

We show in Fig. 12 the Newton constants for the baryonic and dark sectors obtained in this manner, with the function λ used for Fig. 3 such that $d\lambda/da = 0$ at $z=0$. This allows us to reduce $d\ln\mathcal{G}^\phi/Hdt$ at all redshifts below 0.3 and make it smaller than the Lunar Laser Ranging upper bound at $z=0$. On the other hand, for the dark sector, we still have the generic feature $d\ln\mathcal{G}_{\rm dm}^{\phi_{\rm d}}/Hdt$ of order unity at $z=0$. Making $\lambda(a)$ almost constant at low z is not so artificial, in the sense that it is a simple constraint on the coefficients s_ℓ, which are likely to be correlated in any case. Moreover, the plateau for $\mathcal{G}^\phi$ can be reached at $z \gtrsim 1$ and does not need to be tuned at $z=0$ precisely. However, a few numerical tests suggest that it is difficult, or impossible, to make the transition for $\mathcal{G}^\phi$ occur at much higher redshifts,

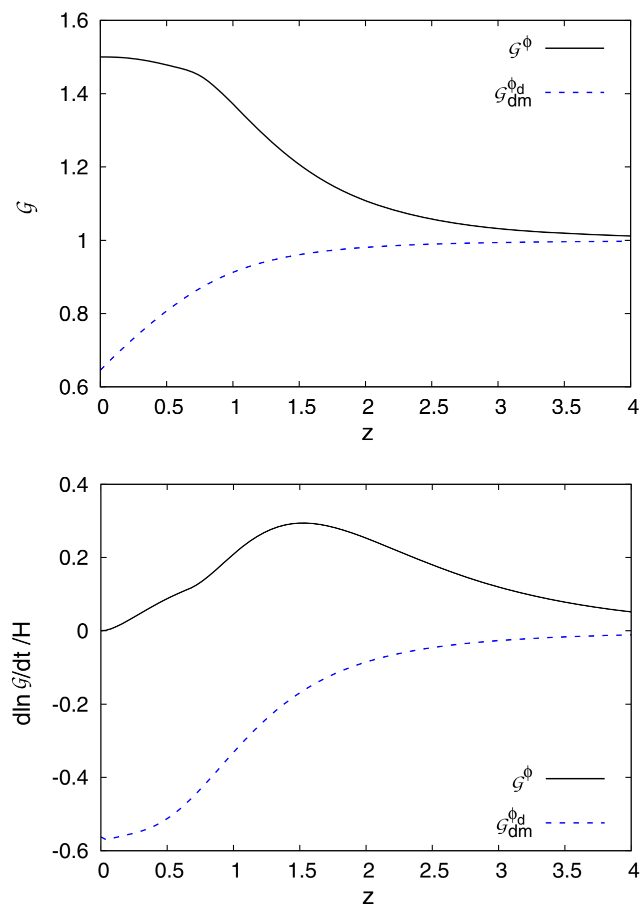

FIG. 12. *Upper panel:* baryonic sector and dark sector Newton constants, normalized to $\mathcal{G}_{\rm N}$. *Middel panel:* time derivatives $d\ln\mathcal{G}/dt$ normalized to H. *Lower panel:* growth factors $f\sigma_8$ and $f_{\rm dm}\sigma_{\rm dm8}$.

such as $z = 10$. This tends to make s_2 negative at intermediate redshifts, amplifying the dip already seen in Fig. 3, and we prefer to keep the coefficients s_ℓ positive (but this requirement may be unnecessary).

From the arguments discussed above, if the sum $s_1^2 + s_2^2$ reaches a constant value at late times, or satisfies a finite upper bound, the decrease of the ratios a_ℓ/a must eventually stop in the future (a simple case is where each coefficient s_ℓ eventually becomes constant). Then, as the gravitational metrics, the baryonic metric must recover an Einstein-de Sitter expansion, unless the energy density and pressure of the scalar field become dominant. Therefore, in this framework where the acceleration of the expansion is not due to an additional dark-energy fluid, the self-acceleration is only a transient phenomenon. An alternative would be that the Newton constant resumes its growth in the future, but this would introduce an additional tuning as the slow down of $d\ln\mathcal{G}^\phi/Hdt$ would be a transient phenomenon that must be set to occur precisely around $z \simeq 0$.

It is interesting to note that the nonsymmetric solutions, such as (53), give rise to behaviors beyond those obtained in models where the baryonic and dark matter metrics are simply given by different conformal rescalings of a single Einstein-frame metric. There, we only have two free functions, $A(\varphi)$ and $A_{\rm d}(\varphi)$, with $g_{\mu\nu} = A^2\tilde{g}_{\mu\nu}$ and $g_{{\rm d}\mu\nu} = A_{\rm d}^2\tilde{g}_{\mu\nu}$. This would correspond for instance to $s_1 = A$ and $s_2 = 0$; that is, there is no second gravitational metric. As there is only one coupling A, both the baryonic scale factor a and the baryonic Newton constant $\mathcal{G}^\phi$ depend on $A(\varphi)$ and run at the same rate. This means that it is not possible to have a self-accelerated expansion, driven by $A(\varphi)$, while keeping $\mathcal{G}^\phi$ constant. In the bimetric scenario, even in the common conformal time case, we can take advantage of the two free functions $s_1(\varphi)$ and $s_2(\varphi)$ to keep a constant Newton strength $\mathcal{G}^\phi$ while having self-acceleration. However, as explained above, this can only happen for a finite time (if we require $s_\ell > 0$), and we cannot reduce the gap $\Delta\mathcal{G} = \mathcal{G}(z=0) - \mathcal{G}(z=\infty)$. Therefore, this scenario is not sufficient to make the model agree with observational constraints. Presumably, increasing the number of metrics, hence of degrees of freedom and free functions of the model, would make it increasingly easy to reconcile a constant Newton strength with self-acceleration.

