Self-acceleration in scalar-bimetric theories

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We describe scalar-bimetric theories where the dynamics of the Universe are governed by two separate metrics, each with an Einstein-Hilbert term. In this setting, the baryonic and dark matter components of the Universe couple to metrics which are constructed as functions of these two gravitational metrics. More precisely, the two metrics coupled to matter are obtained by a linear combination of their vierbeins, with scalar-dependent coefficients. The scalar field, contrary to dark energy models, does not have a potential whose role is to mimic a late-time cosmological constant. The late-time acceleration of the expansion of the Universe can be easily obtained at the background level in these models by appropriately choosing the coupling functions appearing in the decomposition of the vierbeins for the baryonic and dark matter metrics. We explicitly show how the concordance model can be retrieved with negligible scalar kinetic energy. This requires the scalar coupling functions to show variations of order unity during the accelerated expansion era. This leads in turn to deviations of order unity for the effective Newton constants and a fifth force that is of the same order as Newtonian gravity, with peculiar features. The baryonic and dark matter self-gravities are amplified although the gravitational force between baryons and dark matter is reduced and even becomes repulsive at low redshift. This slows down the growth of baryonic density perturbations on cosmological scales, while dark matter perturbations are enhanced. In our local environment, the upper bound on the time evolution of Newton's constant requires an efficient screening mechanism that both damps the fifth force on small scales and decouples the local value of Newton constant from its cosmological value. This cannot be achieved by a quasi-static chameleon mechanism, and requires going beyond the quasi-static regime and probably using derivative screenings, such as Kmouflage or Vainshtein screening, on small scales.

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I. INTRODUCTION

A very common way of reproducing the late time acceleration of the expansion of the Universe [1, 2] is to add a scalar field energy density which would mimic a cosmological constant at small redshifts [3]. Recently it has been proposed that the acceleration could be an illusion due to the different metrics coupled to either the baryons or dark matter [4]. This has been achieved by considering that baryons couple to a metric which can be constructed from both the metric felt by dark matter and the velocity field of the dark matter particles. In the same vein, it has been known for some time that conformally coupled models with a single metric and screening properties, thus evading the local tests of gravity, cannot generate the late time acceleration of the Universe [5]. In this paper, we generalise the latter approach by introducing two gravitational metrics, with an Einstein-Hilbert term each, and we consider that the baryons and dark matter couple to different dynamical metrics. These metrics are obtained by taking linear combinations of the two gravitational vierbeins, with each of the coefficients dependent on a scalar field. Contrary to dark energy models (even coupled), we do not require that the scalar field should play any explicit role in generating an effective cosmological constant at late time. Quite the contrary, the scalar is only a free and massless scalar, with positive pressure. The role of the scalar is to provide a time-dependent mapping and transform the deceleration of the two gravitational metrics into an acceleration for the baryonic metric.

Our approach is inspired by the construction of doubly coupled bigravity models [6, 7] where the late-time acceleration is due to an explicit cosmological constant, albeit related to the mass of the massive graviton [8, 9], and matter couples to a combination (with constant coefficients) of the two dynamical metrics [10]. Here, we remove the potential term of massive gravity and introduce scalar-dependent coupling functions, as our goal is to build self-accelerated solutions. As expected, we find that this leads to major difficulties, as a self acceleration implies effects of order unity on cosmological scales. This generically gives rise to effective Newton constants that evolve on Hubble time scales and a fifth force of the same order as Newtonian gravity.

We mostly focus on the late time Universe, in the matter and dark energy eras. However, doubly coupled bigravity theories suffer from instabilities in the radiation era [11–13] for tensor and vector modes. We briefly rederive these behaviors for our models. Tensor modes have a tachyonic regime that implies an anomalous growth in the early Universe. This has some effect on the Cosmic Microwave Background B-modes which may be amplified [14] in models where there is a nonlinear coupling between the metrics, such as in bigravity theories. Similarly, the vector modes that are decoupled from matter suffer from a gradient instability which could pose serious problems for the viability of the models. However, in our case these instabilities only affect "hidden" modes that are not seen by the matter metrics (at the linear level).

In this paper, we do not perform detailed comparisons with cosmological and astrophysical data, as our goal is only to distinguish which families of solutions offer a realistic framework, which may deserve further investigations. Indeed, imposing a Λ-CDM expansion history for the cosmological background (which ensures consistency with cosmological data at the background level), we find that the tight constraint on the velocity of gravitational waves [15] and the upper bound on the local time dependence of Newton constant [16] already provide significant constraints on the model. In fact, we find that a non-linear screening mechanism [17] must come into play on small scales, to ensure convergence to General Relativity in the Solar System. This must go beyond the quasi-static approximation and probably rely on derivatives of the scalar field (as in Kmouflage [18–20] or Vainshtein mechanisms [21]), while quasi-static chameleon screening [22, 23] cannot occur. We have left the analysis of this regime for future work.

This article is organized as follows. We first define the bi-metric model in section II and next provide the equations of motion in section II C. We describe the cosmological background in section III. We show how to construct solutions that mimic a Λ-CDM expansion and discuss both the simplified cases where all metrics have the same conformal time and the cases where they have different conformal times. We turn to linear perturbations in section IV, for both baryonic and matter density fluctuations. We then describe in section V the early time tensor and vector instabilities, which only occur for "hidden" degrees of freedom decoupled from matter. We then compare our results to doubly coupled bigravity in section VI. We discuss consistency with small-scale tests of General Relativity in section VII and conclude in section VIII.

II. SCALAR-BIMETRIC MODELS

A. Defining the models

In the following we focus on models where the dynamics are driven by two independent metrics coupled to a scalar field. We do not add any non-trivial dynamics for the scalar field which we choose to be massless with canonical kinetic terms. We consider models with the scalar-bimetric action

$$S = S_{\rm grav} + S_{\rm mat}, \tag{1}$$

with

$$S_{\rm grav} = \int d^4x \frac{M_{\rm pl}^2}{2} \left[\sqrt{-g_1} R_1 + \sqrt{-g_2} R_2\right], \tag{2}$$

and

$$S_{\rm mat} = \int d^4x \left[\sqrt{-g_{\rm d}}\, \mathcal{L}_{\rm d}(\varphi, \psi^i_{\rm dm}; g_{\rm d}) + \sqrt{-g_{\rm b}}\, \mathcal{L}_{\rm b}(\psi^i_{\rm b}; g_{\rm b})\right]. \tag{3}$$

The gravitational action $S_{\rm grav}$ contains two Einstein-Hilbert terms for the two gravitational metrics $g_{1\mu\nu}$ and $g_{2\mu\nu}$. The matter action $S_{\rm mat}$ contains the dark sector Lagrangian $\mathcal{L}_{\rm d}$, which includes dark matter fields $\psi^i_{\rm dm}$ and an additional scalar field φ, and the baryonic Lagrangian $\mathcal{L}_{\rm b}$, which includes the ordinary particles of the standard model, both matter and radiation (photons) components. These two matter Lagrangians involve two associated dynamical metrics, $g_{{\rm d}\mu\nu}$ and $g_{{\rm b}\mu\nu}$. In the following we will usually omit the subscript "b", as this is the main sector that is probed by observations and experiments.

We split the dark sector Lagrangian in its scalar field and dark matter components,

$$\mathcal{L}_{\rm d} = \mathcal{L}_\varphi(\varphi; g_{\rm d}) + \mathcal{L}_{\rm dm}(\psi^i_{\rm dm}; g_{\rm d}), \tag{4}$$

and for simplicity we only keep the kinetic term in the scalar field Lagrangian:

$$\mathcal{L}_\varphi(\varphi) = -\frac{1}{2}\, g_{\rm d}^{\mu\nu} \partial_\mu\varphi \partial_\nu\varphi, \tag{5}$$

as we wish to recover the late-time acceleration of the expansion of the Universe through a dynamical mechanism, rather than through an effective cosmological constant associated with a non-zero minimum of the scalar-field potential.

The dark and baryonic metrics are functions of the two gravitational metrics $g_{1\mu\nu}$ and $g_{2\mu\nu}$. We write this relationship in terms of the vierbeins of these four metrics. Thus, introducing the vierbeins $e^a_{1\mu}$ and $e^a_{2\mu}$ of the metrics $g_{1\mu\nu}$ and $g_{2\mu\nu}$,

$$g_{1\mu\nu} = e^a_{1\mu} e^b_{1\nu} \eta_{ab}, \quad g_{2\mu\nu} = e^a_{2\mu} e^b_{2\nu} \eta_{ab}, \tag{6}$$

we define the dark and baryonic metrics as

$$g_{{\rm d}\mu\nu} = e^a_{{\rm d}\mu} e^b_{{\rm d}\nu} \eta_{ab}, \quad g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}, \tag{7}$$

with

$$\begin{aligned} e^a_{{\rm d}\mu} &= s_{\rm d1}(\varphi) e^a_{1\mu} + s_{\rm d2}(\varphi) e^a_{2\mu}, \\ e^a_\mu &= s_1(\varphi) e^a_{1\mu} + s_2(\varphi) e^a_{2\mu}. \end{aligned} \tag{8}$$

Thus, both dynamical matter metrics are a combination of the two gravitational metrics that depends on the scalar field φ. This leads to non-minimal couplings between the matter sectors and the scalar field.