Scenarios with different conformal times.—In the case of the scenario (62), with $r_\ell \neq 1$, we explicitly checked that we can build solutions such that $\mathcal{G}^\phi$ remains constant at all times, by tuning the factors r_ℓ. More precisely, from Eq. (84), a constant $\mathcal{G}^\phi$ corresponds to

$$\frac{d\mathcal{G}^\phi}{d\ln a} = 0:\ \sum_\ell 2 s_\ell r_\ell \frac{ds_\ell}{d\ln a} + s_\ell^2 \frac{dr_\ell}{d\ln a} = 0. \qquad (154)$$

Using the expressions (60), we can write $\{ds_\ell/d\ln a\}$ in terms of $\{dr_\ell/d\ln a\}$. This determines for instance the derivative $dr_2/d\ln a$ while keeping r_1 free, so that this family of solution is still parametrized by a free function $r_1(a)$. However, this usually gives $\mathcal{G}^\psi \neq \mathcal{G}^\phi$, see Eq. (84), with a relative deviation of order unity. To be consistent with Solar System data, in particular with the Shapiro time delay that measures the travel time of light rays in gravitational potentials, we must have $|\psi/\phi - 1| \leq 5 \times 10^{-5}$ [36]. On the other hand, as explained in Sec. III G, we need $r_1 = 1$ (or $r_2 = 1$) at $z = 0$ to comply with the multimessenger gravitational waves event GW170817. This would give both $s_2 = 0$ and $\mathcal{G}^\psi = \mathcal{G}^\phi$ at $z = 0$. However, when we try to combine Eq. (154) with $r_1 \to 1$ at $z = 0$ in a few numerical tests, we find singular behaviors with b_2 becoming negative before $z = 0$ and $a_2 \to 0$ at $z = 0$. This is somewhat reminiscent of the impossibility to achieve a constant $\mathcal{G}^\phi$ in the simpler case $r_1 = r_2 = 1$ shown in Fig. 12. Because the scenarios $r_\ell \neq 1$ already require some tuning, with $|r_1 - 1| < 3 \times 10^{-15}$ at $z = 0$, we do not investigate further this family of solutions.

2. Constant s_ℓ at late times

A natural solution to obtain a small $d\ln\mathcal{G}/dt$ at low z is to consider models where the coefficients s_ℓ reach a constant at late times. This also removes any fifth force on baryons, as $\beta = 0$ from Eq. (66). However, this also makes the baryonic metric expansion rate converge again to an Einstein-de Sitter behavior, in agreement with the simple solution of Sec. III D. The deviation of $s_1^2 + s_2^2$ from unity in this late-time asymptote again corresponds to a different value for the associated Newton's constant, as compared with the one obtained at high redshift.

We show in Fig. 13 our results for the symmetric solution of Fig. 1, which is modified at late times so that the baryonic coefficients are constant for $a > 0.9$. In terms of these coefficients, this model is rather simple as the accelerated expansion of the Universe is a transient phenomenon, due to the transition of the coefficients s_i between two constant asymptotes. By requiring the Hubble expansion rate to follow the Λ-CDM history until $z \gtrsim 0.1$, we make the transition to the final Einstein-de Sitter behavior occur in a very small redshift interval. This leads to a sharp decrease for the baryonic expansion rate $\mathcal{H}(z)$, which suddenly drops to the expansion rate $\mathcal{H}_{a_1} = \mathcal{H}_{a_2}$ of the gravitational metrics. This also leads to a sudden increase in the growth rate of large-scale structures, which resumes the faster growth associated with Einstein-de Sitter cosmologies.

Even though the change of the coefficients s_ℓ is very small, as compared with the solution of Fig. 1, this leads to a change for the Hubble expansion rate of order unity. Indeed, by making the coefficients s_i constant at late times, we change their time derivative $ds_i/d\tau$ from a quantity of order $1/H_0$ to zero over a small time $\Delta\tau$. This yields a

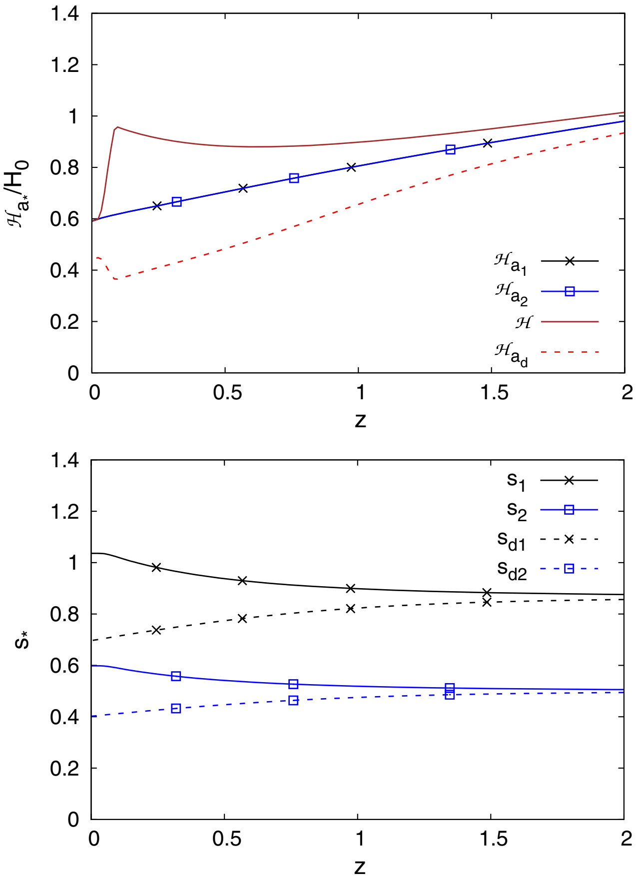

FIG. 13. Conformal Hubble expansion rates (upper panel) and coefficients s_* (lower panel) as a function of redshift, for a solution where the baryonic coefficients s_ℓ are constant at late times.

divergent second derivative $d^2 s_\ell/d\tau^2 \propto 1/(\Delta\tau)$. However, from Eq. (26), we can see that $d\ln\mathcal{H}/d\ln a$, being a second derivative of the scale factor, contains a term such as $d^2 s_\ell/d\tau^2$ and also grows as $1/(\Delta\tau)$. Then, even if we let the transition time $\Delta\tau$ go to zero, the change of $\mathcal{H}$ remains finite and of order unity, in agreement with Fig. 13. The drop of $H(z)$ at low z to about 60% of the Λ-CDM extrapolation H_0 implies a deviation of the distance modulus, $\mu = 5\log_{10}(d_L/10\text{ pc})$, of $\Delta\mu = -5\log_{10}(0.6) \simeq 1.1$. However, the dispersion of the distance modulus of observed type Ia supernovae in the range $0.01 < z < 1$ is of order 0.3, before binning [37], and does not show such a steep step. Therefore, the Hubble diagram shown in Fig. 13 is ruled out by low-redshift supernovae.

In addition, we still have a total increase of $\mathcal{G}$ of about 50% between the high-z and low-z values of the effective Newton constant. Therefore, this scenario would not solve this third problem in any case.

B. Need for screening beyond quasistatic chameleon mechanisms

We have seen in the previous section that the coefficients s_ℓ are unlikely to have reached constant values by $z = 0$, to be consistent with the low-z Hubble diagram. This yields a fifth force that is of the same order as the Newtonian force on cosmological scales. All scenarios also imply a decrease of order 50% of the effective Newton constant at higher redshifts, which makes it impossible to recover the reference Λ-CDM expansion rate unless the matter and radiation densities are also modified. This means that such scalar-bimetric models can only satisfy observational constraints if gravity in the Solar System is decoupled from its behavior on cosmological scales.

Within modified-gravity scenarios, the recovery of General Relativity on small scales is often achieved by introducing nonlinear screening mechanisms that damp the effect of the fifth force. For instance, chameleon screening makes the scalar field short ranged in high-density environments, because its effective potential and its mass depend on the matter density. In a similar fashion, dilaton and symmetron scenarios damp the fifth force by making its coupling vanish in high-density environments, following the Damour-Polyakov screening.

It is interesting to note that these screening mechanisms cannot appear in the models considered in this paper, because the scalar field always remains in the linear regime. A first way to see this is from Eq. (67), which yields $\delta\varphi/M_{\rm Pl} \sim v^2$ for a structure of virial velocity $v^2 \sim \mathcal{G}M/r$, mass M, and radius r. Then, in nonrelativistic environments, from clusters of galaxies to the Solar System, where $v^2 \ll 1$, we have $\delta\varphi \ll \bar\varphi$ as we found in Fig. 1 that $\bar\varphi \sim M_{\rm Pl}$. This also implies that $\delta s_\ell \ll \bar s_\ell$. Thus, from clusters of galaxies to the Solar System, the fluctuations of the scalar field remain small and are not sufficient to significantly modify the coefficients s_ℓ. This means that the effective Poisson equation (i.e., the coefficients μ_*^*) keeps the same deviation from General Relativity on all these scales.

This configuration can be compared with the usual chameleon or Damour-Polyakov screenings, shown by $f(R)$ or Dilaton and Symmetron models. There, the Jordan-frame metric is typically related to the Einstein-frame metric by a conformal coupling, $g_{\mu\nu} = A^2(\varphi)\tilde g_{\mu\nu}$. The fifth force $c^2\nabla\ln A$ again arises from the fluctuations of this metric coefficient A, through the fluctuations of the scalar field. However, in these models which typically include a cosmological constant, either explicitly or as the nonzero minimum of some potential, the conformal coupling always remains very close to unity, $|A-1| \lesssim 10^{-5}$. This ensures that one follows the Λ-CDM background while having effects on cosmological structures that can be of order unity, with $\delta A \sim \phi$. The very small variation of the background value of $\bar A$ also means that it is easy to introduce a screening mechanism, because the spatial perturbations of $\delta\varphi$ and δA can be of the same order as those of the cosmological background over $\delta z \sim 1$, so that the nonlinear regime is easily reached (this may be more easily understood from a tomographic point of view). In the model considered in this paper, the difficulty arises from the fact that we require background variations of order unity for the coefficients s_ℓ, which play a role similar to $A^2(\varphi)$ in the conformal coupling models, whereas spatial variations should remain of order 10^{-5} of the same order as the standard Newtonian potential. This implies that spatial fluctuations of the scalar-field value are not sufficient to reach the nonlinear regime. This analysis agrees with the "no-go" theorem of Ref. [5], which concludes from the same arguments that usual chameleon models cannot provide a self-acceleration of the Universe, and must rely on a form of dark energy (typically a hidden cosmological constant, written as the nonzero minimum of some potential).

A way out of this difficulty is to introduce screening mechanisms that do not rely on the scalar-field value but on its derivatives. Then, even though $\delta\varphi$ remains small, its spatial derivatives $\partial^n\delta\varphi$ can be large on small enough scales. This corresponds to K-mouflage and Vainshtein mechanisms. This can be achieved by adding terms in $(\partial\varphi)^4/M^4$ or $\Box\varphi(\partial\varphi)^2/M^3$. In this case, these nonlinear terms dominate over the simple kinetic terms at short distance depending on the value of M. As a result, the coupling of the scalar field to the baryons (and incidentally the one to dark matter) is reduced, and local tests of gravity are satisfied. However, this only solves the fifth-force problem, and it does not solve the problems associated with the value of Newton constant and its time drift. (In these screening scenarios, they are usually assumed to be set by the cosmological background, which acts as a boundary condition.)

The analysis above implicitly assume the quasistatic approximation, where the scalar field relaxes to its environment-dependent equilibrium and screening appears through the spatial variations of its mass, coupling, or inertia. If the quasistatic approximation is violated, the configuration may be more complex. In fact, from the analysis of Sec. VIII A, we can see that we need a local value of Newton constant that is decoupled from the one on cosmological scales. More precisely, we need its local value to remain equal to its background value at high z, before the dark-energy era. This calls for a new screening mechanism, or a more efficient implementation of K-mouflage or Vainshtein screening, that goes beyond the quasistatic approximation and decouples the small-scale Newton constant from its current large-scale cosmological value. For instance, the local Newton constant should remain equal to the one at the formation of the solar system. All this requires altering the models and imposing stringent restrictions on the possible UV completions of the models that must be introduced in the Solar System below 1 AU.

IX. CONCLUSION

We have seen in this paper that the scalar-bimetric model allows one to recover an accelerated expansion without introducing a cosmological constant or an almost constant dark-energy density. This relies on the time-dependent mapping between the gravitational metrics g_1 and g_2 and the baryonic and dark matter metrics g and $g_{\rm d}$. Because at late times the deviation between the Λ-CDM and Einstein-de Sitter backgrounds is of order unity, the coefficients s_ℓ that define this mapping must show variations of order unity.

When all metrics have the same conformal time, the expansion rates of the gravitational and dark matter metrics are almost independent of the details of the model [e.g., the shape of the functions $s_\ell(\varphi)$], once we require a Λ-CDM expansion for the baryonic metric. Then, the gravitational metrics remain close to an Einstein-de Sitter expansion (because there is no dark energy), while the dark matter metric behaves in a way opposite to the baryonic metric, with a stronger deceleration than in the Einstein-de Sitter case. When the conformal times are different, the scale factors a_* can show slightly different behaviors, and even more so the lapse factors b_*. This scenario is very strongly constrained by the multimessenger event GW170817, which requires that at least one of the two gravitons propagates at the speed of light at $z < 0.01$. This implies that at least one of the ratios b_ℓ/a_ℓ must be unity at low z. This also implies that the baryonic metric becomes independent at low z of the gravitational metric where $c_g \neq 1$, but the dark matter metric still remains sensitive to both gravitational metrics.