B. Degrees of freedom

Although we have defined the model in terms of the vierbiens of the two dynamical metrics, we treat the theory as a metric theory, which is a function of the two metrics $g_{\ell\mu\nu}$, $i = 1, 2$. The two Einstein-Hilbert terms are invariant under two copies of the diffeomorphism group. The coupling to matter which involves the diagonal subgroup reduces the diffeomorphism invariance to one diagonal copy. The two metrics involve 20 degrees of freedom, which can be reduced to 16 when the diagonal gauge invariance under reparameterisation of coordinates has been used. The vierbeins are four by four matrices, which involves $2 \times 16 = 32$ degrees of freedom. This is redundant even when the diagonal diffeomorphism invariance has been used, reducing to 28 the number of degrees of freedom. The two vierbeins have two copies of the local Lorentz symmetry group as an invariance group. Again this is broken to the diagonal Lorentz group by the coupling to matter. This removes six degrees of freedom bringing it down to 22. This is still more than the 16 degrees of freedom of the metric description. This can be made to coincide by imposing the symmetric condition [24]

$$Y_{\mu\nu} = Y_{\nu\mu} \tag{9}$$

where we have introduced the tensor

$$Y_{\mu\nu} = \eta_{ab} e^a_{1\mu} e^b_{2\nu} = e^a_{1\mu} e_{2a\nu}. \tag{10}$$

The 4×4 tensor $Y_{\mu\nu}$ can be decomposed into ten symmetric components and six antisymmetric ones which are imposed to be vanishing. This brings the number of vierbein degrees of freedom down to 16, matching the ones for a bi-metric theory.

The consequences of the symmetric conditions are well-known, let us recall some salient features here. First of all let us define

$$X^\mu_{2\nu} = g_2^{\mu\lambda} Y_{\lambda\nu} = e^\mu_{2a} e^a_{1\nu}, \quad X^\mu_{1\nu} = g_1^{\mu\lambda} Y_{\lambda\nu} = e^\mu_{1a} e^a_{2\nu}, \tag{11}$$

then we have that

$$X^\theta_{2\nu} X^\nu_{2\lambda} = g_2^{\theta\nu} g_{1\nu\lambda}, \quad X^\theta_{1\nu} X^\nu_{1\lambda} = g_1^{\theta\nu} g_{2\nu\lambda}, \tag{12}$$

which implies that in matrix notation

$$Y = g_2 (g_2^{-1} g_1)^{1/2} = g_1 (g_1^{-1} g_2)^{1/2}, \tag{13}$$

with an appropriate definition for the square root of a matrix [24]. As a result, $Y_{\mu\nu}$ becomes a function of the two metrics $g_{\ell\mu\nu}$, which implies that the dark and baryonic metrics

$$\begin{aligned} g_{\mu\nu} &= s_1^2(\varphi) g_{1\mu\nu} + 2 s_1(\varphi) s_2(\varphi) Y_{\mu\nu} + s_2^2(\varphi) g_{2\mu\nu}, \\ g_{\mathrm{d}\mu\nu} &= s_{\mathrm{d}1}^2(\varphi) g_{1\mu\nu} + 2 s_{\mathrm{d}1}(\varphi) s_{\mathrm{d}2}(\varphi) Y_{\mu\nu} + s_{\mathrm{d}2}^2(\varphi) g_{2\mu\nu}, \end{aligned} \tag{14}$$

are simply functions of the two metrics too.

C. Equations of motion

1. Einstein's equations

We cannot obtain the Einstein equations by requiring the functional derivatives of the action with respect to the vierbeins $e^a_{1\mu}$ and $e^a_{2\mu}$ to vanish. Indeed, because of the symmetry condition (9), which reduces the number of degrees of freedom to those of $g_{1\mu\nu}$ and $g_{2\mu\nu}$, the vierbeins are correlated and constrained by Eq.(9). This means that we must take the variations along the directions that span the subspace defined by the constraint (9). If we vary the metric g_1 while keeping g_2 fixed, hence we vary $e^a_{1\mu}$ at fixed $e^a_{2\mu}$, the symmetric constraint (9) reads as

$$\delta e^a_{1\mu} e_{2a\nu} = \delta e^a_{1\nu} e_{2a\mu} \quad \text{for all} \quad \{\mu, \nu\}. \tag{15}$$

We can check that these constraints are satisfied if the variations $\delta e^a_{1\mu}$ are of the form

$$\delta e^a_{1\mu} = \delta Z_{1\mu\nu} e^{a\nu}_2, \tag{16}$$

where $\delta Z_{1\mu\nu}$ is an arbitrary infinitesimal symmetric matrix, $\delta Z_{1\mu\nu} = \delta Z_{1\nu\mu}$. As expected, the matrix $\delta Z_{1\mu\nu}$ provides the same number of degrees of freedom as the metric $g_{1\mu\nu}$. This also gives $\delta g_{1\mu\nu} = \delta Z_{1\mu\lambda} X^\lambda_{2\nu} + \delta Z_{1\nu\lambda} X^\lambda_{2\mu}$. Then, the Einstein equations follow from the variation of the action with respect to $\delta Z_{1\mu\nu}$. Using $\delta e^a_{1\lambda} / \delta Z_{1\mu\nu} = \delta^\mu_\lambda e^{a\nu}_2 + \delta^\nu_\lambda e^{a\mu}_2$ when $\mu \neq \nu$ (because $\delta Z_{1\mu\nu} = \delta Z_{1\nu\mu}$ hence they vary simultaneously), this gives

$$\text{for all} \quad \{\mu, \nu\} : \quad \frac{\delta S}{\delta e^a_{1\mu}} e^{a\nu}_2 + \frac{\delta S}{\delta e^a_{1\nu}} e^{a\mu}_2 = 0. \tag{17}$$

This provides the expected 16 symmetric Einstein equations (hence 10 equations, before we use diffeomorphism invariance), which read as

$$\begin{aligned} & M_{\mathrm{Pl}}^2 \sqrt{-g_1} \left[G_1^{\mu\sigma} X^\nu_{2\sigma} + G_1^{\nu\sigma} X^\mu_{2\sigma} \right] = \\ & \quad s_1 \sqrt{-g} \left[T^{\mu\sigma} (s_1 X^\nu_{2\sigma} + s_2 \delta^\nu_\sigma) + T^{\nu\sigma} (s_1 X^\mu_{2\sigma} + s_2 \delta^\mu_\sigma) \right] \\ & \quad + s_{\mathrm{d}1} \sqrt{-g_{\mathrm{d}}} \left[T_{\mathrm{d}}^{\mu\sigma} (s_{\mathrm{d}1} X^\nu_{2\sigma} + s_{\mathrm{d}2} \delta^\nu_\sigma) + T_{\mathrm{d}}^{\nu\sigma} (s_{\mathrm{d}1} X^\mu_{2\sigma} + s_{\mathrm{d}2} \delta^\mu_\sigma) \right] \end{aligned} \tag{18}$$

The Einstein equations with respect to the second metric g_2 are obtained by exchanging the indices $1 \leftrightarrow 2$ [31]. Here $T^{\mu\nu}$ and $T_{\mathrm{d}}^{\mu\nu}$ are the baryonic and dark-energy energy-momentum tensors, defined with respect to their associated metrics,

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_{\mathrm{b}})}{\delta g^{\mu\nu}}, \quad T_{\mathrm{d}\mu\nu} = \frac{-2}{\sqrt{-g_{\mathrm{d}}}} \frac{\delta(\sqrt{-g_{\mathrm{d}}} \mathcal{L}_{\mathrm{d}})}{\delta g_{\mathrm{d}}^{\mu\nu}}. \tag{19}$$

We recover the standard Einstein equations when the two metrics are identical (case of a single-metric model) with $s_1 + s_2 = 1$ and $s_{\mathrm{d}1} + s_{\mathrm{d}2} = 1$, as it yields $X^\mu_{2\nu} = X^\mu_{1\nu} = \delta^\mu_\nu$.

When the metrics are diagonal, that is, we have

$$g_{*\mu\nu} \supset \delta_{\mu\nu}, \ \ e^a_{*\mu} \supset \delta^a_\mu, \ \ Y_{\mu\nu} \supset \delta_{\mu\nu}, \ \ X^\mu_{\ell\nu} \supset \delta^\mu_\nu,$$

where $* = \{1, 2, \mathrm{d}, \mathrm{b}\}$ and $g_{*\mu\nu} \supset \delta_{\mu\nu}$ means that $g_{*\mu\nu} = 0$ for $\mu \neq \nu$, the Einstein equations (18) simplify along the diagonal as (no summation over μ)

$$M_{\mathrm{Pl}}^2 \sqrt{-g_1} G_1^{\mu\mu} e^a_{1\mu} = s_1 \sqrt{-g} T^{\mu\mu} e^a_\mu + s_{\mathrm{d}1} \sqrt{-g_{\mathrm{d}}} T_{\mathrm{d}}^{\mu\mu} e^a_{\mathrm{d}\mu}$$

with $a = \mu$. This coincides with the Einstein equations that would have been obtained by taking derivatives with respect to the vierbeins without taking care of the symmetric constraint (9). However, the off-diagonal Einstein equations remain modified.

2. Scalar field equation of motion

The dependence of the matter metrics $g_{\mathrm{d}\mu\nu}$ and $g_{\mu\nu}$ on the scalar field φ, through Eq.(8), gives rise to source terms in the Klein-Gordon equation that governs the scalar field dynamics,

$$\partial_\mu\left[\sqrt{-g_{\mathrm{d}}}g_{\mathrm{d}}^{\mu\nu}\partial_\nu\varphi\right]+\sqrt{-g_{\mathrm{d}}}T_{\mathrm{d}}^{\mu\nu}\sum_{\ell=1}^{2}\frac{ds_{\mathrm{d}\ell}}{d\varphi}e^a_{\ell\mu}e^b_{\mathrm{d}\nu}\eta_{ab}$$
$$+\sqrt{-g}T^{\mu\nu}\sum_{\ell=1}^{2}\frac{ds_\ell}{d\varphi}e^a_{\ell\mu}e^b_\nu\eta_{ab}=0. \tag{20}$$

3. Matter equations of motion

The equations of motion of the dark and baryonic matter components take their standard form in their Jordan frames,

$$\nabla_{\mathrm{d}\mu}T^\mu_{\mathrm{dm}\nu}=0,\quad \nabla_\mu T^\mu_\nu=0, \tag{21}$$

where $\nabla_{\mathrm{d}\mu}$ and ∇_μ are the covariant derivatives with respect to the metrics $g_{\mathrm{d}\mu\nu}$ and $g_{\mu\nu}$.