As the coefficients s_ℓ must show variations of order unity to provide a self-acceleration, we generically have deviations of order unity for the effective Newton constants and for the contribution from the fifth force to the dynamical potential seen by particles. The dynamics of baryonic and dark matter perturbation show distinctive features, due to the fact that they couple to different metrics and that their mappings evolve in opposite fashions. While the total force (Newtonian gravity and fifth force) from baryons onto baryons, and from dark matter onto dark matter, is typically amplified at low redshift, the cross-force between baryons and dark matter is damped and even turns negative. This means that dark matter and baryons would tend to segregate (although this does not have the time to happen by $z = 0$ on large scales). Then, the growth of dark matter density fluctuations is amplified (because of the stronger self-gravity) while the growth of baryonic density fluctuations is decreased on cosmological scales (because of the lower cross-gravity, as dark matter is dominant on large scales). This could provide interesting features; for instance, most modified-gravity models predict instead an amplification of baryonic density perturbations.

However, before a detailed comparison with cosmological observations, these models present major difficulties with small-scale tests of gravity. First, the fifth force is of the same order as Newtonian gravity. Second, the baryonic effective Newton constant generically evolves on Hubble time scales. Third, it is greater than its high-z value by about 50%. These features are related to the self-acceleration, which implies modifications of order unity on Hubble timescales.

Thanks to the two couplings associated with the two gravitational metrics, it is possible to keep the baryonic effective Newton strength almost constant at low z (by keeping the sum $s_1^2 + s_2^2$ constant while the two coefficients vary). This is beyond the reach of simpler models where the baryonic metric would be given by a conformal rescaling of a single Einstein-frame metric [which provides a single coupling $A(\varphi)$]. However, this can only work for a finite time. Either the baryonic and dark matter metrics eventually recover an Einstein-de Sitter expansion in the future or the Newton coupling resumes its growth in the future. In this framework, it is more natural to make the self-acceleration only a transient phenomenon, associated with the running of the couplings $s_\ell(\varphi)$ between two constant asymptotes (where the fifth force and the running of Newton constants disappear). [The alternative scenario, where the coefficients $s_\ell(\varphi)$ have already reached their constant asymptote at low z, is rejected by measurements of the Hubble expansion rate, from low-z supernoave or local standard candles such as cepheids.] However, this cannot reduce the gap between the high-z and low-z values of Newton's constant.

On small scales, Solar System tests of gravity imply that we must recover General Relativity. In modified-gravity scenarios, this is often achieved by introducing nonlinear screening mechanisms that damp the effect of the fifth force. As in the case of single-metric and single-field models, we explain that a chameleon mechanism cannot work. It cannot efficiently screen the fifth force in a self-accelerated model. This leaves derivative screening mechanisms, such as K-mouflage and Vainshtein screenings. Therefore, the scalar-field Lagrangian must be supplemented by higher-order derivative terms, that become dominant on small scales and provide the convergence to General Relativity. on small scales by damping the fifth force. However, we need to go beyond usual implementations, as we also require the local Newton constant to be decoupled from its cosmological value and to remain equal to its high-redshift value. Then, the sum $s_1^2 + s_2^2$ is no longer required to be almost constant at low z, and this extends the family of realistic models to all solutions with common conformal time. As the cutoff scale of the model is of order 1 AU, the compliance with Solar System tests for the bimetric models would have to be analyzed thoroughly once UV completions have been constructed. In particular, they would have to avoid all the local issues that we have detailed here. This is beyond the present work.

This paper only provides a first study of such bimetric models with self-acceleration. We have shown that basic requirements already strongly constrain these scenarios. We leave for future works a detailed study to determine whether such scenarios can be consistent with cosmological data at the perturbative level. However, the main challenge is to devise adequate screening mechanisms within appropriate UV completions, if they exist. This would also have a great impact on other modified-gravity models, by providing an explicit scenario where gravity on cosmological scales could be decoupled from Solar System tests. Finally, another issue concerns the stability of the hidden vector modes, as one would like to go beyond the linear regime and guarantee that they do not mix with the matter metrics. This is beyond the scope of the present work.

APPENDIX A: LINEAR PERTURBATIONS IN THE GENERAL CASE $s_{d\ell} \neq s_\ell$

We provide in this Appendix the Einstein equations for linear perturbations in the general case, where we no longer assume $s_{d\ell}$ and s_ℓ to be identical. This allows us to go beyond the early-time Einstein-de Sitter phase (41). In particular, we no longer have $s_1^2 + s_2^2 = 1$ nor $a_\ell = s_\ell a$ and $\mathcal{H}_\ell = \mathcal{H}$. However, we restrict to the case $a_\ell = b_\ell$, to ensure that the graviton speeds remain equal to the speed of light.

Because the two types of matter (baryons and dark matter) now follow different metrics, the quadratic action can no longer be neatly split in a sector S_+, which contains all matter variables and remains identical to General Relativity, and a sector S_-, which is completely decoupled from matter and deviates from General Relativity (and can include new degrees of freedom due to the loss of one diffeomorphism invariance). Then, in this Appendix, we directly work at the level of the Einstein equations. The vierbein and metric perturbations are again defined as in Eqs. (97) and (98).

1. Tensor modes

For tensors, the Einstein equations (19) give

$$h''_{1ij} + 2\mathcal{H}_1 h'_{1ij} - \nabla^2 h_{1ij} = \frac{a_2 \sum_* s_{*1} s_{*2} a_*^2 \bar{p}_*}{a_1 M_{\rm Pl}^2} (h_{1ij} - h_{2ij}) \tag{A1}$$

and a symmetric equation with respect to $1 \leftrightarrow 2$. Here, $* = \mathrm{b}, \mathrm{d}$ stands for the baryonic and dark matters, and we sum over both matter sectors. In the early-time regime (41), these two equations can be diagonalized as in (107) and (108). At high frequencies, $\omega \gg \mathcal{H}$, and high wave numbers, $k \gg \mathcal{H}$, we recover the Minkowski limit of General Relativity, with two massless gravitons that propagate as in Minkowski vacuum, $h''_{1ij} - \nabla^2 h_{1ij} = 0$. This is not surprising, as the bimetric theory (1) reduces to two copies of General Relativity in vacuum. In particular, we recover 2×2 dynamical degrees of freedom.