III. COSMOLOGICAL BACKGROUND

A. Friedmann's equations

We consider diagonal metrics of the form

$$g_{*\mu\nu}(\tau)=\mathrm{diag}(-b_*^2(\tau),a_*^2(\tau),a_*^2(\tau),a_*^2(\tau)), \tag{22}$$

where $*=\{1,2,\mathrm{d},\mathrm{b}\}$, with the vierbeins

$$e^a_{*\mu}=\mathrm{diag}(b_*,a_*,a_*,a_*), \tag{23}$$

and we denote by $\mathcal{H}_{b_*}=d\ln b_*/d\tau$, $\mathcal{H}_{a_*}=d\ln a_*/d\tau$, the conformal expansion rates of the time and spatial components. We can choose to define the conformal time τ with respect to the baryonic metric $g_{\mu\nu}$, so that

$$b=a,\quad g_{\mu\nu}(\tau)=\mathrm{diag}(-a^2,a^2,a^2,a^2), \tag{24}$$

and we use either τ or $\ln(a)$ as the time variable. From the definitions (8) we obtain the constraints

$$b=a=s_1b_1+s_2b_2,\quad a=s_1a_1+s_2a_2,$$
$$b_{\mathrm{d}}=s_{\mathrm{d}1}b_1+s_{\mathrm{d}2}b_2,\quad a_{\mathrm{d}}=s_{\mathrm{d}1}a_1+s_{\mathrm{d}2}a_2. \tag{25}$$

The $(0,0)$ component of the Einstein equations (18) reads

$$3M_{\mathrm{Pl}}^2a_\ell^3b_\ell^{-2}\mathcal{H}_{a_\ell}^2=s_\ell a^3(\bar\rho+\bar\rho_\gamma)+s_{\mathrm{d}\ell}a_{\mathrm{d}}^3(\bar\rho_{\mathrm{dm}}+\bar\rho_\varphi), \tag{26}$$

while the (i,i) components read

$$M_{\mathrm{Pl}}^2a_\ell^2b_\ell^{-1}\left[2\mathcal{H}'_{a_\ell}+3\mathcal{H}_{a_\ell}^2-2\mathcal{H}_{a_\ell}\mathcal{H}_{b_\ell}\right]=-s_\ell a^3\frac{\bar\rho_\gamma}{3}$$
$$-s_{\mathrm{d}\ell}a_{\mathrm{d}}^2b_{\mathrm{d}}\bar\rho_\varphi. \tag{27}$$

Here we assumed non-relativistic matter components, $p_{\mathrm{dm}}=p=0$, and we used $p_\gamma=\rho_\gamma/3$ and $\bar p_\varphi=\bar\rho_\varphi$ for the radiation and scalar pressure.

B. Conservation equations

The Jordan-frame equations of motion (21) lead to the usual conservation equations, hence

$$\bar\rho_{\mathrm{dm}}=\frac{\bar\rho_{\mathrm{dm}0}}{a_{\mathrm{d}}^3},\quad \bar\rho=\frac{\bar\rho_0}{a^3},\quad \bar\rho_\gamma=\frac{\bar\rho_{\gamma0}}{a^4}. \tag{28}$$

We define the cosmological parameters associated with these characteristic densities by

$$\Omega_{\mathrm{b}0}=\frac{\bar\rho_{\mathrm{b}0}}{3M_{\mathrm{Pl}}^2H_0^2},\quad \Omega_{\gamma0}=\frac{\bar\rho_{\gamma0}}{3M_{\mathrm{Pl}}^2H_0^2},\quad \Omega_{\mathrm{dm}0}=\frac{\bar\rho_{\mathrm{dm}0}}{3M_{\mathrm{Pl}}^2H_0^2}, \tag{29}$$

where H_0 is the physical expansion rate associated with the baryonic metric today, at $a=1$. We also define the rescaled scalar field energy density ξ as the ratio of the scalar field to dark matter energy densities

$$\xi(a)=\frac{a_{\mathrm{d}}^3\bar\rho_\varphi}{3M_{\mathrm{Pl}}^2H_0^2},\quad \frac{\xi(a)}{\Omega_{\mathrm{dm}0}}=\frac{\bar\rho_\varphi(a)}{\bar\rho_{\mathrm{dm}}(a)}. \tag{30}$$

It is convenient to introduce the dimensionless combination

$$\ell=1,2:\quad \omega_\ell=a_\ell^3b_\ell^{-2}\frac{\mathcal{H}_{a_\ell}^2}{H_0^2}. \tag{31}$$

Then, the Friedmann equations (26)-(27) simplify as

$$\omega_\ell=s_\ell\left(\Omega_{\mathrm{b}0}+\frac{\Omega_{\gamma0}}{a}\right)+s_{\mathrm{d}\ell}\left(\Omega_{\mathrm{dm}0}+\xi(a)\right), \tag{32}$$

$$\frac{b_\ell}{a_\ell}\frac{\mathcal{H}}{\mathcal{H}_{a_\ell}}\frac{d\omega_\ell}{d\ln a}=-s_\ell\frac{\Omega_{\gamma0}}{a}-s_{\mathrm{d}\ell}\,3\frac{b_{\mathrm{d}}}{a_{\mathrm{d}}}\xi. \tag{33}$$

We recover the usual Friedmann equations of General Relativity with $\xi=0$, $b_*=a_*$, $\mathcal{H}=\mathcal{H}_{a_\ell}$, $s_\ell=1$ and $s_{\mathrm{d}\ell}=1$. In this case, we can check that the second Friedmann equation is a consequence of the first Friedmann equation and of the conservation equations, as it the derivative of Eq.(32) with respect to $\ln a$.

By taking the first derivative of Eq.(32) and combining with Eq.(33), we obtain the useful combinations

$$\frac{ds_\ell}{d\ln a}\left(\Omega_{\mathrm{b}0}+\frac{\Omega_{\gamma0}}{a}\right)+\frac{ds_{\mathrm{d}\ell}}{d\ln a}\left(\Omega_{\mathrm{dm}0}+\xi(a)\right)=$$
$$s_\ell\frac{\Omega_{\gamma0}}{a}\left(1-\frac{a_\ell}{b_\ell}\frac{\mathcal{H}_{a_\ell}}{\mathcal{H}}\right)-s_{\mathrm{d}\ell}\xi\left(\frac{d\ln\xi}{d\ln a}+3\frac{b_{\mathrm{d}}a_\ell}{a_{\mathrm{d}}b_\ell}\frac{\mathcal{H}_{a_\ell}}{\mathcal{H}}\right). \tag{34}$$

This shows that the evolutions of the baryonic and dark matter couplings are correlated and related to the running of the scalar field ($\xi>0$) and the deviations between the different expansion rates $\mathcal{H}_*$. In the absence of the scalar field in the dynamics the relation (34) reduces to the branch of solutions

$$\frac{a_\ell}{b_\ell}\mathcal{H}_{a_\ell}=\mathcal{H}, \tag{35}$$

which appears in doubly coupled bigravity [13].

C. Scalar field equation of motion

The scalar field energy density reads as

$$\bar{\rho}_\varphi = \frac{1}{2b_{\rm d}^2}\left(\frac{d\bar{\varphi}}{d\tau}\right)^2 . \tag{36}$$

Then, we can check that the background Klein-Gordon equation (20) can be written in terms of $\bar{\rho}_\varphi$. Using the rescaled scalar field density ξ of Eq.(30), this gives

$$\begin{aligned}&\frac{b_{\rm d}}{a_{\rm d}^3}\frac{d}{d\ln a}\left[a_{\rm d}^3\xi\right] + (\Omega_{\rm dm0}+\xi)\sum_\ell \frac{ds_{{\rm d}\ell}}{d\ln a}b_\ell \\ &-3\frac{b_{\rm d}}{a_{\rm d}}\xi\sum_\ell\frac{ds_{{\rm d}\ell}}{d\ln a}a_\ell + \left(\Omega_{\rm b0}+\frac{\Omega_{\gamma 0}}{a}\right)\sum_\ell\frac{ds_\ell}{d\ln a}b_\ell \\ &-\frac{\Omega_{\gamma 0}}{a}\sum_\ell\frac{ds_\ell}{d\ln a}a_\ell = 0.\end{aligned} \tag{37}$$

From $\xi(a)$ we obtain the evolution of the scalar field $\bar{\varphi}(a)$ by integrating Eq.(36). With the initial condition $\bar{\varphi}(0)=0$, this gives

$$\bar{\varphi}(a) = M_{\rm Pl}\int_0^a \frac{da}{a}\frac{b_{\rm d}H_0}{\mathcal{H}}\sqrt{\frac{6\xi}{a_{\rm d}^3}} . \tag{38}$$

We can actually check that the Klein-Gordon equation (37) is also a consequence of the Friedmann equations (32)-(33), supplemented with the constraints (25). Therefore, as in General Relativity the Friedmann equations and the conservation equations are not independent. In General Relativity, it is customary to work with the first Friedmann equation and the conservation equations of the various matter components, leaving aside the second Friedmann equation that is their automatic consequence. In this paper, because we have two symmetric sets of Friedmann equations (32)-(33), for $\ell = 1,2$, and the Klein-Gordon equation (37) takes a complicated form with its new source terms, we instead work with the four Friedmann equations and we discard the Klein-Gordon equation (37), which is their automatic consequence.