2. Vector modes

For vectors, the Einstein equations (19) and the continuity equations give

$$\nabla^2(V'_{1i}+C_{1i}) = \frac{a_2\sum_* s_{*1}s_{*2}a_*^2(3\bar{\rho}_*+\bar{p}_*)}{2a_1M_{\rm Pl}^2}\times(C_{1i}-C_{2i}), \tag{A2}$$

and

$$V''_{1i}+C'_{1i}+2\mathcal{H}_1(V'_{1i}+C_{1i}) = \frac{a_2\sum_* s_{*1}s_{*2}a_*^2\bar{p}_*}{a_1M_{\rm Pl}^2}\times(V_{1i}-V_{2i}) \tag{A3}$$

and the symmetric equations with respect to $1 \leftrightarrow 2$. Again, the left-hand side corresponds to General Relativity, and the right-hand side is a new mass coupling term between the two gravitational metrics that is proportional to the background matter content ($\bar{\rho}_*$, $\bar{p}_*$, hence to $\mathcal{H}^2$) and to the products $s_{*1}s_{*2}$. It vanishes in vacuum or when one coupling $s_{*\ell}$ is zero. In the early-time regime (41), this system can be diagonalized as in (115) and (116). By combining Eq. (A2), multiplied by a_1^2, with its symmetric, we obtain

$$a_1^2(V'_{1i}+C_{1i})+a_2^2(V'_{1i}+C_{1i}) = 0. \tag{A4}$$

This automatically implies that the same combination obtained from Eq. (A3) is also satisfied. This "loss" of one equation is related to the diagonal vector gauge freedom. Here, $C_{+i} = a_1^2C_{1i}+a_2^2C_{2i}$, which generalizes Eq. (100) beyond the early-time regime. Defining again $C_{-i} = C_{1i}-C_{2i}$ and $V_{-i} = V_{1i}-V_{2i}$, we find that Eq. (117) generalizes to

$$C_{-i} = \frac{-2a_1a_2M_{\rm Pl}^2k^2V'_{-i}}{2a_1a_2M_{\rm Pl}^2k^2+(a_1^2+a_2^2)\sum_* s_{*1}s_{*2}a_*^2(3\bar{\rho}_*+\bar{p}_*)}, \tag{A5}$$

and at high frequencies and wave numbers, we obtain the equation of motion

$$V''_{-i} = \frac{\sum_* s_{*1}s_{*2}a_*^2\bar{p}_*}{\sum_* s_{*1}s_{*2}a_*^2(3\bar{\rho}_*+\bar{p}_*)}2k^2V_{-i}, \tag{A6}$$

which generalizes Eq. (119). In particular, we recover the same gradient instability (120) as in the Einstein-de Sitter phase, whatever the values of the coefficients $s_{*\ell}$.

In contrast with the case of tensors, the high frequency and high wave number limit is not so straightforward and does not coincide with a naive Minkowski limit where we put $\bar{\rho}_* = \bar{p}_* = 0$ and $\mathcal{H}_\ell = 0$ in Eqs. (A2) and (A3). This is because the loss of the nondiagonal gauge invariance leads to a new vector degree of freedom (here, V_{-i}) that cannot be "forgotten" and implies a different limit than the naive expectation of two Minkowski copies.

3. Scalar modes

For scalars, the Einstein equations (19) give

$$-6\mathcal{H}_1^2\phi_1-6\mathcal{H}_1\psi'_1+2\nabla^2\psi_1-4\mathcal{H}_1\nabla^2V_1+2\mathcal{H}_1\nabla^2U'_1 = \frac{\sum_* s_{*1}a_*^3\delta\rho_*}{a_1M_{\rm Pl}^2}+\frac{a_2\sum_* s_{*1}s_{*2}a_*^2\bar{\rho}_*}{a_1M_{\rm Pl}^2}[3(\psi_1-\psi_2) - \nabla^2(U_1-U_2)], \tag{A7}$$

$$-4\mathcal{H}_1\phi_1-4\psi'_1+8(\mathcal{H}'_1-\mathcal{H}_1^2)V_1 = \frac{2\sum_* s_{*1}a_*^3(\bar{\rho}_*+\bar{p}_*)v_*}{a_1M_{\rm Pl}^2} - \frac{a_2\sum_* s_{*1}s_{*2}a_*^2(\bar{\rho}_*+3\bar{p}_*)}{a_1M_{\rm Pl}^2}(V_1-V_2), \tag{A8}$$

$$U''_1-2V'_1+2\mathcal{H}_1(U'_1-2V_1)+\psi_1-\phi_1 = \frac{a_2\sum_* s_{*1}s_{*2}a_*^2\bar{p}_*}{a_1M_{\rm Pl}^2}(U_1-U_2), \tag{A9}$$

$$2(2\mathcal{H}'_1+\mathcal{H}_1^2)\phi_1+2\psi''_1+4\mathcal{H}_1\psi'_1+2\mathcal{H}_1\phi'_1 = \frac{\sum_* s_{*1}a_*^3\delta p_*}{a_1M_{\rm Pl}^2} + \frac{a_2\sum_* s_{*1}s_{*2}a_*^2\bar{p}_*}{a_1M_{\rm Pl}^2}[-(\phi_1-\phi_2)+2(\psi_1-\psi_2)] \tag{A10}$$

and the symmetric equations with respect to $1 \leftrightarrow 2$. Again, the left-hand side and the matter source terms on the right-hand side are identical to General Relativity. There are new mass coupling terms on the right-hand side that are proportional to ($\bar{\rho}_*$, $\bar{p}_*$, i.e., $\mathcal{H}^2$) and $s_{*1}s_{*2}$. Here, we did not include the perturbations of the scalar field φ, which corresponds to $\gamma_*^* = 0$ in the quasistatic equations (77) and (78).

4. Nonpropagation of the Goldstone mode

In the Einstein-de Sitter phase, where the quadratic action can be split over the two sectors S_+ and S_-, we could see from the explicit action (123) or from the Stückelberg analysis in Sec. VI A that the scalar mode associated with the breaking of the nondiagonal diffeomorphism does not propagate. Here, we provide an alternative check that such a mode cannot sustain decoupled propagation at high frequencies and wave numbers, even beyond the Einstein-de Sitter phase.