D. Einstein - de Sitter reference

When the scalar field is a constant, it should not play any role and we expect to recover a standard cosmology. Because we did not introduce any cosmological constant, this must be an Einstein-de Sitter Universe without late-time acceleration (more precisely, a Universe with only matter and radiation components). In this reference Universe, obtained within General Relativity with only one metric, the Friedmann equations (32)-(33) read as

$$\begin{aligned}&\omega^{(0)} = a\frac{\mathcal{H}^{(0)2}}{H_0^2} = \Omega_{\rm dm0}+\Omega_{\rm b0}+\frac{\Omega_{\gamma 0}}{a}, \\ &\frac{d\omega^{(0)}}{d\ln a} = -\frac{\Omega_{\gamma 0}}{a} .\end{aligned} \tag{39}$$

Here and in the following, we denote with the superscript "(0)" quantities associated with this Einstein-de Sitter reference Universe, which follows General Relativity. As noticed above, here the second Friedmann equation is trivial as it is a mere consequence of the first Friedmann equation and of the conservation equations, which have already been used in the first Eq.(39).

We can recover the standard cosmology (39) within the bi-metric model (1) by the simple solution

$$\begin{aligned}&a_\ell^{(0)} = s_\ell^{(0)}a, \;\; a_{\rm d}^{(0)} = a, \;\; b_*^{(0)} = a_*^{(0)}, \;\; \mathcal{H}_*^{(0)} = \mathcal{H}^{(0)}, \\ &\omega_\ell^{(0)} = s_\ell^{(0)}\omega^{(0)}, \;\; \xi^{(0)} = 0, \;\; s_{{\rm d}\ell}^{(0)} = s_\ell^{(0)},\end{aligned} \tag{40}$$

where the coefficients $s_\ell^{(0)}$ are constants that obey the condition

$$\left(s_1^{(0)}\right)^2 + \left(s_2^{(0)}\right)^2 = 1. \tag{41}$$

The scalar field φ is also constant, as the derivatives in the source terms of Eq.(20) vanish. In this solution, all four metrics are essentially equivalent, as $b_* = a_*$ and all scale factors a_* are proportional. The common expansion rate $\mathcal{H}_*(a)$ follows the standard Einstein-de Sitter reference $\mathcal{H}^{(0)}(a)$ of Eq.(39).

E. Λ-CDM reference

To match observations, the expansion rate $\mathcal{H}(a)$ must deviate from the Einstein-de Sitter reference (39) and remain close to the concordance Λ-CDM cosmology. To ensure that this is the case, in this paper we constrain the baryonic expansion rate $\mathcal{H}(a)$ to follow exactly a reference Λ-CDM cosmology. Of course, in practice small deviations from the Λ-CDM limit are allowed by the data, and we could also generalize the solutions that we consider in this paper by adding small deviations. However, by definition this would not significantly modify the properties of these solutions. Besides, being able to recover a Λ-CDM expansion rate is sufficient to show that the bi-metric model can be made consistent with data at the level of the cosmological background.

In the Λ-CDM cosmology, we add a cosmological constant to the components of the Universe. The usual Friedmann equation reads as

$$\frac{\mathcal{H}^2}{H_0^2} = \frac{\Omega_{\rm dm0}+\Omega_{\rm b0}}{a} + \frac{\Omega_{\gamma 0}}{a^2} + \Omega_{\Lambda 0}a^2, \tag{42}$$

where $\Omega_{\Lambda 0}$ is the cosmological parameter associated with the cosmological constant. In terms of the variable ω this gives

$$\omega(a) = \omega^{(0)}(a) + \Omega_{\Lambda 0}a^3, \tag{43}$$

which explicitly shows the deviation from the Einstein-de Sitter reference (39). [Here the Einstein-de Sitter reference is normalized with $\Omega_{\rm dm0}+\Omega_{\rm b0}+\Omega_{\gamma 0} = 1-\Omega_{\Lambda 0} \neq 1$, because we normalize the cosmological densities by H_0 instead of $H_0^{(0)}$.]

The bi-metric solution with a constant scalar field, which was able to reproduce the Einstein-de Sitter cosmology (39), cannot mimic the Λ-CDM cosmology (43) because of the extra term on the right-hand side of the Friedmann equation [a constant scalar field implies $\xi = 0$ in Eq.(32)]. Therefore, to recover a Λ-CDM expansion rate we must consider more general solutions with a non-constant scalar field. In particular, even if the scale factors a_i of the gravitational metrics keep decelerating at late times, the baryonic scale factor $a = s_1 a_1 + s_2 a_2$ can accelerate at late times if s_1 or s_2 grows sufficiently fast. Then, the acceleration experienced by the baryonic metric is a dynamical effect due to the time dependent relationship between this metric and the two gravitational metrics.

On the other hand, at early times where data show that the dark energy density is negligible, we converge to the simple Einstein - de Sitter solution (40). This will be the common early time behavior of all the solutions that we consider in this paper. We can check that the integral in Eq.(38) is indeed finite and goes to zero for $a \to 0$, both in the radiation and dark matter eras, provided

$$\xi \to 0 \quad \text{for} \quad a \to 0. \tag{44}$$

This also ensures that the dark energy density is negligible as compared with the dark matter density. As we shall see below, the families of solutions that we build in this paper are parameterized by $\xi(a)$, which is treated as a free function of the model. Therefore, the condition (44) is easily satisfied, by choosing functions $\xi(a)$ that exhibit a fast decay at high redshift.

F. Solutions with common conformal time

To illustrate how we can build bi-metric solutions that follow a Λ-CDM expansion rate, we first consider solutions with

$$a_* = b_*, \tag{45}$$

that is, the conformal time τ is the same for all metrics. Then, at the background level each metric is defined by a single scale factor a_* and the two constraints in the first line in Eq.(25) reduce to one, $a = s_1 a_1 + s_2 a_2$. As all metrics are proportional, at the background level this scenario is similar to a single gravitational metric model, $\tilde{g}_{\mu\nu}$, where the baryonic and the dark matter metrics are given by different conformal rescalings, $g_{\mu\nu} = A^2(\varphi)\tilde{g}_{\mu\nu}$ and $g_{\mu\nu} = A_{\rm d}^2(\varphi)\tilde{g}_{\mu\nu}$.

1. Symmetric solution

We first consider a simple symmetric solution where we split the single constraint $a = s_1 a_1 + s_2 a_2$ into two symmetric constraints:

$$\frac{s_1 a_1}{a} = \left(s_1^{(0)}\right)^2, \quad \frac{s_2 a_2}{a} = \left(s_2^{(0)}\right)^2. \tag{46}$$

This is consistent with the initial conditions defined by the early time solution (40)-(41). Then, Eq.(46) gives $s_\ell(a)$ as an explicit function of $\{a, a_\ell(a)\}$, and we solve for the two sets $\{a_\ell(a), \omega_\ell(a), s_{{\rm d}\ell}(a)\}$. Thanks to the splitting (46), these two sets of variables can be solved independently. Then, the three functions $\{a_\ell(a), \omega_\ell(a), s_{{\rm d}\ell}(a)\}$ are determined by the two Friedmann equations (32)-(33) and the definition (31). The definition (31) provides $\mathcal{H}_\ell$ at each time step, hence $d\ln a_\ell/d\ln a$. The second Friedmann equation (33) gives $d\omega_\ell/d\ln a$. The first Friedmann equation (32) provides $s_{{\rm d}\ell}(a)$. The dark sector scale factor $a_{\rm d}$ is given by Eq.(25), $a_{\rm d}(a) = s_{\rm d1}a_1 + s_{\rm d2}a_2$. The scalar field energy density $\xi(a)$ is an arbitrary function, which is a free function of the bi-metric model. It must be positive and we only request that it vanishes at early times to recover the high-redshift cosmology (40).

This procedure provides a family of solutions that are parameterized by the initial coefficients $s_\ell^{(0)}$ and the scalar energy density $\xi(a)$, and which follow the Λ-CDM expansion history for $\mathcal{H}(a)$. The latter enters the dynamical equations through the factors $\mathcal{H}(a)$ in Eqs.(33) and (31) [when we write $d\ln a_\ell/d\tau = \mathcal{H}(d\ln a_\ell/d\ln a)$]. As the coefficients $s_\ell^{(0)}$ do not appear in these equations, the two metrics are actually equivalent, with

$$\frac{a_1}{a_2} = \frac{s_1}{s_2} = \frac{s_1^{(0)}}{s_2^{(0)}}, \quad \mathcal{H}_1 = \mathcal{H}_2. \tag{47}$$

We show in Figs. 1 and 2 the evolution with redshift of the main background quantities, in such a solution with $s_1^{(0)} = \sqrt{3}/2, s_2^{(0)} = 1/2$. The scalar field energy density $\xi(z)$ is chosen to vanish at high z and to remain much smaller than the dark matter energy density at all times, $\xi \ll 1$. More specifically, we use the simple form

$$\xi(a) \propto \frac{u^{3/2}}{1+u^{3/2}}, \quad u = \frac{\Omega_{\Lambda 0} a^4}{\Omega_{\gamma 0} + (\Omega_{\rm dm0} + \Omega_{\rm b0})a}. \tag{48}$$

From Eq.(42), the quantity $u(a)$ is a natural measure of the deviation of the Λ-CDM cosmological background from the Einstein-de Sitter background. It is also the ratio of the effective dark energy density to the matter and radiation energy densities and we have $\omega = \omega^{(0)}(1+u)$. In this paper, we write the free functions of the models in terms of powers of $u(a)$, to ensure that we recover the Einstein-de Sitter reference of Section III D at early times. This also means that the effects of the scalar field only appear at low redshifts, where the departure from the Einstein-de Sitter reference is associated with a running of the scalar field.