As in Eq. (126), we introduce the Stückelberg scalar π associated with the nondiagonal diffeomorphism by writing

$$\phi_\ell = \hat{\phi}_\ell-\mathcal{H}_\ell\gamma_\ell\pi'-\gamma_\ell\pi'', \quad \psi_\ell = \hat{\psi}_\ell+\mathcal{H}_\ell\gamma_\ell\pi', \tag{A11}$$

$$V_\ell = \hat{V}_\ell+\gamma_\ell\pi', \quad U_\ell = \hat{U}_\ell+\gamma_\ell\pi, \tag{A12}$$

where $\gamma_1 \neq \gamma_2$ are constant. The case $\gamma_1 = \gamma_2$ would be associated with the diagonal diffeomorphism. Substituting into Eqs. (A7)–(A10) and only keeping the π terms, we obtain

$$6\mathcal{H}_1(\mathcal{H}_1' - \mathcal{H}_1^2)\gamma_1\pi' = \frac{a_2\sum_* s_{*1}s_{*2}a_*^2\bar{\rho}_*}{a_1 M_{\rm Pl}^2}[(\gamma_1-\gamma_2)\nabla^2\pi - 3(\mathcal{H}_1\gamma_1 - \mathcal{H}_2\gamma_2)\pi'], \tag{A13}$$

$$4(\mathcal{H}_1^2 - \mathcal{H}_1{}')\gamma_1\pi' = \frac{a_2\sum_* s_{*1}s_{*2}a_*^2(\bar{\rho}_* + 3\bar{p}_*)}{a_1 M_{\rm Pl}^2}(\gamma_1-\gamma_2)\pi', \tag{A14}$$

$$0 = \frac{a_2\sum_* s_{*1}s_{*2}a_*^2\bar{p}_*}{a_1 M_{\rm Pl}^2}(\gamma_1-\gamma_2)\pi, \tag{A15}$$

$$2(\mathcal{H}_1'' - \mathcal{H}_1^3 - \mathcal{H}_1\mathcal{H}_1')\gamma_1\pi' = \frac{a_2\sum_* s_{*1}s_{*2}a_*^2\bar{p}_*}{a_1 M_{\rm Pl}^2} \times [(\gamma_1-\gamma_2)\pi'' + 3(\mathcal{H}_1\gamma_1 - \mathcal{H}_2\gamma_2)\pi']. \tag{A16}$$

They take the expected form involving $\mathcal{H}^2\partial^2\pi = \frac{s_{*1}s_{*2}a_*^2\bar{\rho}_*}{M_{\rm Pl}^2}\partial^2\pi$ (where some derivatives ∂ can be replaced by factors $\mathcal{H}$), as these terms must disappear in the naive Minkowski limit $\mathcal{H}\to 0$ and $\bar{\rho}_* \to 0$.

In the limit of high frequencies and wave numbers, $\omega \gg \mathcal{H}$ and $k \gg \mathcal{H}$, the last relation (A16) gives $\pi' = 0$ if $\bar{p}_* = 0$ or $\pi'' \sim \mathcal{H}\pi'$ if $a_*^2\bar{p}_* \sim M_{\rm Pl}^2\mathcal{H}^2$. Therefore, the scalar π cannot develop decoupled high frequency modes and does not propagate.

APPENDIX B: COUPLING OF THE GOLDSTONE MODE TO THE METRICS

1. General case

In this Appendix, we explore the role played by the Goldstone boson π in the modification of gravity. More precisely, we derive the coupling $h\partial^2\pi$ that was neglected in Sec. VI B, and we check that it agrees with the explicit expression (127) in the early-time regime where $s_{{\rm d}\ell} = s_\ell$. As in Sec. VI B, we go beyond this early-time regime, and we allow the baryonic and dark matter metrics to be different, but we focus on short lengths and timescales as compared with the Hubble parameter, using the approximation (130).

The Stückelberg fields ϕ_ℓ^μ are introduced as in Eq. (133),

$$e_{\ell\mu}^a = \hat{e}_{\ell\nu}^a \frac{\partial\phi_\ell^\nu}{\partial x^\mu}, \qquad \hat{e}_{\ell\mu}^a = a_\ell(\delta_\mu^a + \hat{h}_{\ell\mu}^a), \tag{B1}$$

where $\hat{h}_{\ell\mu}^a$ parametrize the deviations from the FLRW background. We again separate the diffeomorphisms into the diagonal ones, which are not broken by the presence of matter, and the broken ones in the complementary directions which belong to the group quotient $({\rm diff}_1 \times {\rm diff}_2)/{\rm diff}_{\rm diag}$. We choose in the following the particular combination

$$\phi_\ell^\mu = x^\mu + \xi^\mu + \xi_\ell^\mu \tag{B2}$$

with

$$\xi_1^\mu = \frac{1}{s_1 s_{\rm d1}}\pi^\mu, \qquad \xi_2^\mu = -\frac{1}{s_2 s_{\rm d2}}\pi^\mu. \tag{B3}$$

This corresponds to the choice $\gamma_1 = 1/s_1 s_{\rm d1}$ and $\gamma_2 = -1/s_2 s_{\rm d2}$ in the main text (134). As in Secs. VI A and VI B, we focus on the scalar mode that would be associated with a Boulware-Deser ghost, and we write

$$\pi^\mu = \partial^\mu\pi = \eta^{\mu\nu}\partial_\nu\pi. \tag{B4}$$

To simplify expressions, we always define $\partial^\mu\pi$ by the metric $\eta^{\mu\nu}$ in the following.

The total action does not depend on the diagonal diffeomorphism ξ^μ, which we set to zero in the following. We now derive the terms $\hat{h}\partial^2\pi$ that arise from the Einstein-Hilbert Lagrangians, which we write as

$$L_{\rm EH}(e_{\ell\mu}^a) = \sqrt{-g_\ell}R(g_{\ell\mu\nu}). \tag{B5}$$

Because of the invariance of the Ricci scalar under change of coordinates, we have from Eq. (B1)

$$L_{\rm EH}(e_{\ell\mu}^a) = \det(\partial_\mu\phi_\ell^\nu)L_{\rm EH}(\hat{e}_{\ell\nu}^a). \tag{B6}$$

To obtain the terms $\hat{h}\partial^2\pi$, we only need to work at linear order over $\hat{h}$ and π separately. At linear order over π, we have from Eq. (B2)

$$\det(\partial_\mu\phi_\ell^\nu) = 1 + \partial_\sigma\xi_\ell^\sigma. \tag{B7}$$

On the other hand, at linear order over $\hat{h}$, we have

$$\begin{aligned} L_{\rm EH}(\hat{e}_{\ell\mu}^a) &= L_{\rm EH}(\bar{e}_{\ell\mu}^a) - \sqrt{-\bar{g}_\ell}\bar{G}_\ell^{\mu\nu}\delta\hat{g}_{\ell\mu\nu} \\ &= \bar{L}_{\rm EH} - \sqrt{-\bar{g}_\ell}\bar{G}_\ell^{\mu\mu}\eta_{\mu\mu}2a_\ell^2\hat{h}_{\ell\mu}^\mu. \end{aligned} \tag{B8}$$