We can see in Fig. 1 that a_1/a and a_2/a decrease at low z while s_1 and s_2 increase. Indeed, because of the absence of a cosmological constant, the scale factors $a_i(\tau)$ of the

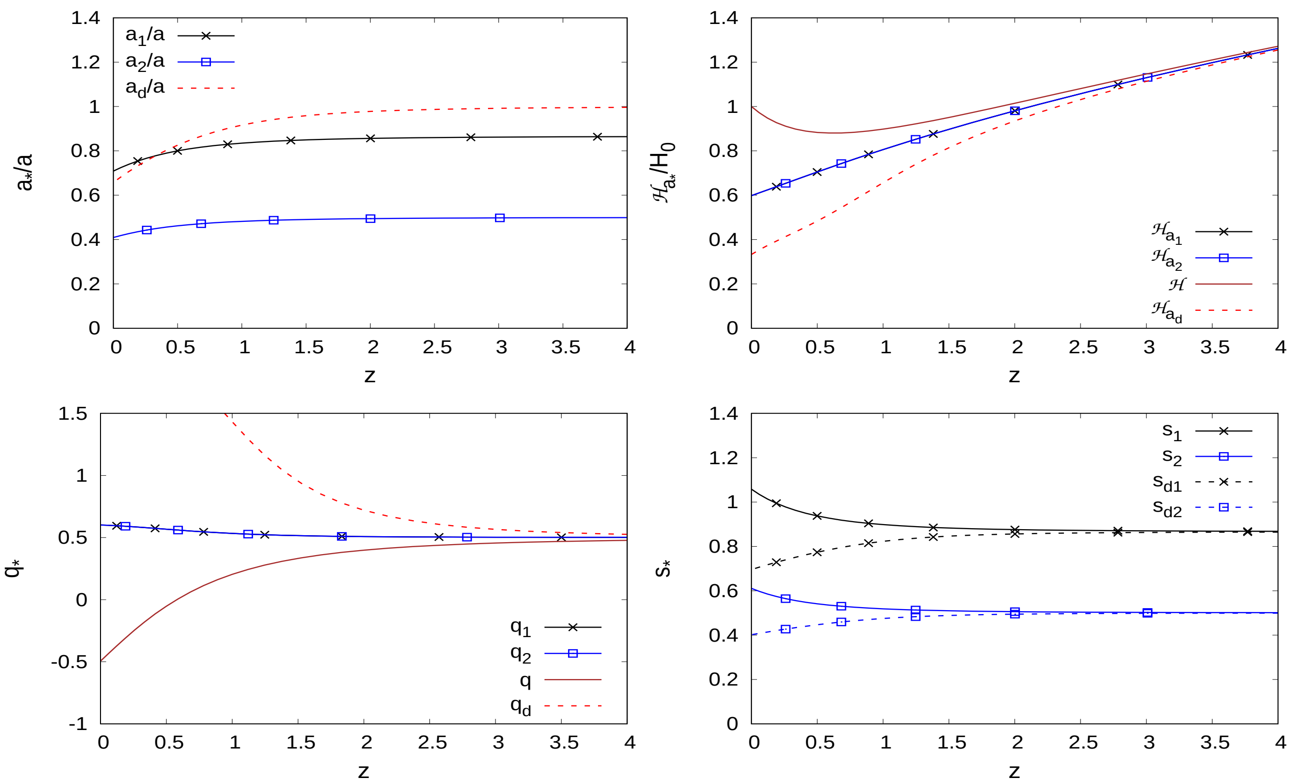

FIG. 1: Background quantities for a symmetric solution of the form (46), as a function of redshift. *Upper left panel:* ratio of the various scale factors a_* to the baryonic scale factor a. *Upper right panel:* the various expansion rates $\mathcal{H}_*$ normalized to H_0. *Lower left panel:* the various deceleration parameters q_*. *Lower right panel:* coefficients s_ℓ and $s_{\mathrm{d}\ell}$ of Eqs.(8) and (25).

gravitational metric tend to follow an Einstein-de Sitter expansion rate, which falls below the Λ-CDM expansion rate of $a(\tau)$. The latter manages to mimic the Λ-CDM history thanks to the late-time growth of the factors s_ℓ in Eq.(25). On the other hand, the dark factors $s_{\mathrm{d}\ell}$ decrease at low z, in a fashion that is opposite to the baryonic factors s_ℓ. This follows from the relationship (34), which gives

$$\xi \ll 1, \quad \Omega_{\gamma 0} \ll 1: \quad \frac{ds_{\mathrm{d}\ell}}{d\ln a} \simeq -\frac{\Omega_{\mathrm{b}0}}{\Omega_{\mathrm{dm}0}} \frac{ds_\ell}{d\ln a} \quad \text{for} \quad a \sim 1. \tag{49}$$

Then, from Eq.(25) the dark sector scale factor $a_{\mathrm{d}}(\tau)$ grows even more slowly than the gravitational scale factors $a_\ell(\tau)$ at late times, and we have $\mathcal{H}_{a_{\mathrm{d}}} < \mathcal{H}_{a_\ell} < \mathcal{H}$.

These different cosmic evolutions are clearly shown by the deceleration factors q_*, defined for each metric with respect to its cosmic time $dt_* = b_* d\tau$ by

$$q_* = -\frac{\ddot{a}_* a_*}{\dot{a}_*^2} = -\frac{d^2 a_*}{dt_*^2} a_* \left(\frac{da_*}{dt_*}\right)^{-2} . \tag{50}$$

Thus, we can see that the gravitational metrics g_1 and g_2 show no acceleration. They keep behaving like an Einstein de Sitter cosmology, except for a slightly stronger deceleration at low z. Only the baryonic metric shows an accelerated expansion with $q < 0$. Because of the opposite behavior of the dark sector coefficients $s_{\mathrm{d}\ell}$, as compared with the baryonic coefficients s_ℓ, the dark sector metric shows instead a stronger deceleration at late times than the Einstein de Sitter cosmology. This clearly shows that the apparent acceleration of the baryonic metric is not due to a dark energy component, associated for instance with the scalar field φ, as the "Einstein-frame" metrics g_1 and g_2 do not accelerate. It is only due to the time-dependent mapping (25) between these metrics and the baryonic metric. Therefore, this provides a "self-accelerated model", in the sense that the acceleration is not due to a hidden cosmological constant (e.g., the non-zero minimum of some potential, or a dark energy fluid with negligible kinetic energy).

As we wish to mimic a Λ-CDM cosmology, with $\Omega_{\Lambda 0} \simeq 0.7$, the deviations from the Einstein-de Sitter cosmology are of order unity at low z. This implies that the deviation of the coefficients s_ℓ and $s_{\mathrm{d}\ell}$ from their initial value is also of order unity at low z, while from Eq.(38) we have $\bar{\varphi} \sim M_{\mathrm{Pl}} \sqrt{\xi}$,

$$z = 0: \quad s_\ell - s_\ell^{(0)} \sim 1, \quad s_{\mathrm{d}\ell} - s_\ell^{(0)} \sim 1, \quad \frac{\bar{\varphi}}{M_{\mathrm{Pl}}} \sim \sqrt{\xi}. \tag{51}$$

As explained below, after Eq.(68), we cannot take ξ too small as this would give rise to a large fifth force. On the other hand, we wish to keep the scalar field

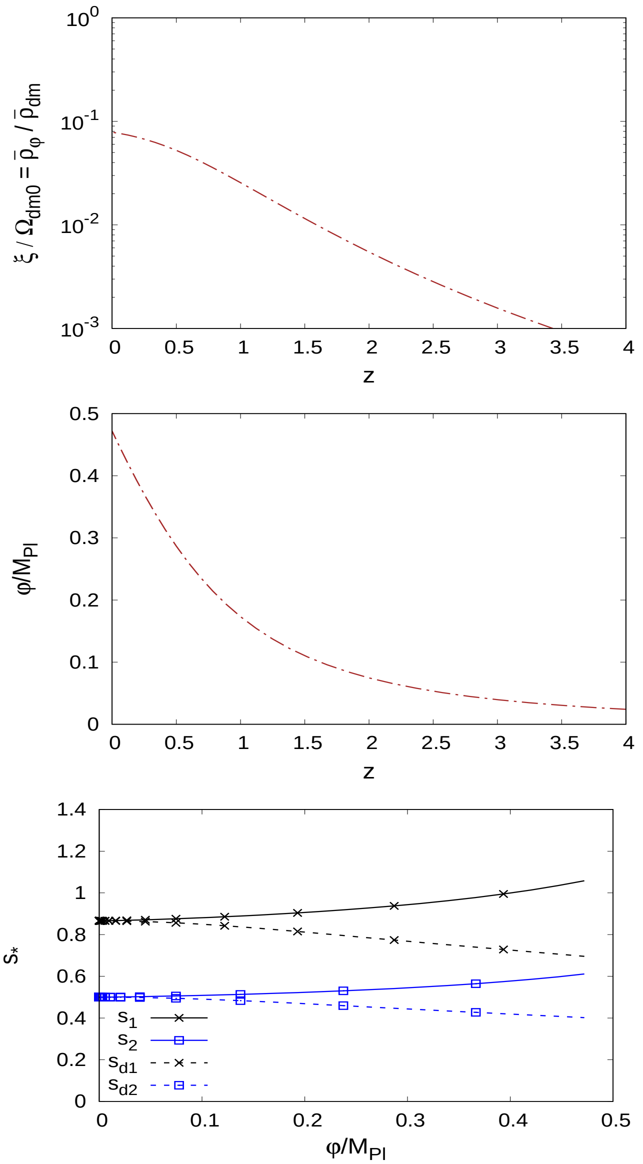

FIG. 2: Background quantities for a symmetric solution of the form (46). *Upper panel:* ratio of the scalar field energy density to the dark matter energy density. *Middle panel:* value of the scalar field in Planck mass unit. *Lower panel:* coefficients s_ℓ and $s_{\mathrm{d}\ell}$ as a function of the scalar field.