In the second line, we used the fact that the background Einstein tensors are diagonal, and we sum over μ. Substituting into Eq. (B6), we find that the term $\hat{h}\partial^2\pi$ that arises from the Einstein-Hilbert Lagrangians is

$$L_{\rm EH}(e_{\ell\mu}^a) \supset -\sqrt{-\bar{g}_\ell}\bar{G}_\ell^{\mu\mu}\eta_{\mu\mu}2a_\ell^2\hat{h}_{\ell\mu}^\mu\partial_\sigma\xi_\ell^\sigma. \tag{B9}$$

Along the diagonal, the background Einstein equations (19) read (no summation over μ)

$$M_{\rm Pl}^2\sqrt{-\bar{g}_\ell}\bar{G}_\ell^{\mu\mu}a_\ell = s_\ell\sqrt{-\bar{g}}\bar{T}^{\mu\mu}a + s_{{\rm d}\ell}\sqrt{-\bar{g}_{\rm d}}\bar{T}_{\rm d}^{\mu\mu}a_{\rm d}. \tag{B10}$$

Then, using Eq. (B3), the term $\hat{h}\partial^2\pi$ arising from the two Einstein-Hilbert actions is

$$\frac{M_{\rm Pl}^2}{2}[L_{\rm EH}(e^a_{1\mu}) + L_{\rm EH}(e^a_{2\mu})] \supset -\sqrt{-\bar{g}}\bar{T}^{\mu\mu}\eta_{\mu\mu}a \times \left(\frac{a_1}{s_{\rm d1}}\hat{h}^\mu_{1\mu} - \frac{a_2}{s_{\rm d2}}\hat{h}^\mu_{2\mu}\right)(\partial_\sigma\partial^\sigma\pi) - \sqrt{-\bar{g}_{\rm d}}\bar{T}^{\mu\mu}_{\rm d}\eta_{\mu\mu}a_{\rm d} \times \left(\frac{a_1}{s_1}\hat{h}^\mu_{1\mu} - \frac{a_2}{s_2}\hat{h}^\mu_{2\mu}\right)(\partial_\sigma\partial^\sigma\pi). \tag{B11}$$

We now turn to the matter actions. The matter vierbeins are given by

$$e^a_{\star\mu} = s_{\star 1}e^a_{1\mu} + s_{\star 2}e^a_{2\mu}, \tag{B12}$$

where $\star$ stands for b (baryons) or d (dark matter). They can be written as

$$e^a_{\star\mu} = \hat{e}^a_{\star\nu}\frac{\partial\phi^\nu_\star}{\partial x^\mu} + \delta\tilde{e}^a_{\star\mu}, \tag{B13}$$

where we introduced

$$\hat{e}^a_{\star\mu} = a_\star(\delta^a_\mu + \hat{h}^a_{\star\mu}), \tag{B14}$$

$$a_\star\hat{h}^a_{\star\mu} = s_{\star 1}a_1\hat{h}^a_{1\mu} + s_{\star 2}a_2\hat{h}^a_{2\mu}, \tag{B15}$$

$$\phi^\mu_\star = x^\mu + \xi^\mu_\star, \qquad a_\star\xi^\mu_\star = s_{\star 1}a_1\xi^\mu_1 + s_{\star 2}a_2\xi^\mu_2, \tag{B16}$$

and

$$\delta\tilde{e}^a_{\star\mu} = -a_\star\hat{h}^a_{\star\nu}\partial_\mu\xi^\nu_\star + s_{\star 1}a_1\hat{h}^a_{1\nu}\partial_\mu\xi^\nu_1 + s_{\star 2}a_2\hat{h}^a_{2\nu}\partial_\mu\xi^\nu_2. \tag{B17}$$

As compared with Eq. (B1), there is an additional term $\delta\tilde{e}^a_{\star\mu}$ of the form $\hat{h}\partial\xi$ because the matter vierbeins are defined by the composite expression (B12). Defining the matter Lagrangians as

$$L_\star(e^a_{\star\mu}) = \sqrt{-g_\star}\mathcal{L}_\star(g_{\star\mu\nu}), \tag{B18}$$

we now have

$$L_\star(e^a_{\star\mu}) = L_\star(\hat{e}^a_{\star\nu}\partial_\mu\phi^\nu_\star) + \frac{\sqrt{-\bar{g}_\star}}{2}\bar{T}^{\mu\nu}_\star\delta\tilde{g}_{\star\mu\nu}, \tag{B19}$$

where $\delta\tilde{g}_{\star\mu\nu}$ is the metric perturbation associated with $\delta\tilde{e}^a_{\star\mu}$ in Eq. (B13). On the other hand, as for the Einstein-Hilbert terms (B9), the term $L_\star(\hat{e}^a_{\star\nu}\partial_\mu\phi^\nu_\star)$ gives rise to the factor

$$L_\star(\hat{e}^a_{\star\nu}\partial_\mu\phi^\nu_\star) \supset \sqrt{-\bar{g}_\star}\bar{T}^{\mu\mu}_\star\eta_{\mu\mu}a^2_\star\hat{h}^\mu_{\star\mu}\partial_\sigma\xi^\sigma_\star. \tag{B20}$$

Collecting all terms, this gives

$$L_\star(e^a_{\star\mu}) \supset \sqrt{-\bar{g}_\star}\bar{T}^{\mu\mu}_\star\eta_{\mu\mu}a_\star[a_\star\hat{h}^\mu_{\star\mu}\partial_\sigma\xi^\sigma_\star - a_\star\hat{h}^\mu_{\star\sigma}\partial_\mu\xi^\sigma_\star + s_{\star 1}a_1\hat{h}^\mu_{1\sigma}\partial_\mu\xi^\sigma_1 + s_{\star 2}a_2\hat{h}^\mu_{2\sigma}\partial_\mu\xi^\sigma_2]. \tag{B21}$$

Using Eq. (B3), this yields for the baryonic matter Lagrangian

$$L(e^a_\mu) \supset \sqrt{-\bar{g}}\bar{T}^{\mu\mu}\eta_{\mu\mu}\left[\left(\frac{a_1}{s_{\rm d1}} - \frac{a_2}{s_{\rm d2}}\right)(s_1a_1\hat{h}^\mu_{1\mu} + s_2a_2\hat{h}^\mu_{2\mu}) \times(\partial_\sigma\partial^\sigma\pi) + \frac{a_1a_2(s_1s_{\rm d1}+s_2s_{\rm d2})}{s_{\rm d1}s_{\rm d2}}(\hat{h}^\mu_{1\sigma} - \hat{h}^\mu_{2\sigma})(\partial_\mu\partial^\sigma\pi)\right] \tag{B22}$$

and for the dark matter Lagrangian

$$L_{\rm d}(e^a_{{\rm d}\mu}) \supset \sqrt{-\bar{g}_{\rm d}}\bar{T}^{\mu\mu}_{\rm d}\eta_{\mu\mu}\left[\left(\frac{a_1}{s_1} - \frac{a_2}{s_2}\right)(s_{\rm d1}a_1\hat{h}^\mu_{1\mu} + s_{\rm d2}a_2\hat{h}^\mu_{2\mu}) \times(\partial_\sigma\partial^\sigma\pi) + \frac{a_1a_2(s_1s_{\rm d1}+s_2s_{\rm d2})}{s_1s_2} \times(\hat{h}^\mu_{1\sigma} - \hat{h}^\mu_{2\sigma})(\partial_\mu\partial^\sigma\pi)\right]. \tag{B23}$$