energy density to be subdominant. We choose for all the solutions that we consider in this paper the same scalar field energy density, shown in the upper panel in Fig. 2. It is of order $\Omega_{\mathrm{dm}0}/10$ at $z=0$ and decreases at higher z. The $u^{3/2}$ falloff of $\xi(a)$ is fast enough to make the scalar field subdominant and to converge to the Einstein - de Sitter solution (40). It is also slow enough to enforce $ds_*/d\varphi \to 0$, as we have $ds_*/d\varphi = (ds_*/d\ln a)/(d\varphi/d\ln a) \sim u\mathcal{H}\sqrt{a/\xi}$. This yields vierbein coefficients $s_\ell(\varphi)$ that look somewhat more natural than functions with a divergent slope at the origin. We can see in the lower panel that the functions $s_*(\varphi)$ built by this procedure have simple shapes and do not develop fine-tuned features. The model chosen for $\xi(a)$ gives scalar field excursions of about $M_{\mathrm{Pl}}/2$ at $z=0$.

2. *Non-symmetric solution*

We can also build non-symmetric solutions, which do not obey Eq.(46). Instead of splitting the constraint $a = s_1 a_1 + s_2 a_2$ into the two conditions (46), we can add another condition, such as requiring the ratio s_2/s_1 to follow an arbitrary function of time $\kappa(a)$. Then, the function $\kappa(a)$ parameterizes this extended family of solutions. The symmetric solution of section III F 1 corresponds to the particular case $\kappa(a) = s_2^{(0)}/s_1^{(0)}$. Because the effective Newton constant is given by $s_1^2+s_2^2$, in units of $\mathcal{G}_{\mathrm{N}} = 1/8\pi M_{\mathrm{Pl}}^2$, as we shall see in Eq.(78) below, we choose instead to parameterize the solutions by the sum $s_1^2+s_2^2$, as a function of redshift. Thus, we solve the system

$$s_1 a_1 + s_2 a_2 = a, \quad s_1^2 + s_2^2 = \lambda(a), \tag{52}$$

where $\lambda(a)$ is a new arbitrary function that parameterizes this extended family of solutions. These two equations now provide $\{s_1, s_2\}$ as a function of $\{a, a_1, a_2\}$,

$$\begin{aligned} s_1 &= \frac{a a_1 + \epsilon a_2 \sqrt{\lambda(a_1^2+a_2^2) - a^2}}{a_1^2+a_2^2}, \\ s_2 &= \frac{a a_2 - \epsilon a_1 \sqrt{\lambda(a_1^2+a_2^2) - a^2}}{a_1^2+a_2^2}, \end{aligned} \tag{53}$$

where $\epsilon = \pm 1$. Then, we can again solve for the two sets $\{a_\ell(a), \omega_\ell(a), s_{\mathrm{d}\ell}(a)\}$ from Eqs.(31), (32), and (33), the only difference being that these two sets of variables are now coupled.

We show in Fig. 3 the evolution of the background quantities for a solution of the form (52), where s_2/s_1 is no longer constant and we impose that $d\lambda/da = 0$ at $z=0$. Despite this difference, the scale factors and the Hubble expansion rates are very close to those of Fig. 1. This is because, at late times after the radiation to matter transition, $a \gg a_{\mathrm{eq}}$, and for $\xi \ll 1$, the second Friedmann equation (33) reduces to

$$\frac{d\omega_\ell}{d\ln a} \sim -\frac{\Omega_{\gamma 0}}{a} - \xi, \quad \text{hence} \quad \left| \frac{d\omega_\ell}{d\ln a} \right| \ll 1. \tag{54}$$

Since the dark energy era and the running of the scalar field occur much later than the radiation to matter transition, we can actually see from the first Friedmann equation Eq.(32) that we must have

$$\omega_\ell \simeq s_\ell \Omega_{\mathrm{b}0} + s_{\mathrm{d}\ell}\Omega_{\mathrm{dm}0} \simeq s_\ell^{(0)}\Omega_{\mathrm{b}0} + s_\ell^{(0)}\Omega_{\mathrm{dm}0}. \tag{55}$$

Thus, we recover the relationship (49) and we also find that for the general class of solutions with a common conformal time the quantities ω_ℓ are set by the initial conditions and show a negligible dependence on the late-time

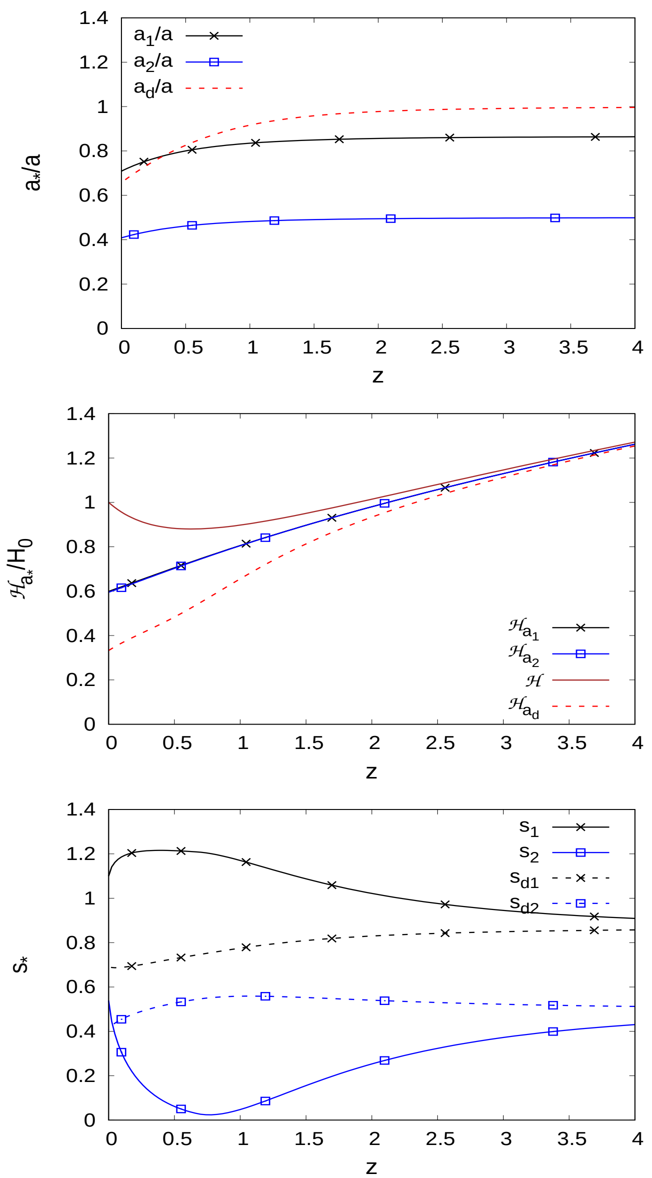

FIG. 3: Background quantities for a solution of the form (52), where s_2/s_1 is not constant.

evolution of the coefficients s_ℓ and $s_{\mathrm{d}\ell}$ and on the scalar field (as long as it remains subdominant). This explains why we recover almost the same evolution for the scale factors a_* and the Hubble expansion rates $\mathcal{H}_*$, which are determined by the definition (31). Then, the deceleration parameters q_* are also close to those obtained in Fig. 1. The change to the factors s_ℓ associated with different solutions is almost fully compensated by the change to the dark coefficients $s_{\mathrm{d}\ell}$ that is implied by the constraint of recovering a Λ-CDM expansion rate for the baryonic metric. By the same mechanism, we also find that in these solutions, despite the different behaviors of s_1 and s_2, the two gravitational metrics are mostly equivalent, with again the same expansion rates $\mathcal{H}_1 \simeq \mathcal{H}_2$ up to negligible deviations.

G. Solutions with different conformal times

We now turn to more general solutions, which still follow the Λ-CDM expansion rate for the baryonic metric, but where

$$a_\ell \neq b_\ell, \quad a_{\mathrm{d}} \neq b_{\mathrm{d}}. \tag{56}$$

Then, the conformal times τ_* of the various metrics are different. As the metrics are not proportional, already at the background level this scenario is different from models where the baryonic and the dark matter metrics are given by different conformal rescalings of a single Einstein-frame metric $\tilde{g}_{\mu\nu}$.