Collecting (B11), (B22), and (B23), we find that the terms $\hat{h}\partial^2\pi$ that arise in the total action are

$$L_{\rm EH+matter} \supset [\alpha\sqrt{-\bar{g}}\bar{T}^{\mu\mu}a^2 + \alpha_{\rm d}\sqrt{-\bar{g}_{\rm d}}\bar{T}^{\mu\mu}_{\rm d}a^2_{\rm d}]\eta_{\mu\mu} \times(\hat{h}^\mu_{-\sigma}\partial_\mu\partial^\sigma\pi - \hat{h}^\mu_{-\mu}\partial_\sigma\partial^\sigma\pi), \tag{B24}$$

where we have

$$\alpha = \frac{a_1a_2}{a^2}\frac{s_1s_{\rm d1}+s_2s_{\rm d2}}{s_{\rm d1}s_{\rm d2}}, \qquad \alpha_{\rm d} = \frac{a_1a_2}{a^2_{\rm d}}\frac{s_1s_{\rm d1}+s_2s_{\rm d2}}{s_1s_2}, \tag{B25}$$

and we introduced the metric combination

$$\hat{h}^a_{-\mu} = \hat{h}^a_{1\mu} - \hat{h}^a_{2\mu}, \tag{B26}$$

which agrees with Eq. (101).

Thus, we find that in all cases, even when the baryonic and dark matter couplings $s_{\star\ell}$ are different, the Stückelberg field π only couples to the same metric combination $\hat{h}_-$. The $\hat{h}_-\partial^2\pi$ terms in the last set of parentheses in Eq. (B24) are the same as in Eqs. (B27) and (B29) below, and they coincide with the result (127) in the main text, where we only keep the dominant terms with $\partial \gg \mathcal{H}$. In particular, by integrating by parts the terms in π'', we can again check that this contribution to the action can be written in terms of first-order time derivatives only. Therefore, it does not give rise to Boulware-Deser ghosts.

In the case where the couplings $s_{{\rm d}\ell}$ and s_ℓ are identical, we can separate the quadratic action in two sectors S_+ and

S_-, as explicitly shown in Sec. V. When the baryonic and dark matter metrics are different, we cannot simultaneously decouple both matter metrics from h_-, as we only have two fundamental metrics h_1 and h_2, so that $h_{\rm d}$ must be a combination of h and h_-. This may give rise to a modification of gravity on Hubble scales, although this is the regime where the derivation presented in this Appendix is no longer valid. On small scales, we have seen in Appendix A 4 that π does not propagate and does not generate a modification of gravity. In the main text, we have described the modification of gravity that is seen by the large-scale structures, which is entirely due to the fluctuations of the scalar field φ of which the effect is to generate a fifth force as described in Sec. IV.

2. Identical couplings

In the early-time regime, where $s_{{\rm d}\ell} = s_\ell$ and $a_\ell = s_\ell a$, we have $\alpha = \alpha_{\rm d} = 1$, and Eq. (B24) simplifies as

$$L_{\rm EH+matter} \supset \sqrt{-\bar{g}}(\bar{T}^{\mu\mu} + \bar{T}^{\mu\mu}_{\rm d})a^2\eta_{\mu\mu} \times (\hat{h}^{\mu}_{-\sigma}\partial_\mu\partial^\sigma\pi - \hat{h}^{\mu}_{-\mu}\partial_\sigma\partial^\sigma\pi). \quad \text{(B27)}$$

Thus, we recover the result of Secs. V A and VI A, obtained from the explicit derivation of the quadratic action, that the Goldstone boson does not couple to matter at the quadratic order in the Lagrangian and at the linear level in the equations of motion. It belongs to the sector S_- of the action, and it is only coupled to the graviton $\hat{h}_-$, which is also decoupled from matter. Explicitly, this metric coupling reads

$$L_{\hat{h}\partial^2\pi} = -a^4\bar{\rho}_T(\hat{h}^0_{-i}\partial_i\pi' - \hat{h}^0_{-0}\nabla^2\pi) + a^4\bar{p}_T \times(-\hat{h}^i_{-0}\partial_i\pi' + \hat{h}^i_{-j}\partial_i\partial_j\pi + \hat{h}^i_{-i}(\pi'' - \nabla^2\pi)), \quad \text{(B28)}$$

where $\bar{p}_T$ and $\bar{\rho}_T$ are the total pressure and energy densities. Now, using $\hat{h}^0_{-0} = \hat{\phi}_-$, $\hat{h}^0_{-i} = -\partial_i\hat{V}_-$, $\hat{h}^i_{-0} = \partial_i\hat{V}_-$, $\hat{h}^i_{-j} = -\hat{\psi}_-\delta^i_j + \partial_i\partial_j\hat{U}_-$, and $a^2\bar{\rho}_T = 3M^2_{\rm Pl}\mathcal{H}^2$, $a^2\bar{p}_T = -M^2_{\rm Pl}(\mathcal{H}^2 + 2\mathcal{H}')$, this gives

$$\frac{L_{\hat{h}\partial^2\pi}}{a^2M^2_{\rm Pl}} = 3\mathcal{H}^2\hat{\phi}_-\nabla^2\pi + 2(2\mathcal{H}^2 + \mathcal{H}')(\nabla\hat{V}_-)\cdot(\nabla\pi') + (\mathcal{H}^2 + 2\mathcal{H}')[3\hat{\psi}_-\pi'' - 2\hat{\psi}_-\nabla^2\pi - (\nabla^2\hat{U}_-)\pi'']. \quad \text{(B29)}$$

This coincides with the result (127) in the main text, when the subdominant terms have been dropped. In particular, integrating by parts the terms in π'', we recover the fact that the quadratic action can be written in terms of first-order time derivatives only, without third- and fourth-order time derivatives left.

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