Defining the scale-factor ratios

$$r_\ell(a) = \frac{b_\ell}{a_\ell}, \quad r_{\mathrm{d}}(a) = \frac{b_{\mathrm{d}}}{a_{\mathrm{d}}}, \tag{57}$$

the two constraints in the first line of Eq.(25) read as

$$a = s_1 a_1 + s_2 a_2, \quad a = s_1 r_1 a_1 + s_2 r_2 a_2, \tag{58}$$

These two linear equations provide $\{s_\ell\}$ as a function of $\{a, a_\ell\}$,

$$s_1 = \frac{a(1-r_2)}{a_1(r_1-r_2)}, \quad s_2 = \frac{a(1-r_1)}{a_2(r_2-r_1)}, \tag{59}$$

when we are given the arbitrary free functions $r_\ell(a)$. As in the previous cases, $\{a_\ell, \omega_\ell, s_{\mathrm{d}\ell}\}$ are obtained from Eqs.(31) and (32)-(33), while a_{d} and b_{d} are obtained from the second line of Eq.(25).

We require $a_* > 0, b_* > 0, s_\ell > 0, s_{\mathrm{d}\ell} > 0$, to avoid singularities. This implies $s_1 s_2 > 0$ and Eq.(59) leads to

$$(1-r_1)(1-r_2) < 0, \tag{60}$$

and we can choose for instance

$$r_1 < 1 < r_2. \tag{61}$$

The recent detections of gravitational waves from a binary neutron star merger by the LIGO-VIRGO collaboration (GW170817) [25], with electromagnetic counterparts in gamma-ray burst [15] and in UV, optical and NIR bands [26], places very stringent limits on the speed of gravitational waves, $|c_g - 1| \leq 3 \times 10^{-15}$ [15]. For the bi-metric action (2), we have two massless gravitons associated with the two Einstein-Hilbert terms R_ℓ. Their equations of motion in the cosmological background are $\partial_\tau^2 h_{\ell ij} - r_\ell^2 \nabla^2 h_{\ell ij} = 0$ for high frequencies, with the propagation speeds $c_{g\ell} = r_\ell$, see Appendix A 2 and Eq.(88) below. Then, to explain the multi-messenger event GW170817, at least one of these two gravitons must propagate at the speed of light (up to an accuracy of 10^{-15}) in the local and recent Universe, $d \lesssim 40$

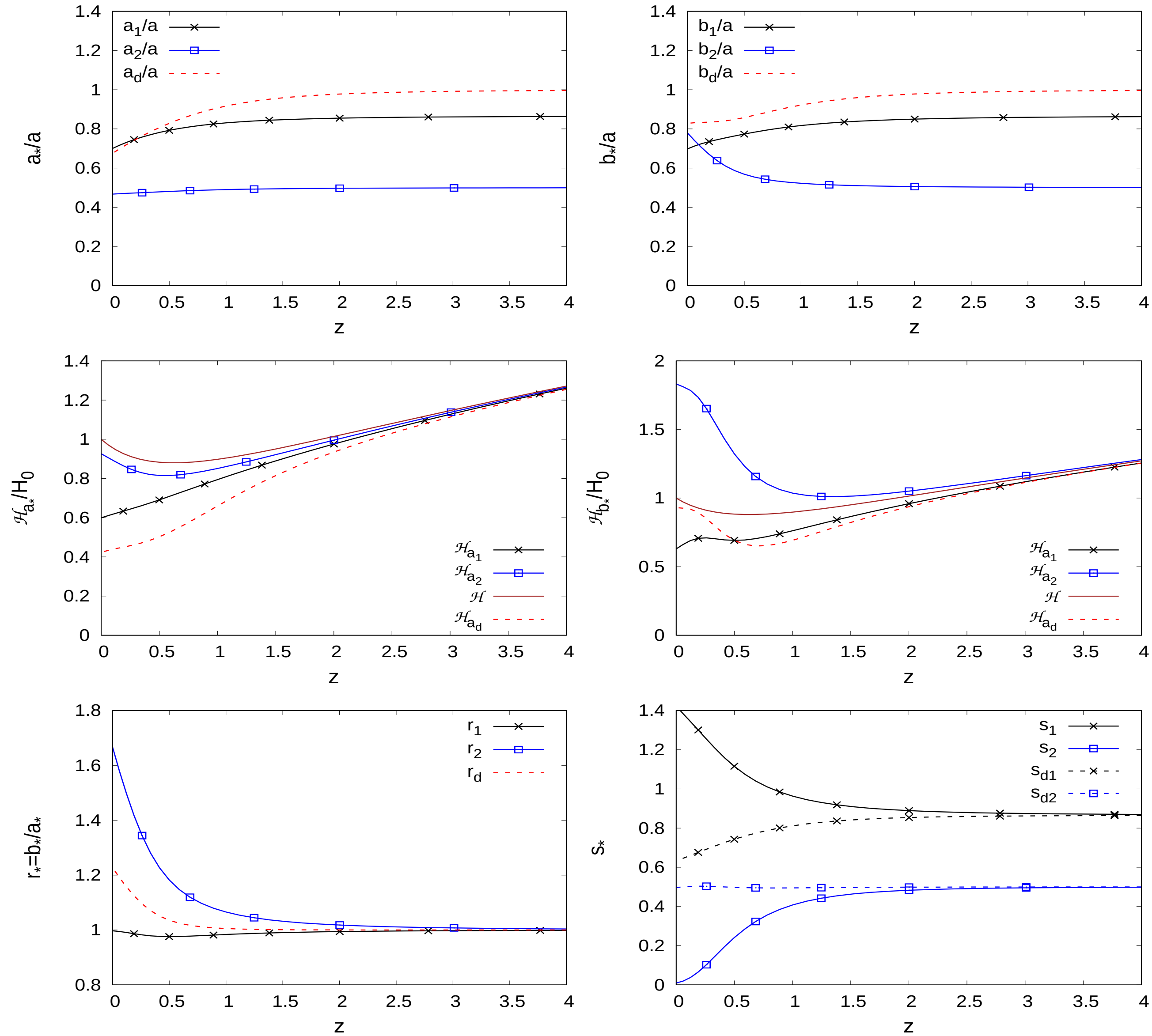

FIG. 4: Background quantities for a solution of the form (61), where the different metrics have different conformal times (i.e., are not proportional), but $r_1 \to 1$ at $z = 0$.

Mpc and $z \lesssim 0.01$. In principle, a non-linear screening mechanism might change the laws of gravity and ensure convergence to General Relativity in the local environment. However, it is unlikely that it would apply over 40 Mpc. Moreover, in most parts of the trajectory, between the host galaxy and the Milky Way, the local density is below or of the order of the cosmological background density. Besides, it would require a fine-tuned cancellation to make the average speed $c_g = 1$ over the full trajectory, inside the two galaxies and the low-density intergalactic medium. Then, at least one of the lapse factors r_ℓ must converge to unity at low z. If both coefficients r_ℓ go to unity, we converge to the solutions studied in section III F. For illustration, we consider in Fig. 4 the case where only one of the coefficients r_ℓ goes to unity at low redshift, for instance r_1 [with again the same initial conditions $\{s_1^{(0)}, s_2^{(0)}\}$ and scalar field energy density $\xi(a)$ as in Fig. 1]. In this limit, the system effectively reduces again to a single metric for the baryonic sector. Indeed, Eq.(59) implies that $s_2 \to 0$ if $r_1 \to 1$ (and $s_1 \to 0$ if $r_2 \to 1$). Then, the baryonic metric $g_{\mu\nu}$ becomes proportional to the metric $g_{1\mu\nu}$. However, the dark matter metric remains sensitive to both gravitational metrics g_1 and g_2, as s_{d2} remains non-zero, so that the baryon+dark matter system remains different from the common conformal time scenarios of section III F. In particular, the baryonic and dark matter metrics are not proportional, so that this scenario remains different from models where the baryonic and the dark matter metrics are given by different conformal rescalings of a single Einstein-frame metric $\tilde{g}_{\mu\nu}$.

We can see that in this scenario the scale factors a_* remain similar to those obtained in Fig. 1 for the symmetric solution (46). However, we can now distinguish the difference between the two expansion rates $\mathcal{H}_{a_1}$ and $\mathcal{H}_{a_2}$ at low z. The main difference with respect to the previous solutions is the behavior of the lapse functions b_ℓ. Thanks to the additional degrees of freedom r_ℓ, the lapses b_ℓ can behave in a significantly different way than the scale factors a_ℓ. In the example shown in Fig. 4, the two lapses even evolve in different directions and cross each other at $z \simeq 0.1$. This leads to rates that are significantly different with $\mathcal{H}_{b_2} > \mathcal{H}$. As explained in Section III F 1, because of the lack of cosmological constant, the gravitational expansion rates $\mathcal{H}_{a_\ell}$ are typically smaller than the Λ-CDM expansion rate $\mathcal{H}$. This remains true for the more general solution shown in Fig. 4. But the lapse functions are not so strongly constrained and it is possible to have one of them growing faster than a. For the choice (61) this corresponds to b_2, with $\mathcal{H}_{b_2} > \mathcal{H}$. This requires a ratio r_2 that significantly departs from unity at low z, as seen in the lower left panel.

The coefficients s_ℓ and $s_{\mathrm{d}\ell}$ follow similar behaviors to those obtained in Figs. 1 and 3, with opposite deviations at low redshift for the baryonic and dark sector coefficients. Because of the constraint $|r_1 - 1| < 3 \times 10^{-15}$ at $z = 0$, the coefficient s_2 almost goes to zero, with $s_2 \lesssim 10^{-15}$ at $z = 0$.

IV. LINEAR PERTURBATIONS

We have seen that it is possible to build several families of solutions that follow a Λ-CDM expansion history for the baryonic metric. In the case of metrics that are not proportional, the multi-messenger neutron star merger GW170817 also implies that at least one of the two gravitational metrics, g_1 and g_2, becomes proportional to the baryonic metric (i.e., $r_\ell = 1$) at low redshift.

We show below that these models are actually severely constrained by the behavior of perturbations. Here we consider the linear regime for the metric and density perturbations. The number of perturbative degrees of freedom in bigravity theories has been discussed in [11–13]. They obtained the behavior of scalar, vector and tensor modes by expanding the action up to quadratic order over the fluctuations. We present an alternative derivation in Appendix A, starting directly from the nonlinear Einstein equations (18) at the level of the vierbeins. This also allows us to explicitly implement the discussion of section II B and to show how the 32 degrees of freedom of the vierbeins can be reduced to the expected 16 degrees of freedom by successive gauge choices, associated with the diagonal Lorentz and diffeomorphism invariances and with the symmetry constraint (9).

In this section we focus on the scalar perturbations in the quasistatic approximation, which applies to the formation of large-scale structures. Then, the relevant metric perturbations are set by the four gravitational potentials $\{\phi_\ell, \psi_\ell\}$ as in the usual Newtonian gauge.

A. Scalar field perturbations

On small scales in the quasi-static approximation, the Klein-Gordon equation (20) becomes

$$\frac{1}{a_{\mathrm{d}}^2}\nabla^2\delta\varphi = m^2\delta\varphi + \frac{\beta_{\mathrm{dm}}}{M_{\mathrm{Pl}}}\delta\rho_{\mathrm{dm}} + \frac{\beta}{M_{\mathrm{Pl}}}\delta\rho, \tag{62}$$

with $\delta\varphi = \varphi - \bar{\varphi}$, $\delta\rho_{\mathrm{dm}} = \rho_{\mathrm{dm}} - \bar{\rho}_{\mathrm{dm}}$, and $\delta\rho = \rho - \bar{\rho}$. Here we assumed non-relativistic matter components, $p_{\mathrm{dm}} = p = 0$, and we neglected radiation fluctuations. As $\delta\rho_\varphi = \delta p_\varphi = b_{\mathrm{d}}^{-2}\frac{d\bar{\varphi}}{d\tau}\frac{\partial\delta\varphi}{\partial\tau}$, we also neglected the linear fluctuations of the scalar field density and pressure in the quasi-static limit. The scalar field mass around the cosmological background is

$$m^2 = (\bar{\rho}_{\mathrm{dm}} + \bar{\rho}_\varphi)\sum_\ell \frac{d^2 s_{\mathrm{d}\ell}}{d\varphi^2}\frac{b_\ell}{b_{\mathrm{d}}} - 3\bar{\rho}_\varphi \sum_\ell \frac{d^2 s_{\mathrm{d}\ell}}{d\varphi^2}\frac{a_\ell}{a_{\mathrm{d}}}$$
$$+(\bar{\rho} + \bar{\rho}_\gamma)\sum_\ell \frac{d^2 s_\ell}{d\varphi^2}\frac{a^3 b_\ell}{a_{\mathrm{d}}^3 b_{\mathrm{d}}} - \bar{\rho}_\gamma \sum_\ell \frac{d^2 s_\ell}{d\varphi^2}\frac{a^3 a_\ell}{a_{\mathrm{d}}^3 b_{\mathrm{d}}}. \tag{63}$$

Using the relation (34) it is possible to express the dark sector derivatives $d^2 s_{\mathrm{d}\ell}/d\varphi^2$ and $ds_{\mathrm{d}\ell}/d\varphi$ in terms of $d^2 s_\ell/d\varphi^2$ and $ds_\ell/d\varphi$. It is then possible to remove the second derivatives $d^2 s_\ell/d\varphi^2$ thanks to the symmetry in $\ell = 1, 2$, using the relations obtained by taking derivatives with respect to $\ln a$ of the constraints $a = s_1 b_1 + s_2 b_2$ and $a = s_1 a_1 + s_2 a_2$. The couplings to matter are

$$\beta = M_{\mathrm{Pl}}\sum_\ell \frac{ds_\ell}{d\varphi}\frac{a^3 b_\ell}{a_{\mathrm{d}}^3 b_{\mathrm{d}}}, \quad \beta_{\mathrm{dm}} = M_{\mathrm{Pl}}\sum_\ell \frac{ds_{\mathrm{d}\ell}}{d\varphi}\frac{b_\ell}{b_{\mathrm{d}}}. \tag{64}$$

In Fourier space, this yields

$$\frac{\delta\varphi}{M_{\mathrm{Pl}}} = -\frac{3H_0^2}{m^2 + k^2/a_{\mathrm{d}}^2}\left[\frac{\Omega_{\mathrm{dm0}}\beta_{\mathrm{dm}}\delta_{\mathrm{dm}}}{a_{\mathrm{d}}^3} + \frac{\Omega_{\mathrm{b0}}\beta\delta}{a^3}\right], \tag{65}$$

where $\delta_{\mathrm{dm}} = \delta\rho_{\mathrm{dm}}/\bar{\rho}_{\mathrm{dm}}$, $\delta = \delta\rho/\bar{\rho}$.

It is interesting to consider the scaling in ξ of the scalar field mass and couplings. From Eq.(51) we have the scalings

$$\frac{ds_*}{d\ln a} \sim 1, \quad \frac{ds_*}{d\varphi} \sim \frac{1}{M_{\mathrm{Pl}}\sqrt{\xi}}, \quad \frac{d^2 s_*}{d\varphi^2} \sim \frac{1}{M_{\mathrm{Pl}}^2\xi}. \tag{66}$$

Then, from Eq.(63) it seems that $m^2 \sim H_0^2/\xi$. However, using the relationship (34) one finds that the terms of order $1/\xi$ cancel out and we obtain

$$m^2 \sim H_0^2(\Omega_{\gamma 0} + \xi). \tag{67}$$

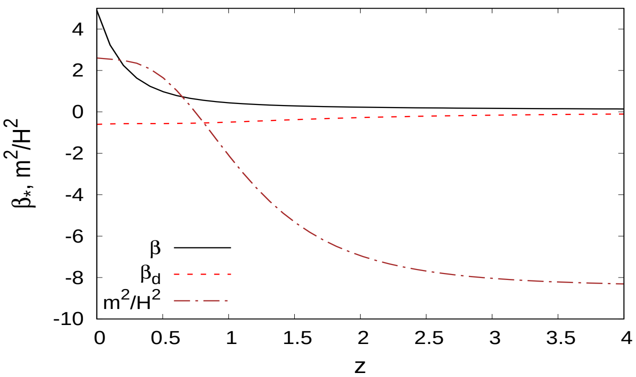

FIG. 5: Scalar field mass and couplings for the symmetric model of Fig. 1.

On the other hand, the couplings scale as

$$\beta \sim \frac{1}{\sqrt{\xi}}, \quad \beta_{\rm dm} \sim \frac{1}{\sqrt{\xi}}. \tag{68}$$

Therefore, very small values of the scalar field energy density ξ yield a very large fifth force. This implies that we cannot take ξ too small, which is why we choose $\xi \sim \Omega_{\rm dm0}/10$ at $z=0$ in the models that we consider in this paper. This feature comes from the fact that we require effects of order one from the scalar field onto the background at low redshift, $ds_*/d\ln a \sim 1$, to generate the apparent acceleration of the baryonic metric. This implies $ds_*/d\varphi \propto 1/\bar{\varphi}' \propto 1/\sqrt{\xi}$.

We show in Fig. 5 the scalar field mass and couplings for the symmetric model of Fig. 1. As expected from the expression (63), the squared mass evolves as $\bar{\rho}/M_{\rm Pl}^2 \sim H^2$ and it is of order H^2. This means that it is negligible on scales much below the horizon, where the quasi-static approximation (62) applies, and does not lead to small-scale instabilities, even when it is negative. The couplings β and $\beta_{\rm d}$ are of order unity and decrease at high z, because $ds_*/d\varphi \to 0$. This is because we choose the high-z decay of the scalar field energy density, determined by Eq.(48), to be slow enough so that $ds_*/d\varphi \to 0$ at early times. The baryonic and dark matter couplings have opposite signs, with $\beta > 0 > \beta_{\rm dm}$, because we typically have $ds_\ell/d\ln a > 0$ and $ds_{\rm d\ell}/d\ln a < 0$, as explained in Section III F and in agreement with Eq.(49).

The other solutions considered in Sections III F and III G give similar results to those found in Fig. 5.

B. Einstein equations

1. Gravitational potentials ϕ_ℓ and ψ_ℓ

As described in Appendix A, the scalar perturbations of the metrics, or of the vierbeins, can be reduced to the six degrees of freedom $\{\phi_\ell, \psi_\ell, V_1, U_1\}$, and $W_1 = -(r_2/r_1)V_1$, in the general forms (A6) and (A7). This choice of gauge follows from the invariances over diagonal Lorentz transforms and diffeomorphisms, together with the symmetry constraint (9). In the quasistatic approximation only the four gravitational potentials $\{\phi_\ell, \psi_\ell\}$ remain. Then, the perturbed metrics take the usual Newtonian gauge form

$$g_{*00} = -b_*^2(1+2\phi_*), \quad g_{*ii} = a_*^2(1-2\psi_*), \tag{69}$$

while the vierbeins are diagonal with

$$e_{*0}^0 = b_*(1+\phi_*), \quad e_{*i}^i = a_*(1-\psi_*). \tag{70}$$

For non-relativistic matter components, the Einstein equations (18) give in Fourier space (see Appendix A 4)

$$\begin{aligned} -\frac{2}{3} a_\ell \frac{k^2}{H_0^2} \psi_\ell \;&=\; \left(1+\gamma^{\psi_\ell}\right) s_\ell \Omega_{\rm b0} \delta \\ &\quad + \left(1+\gamma_{\rm dm}^{\psi_\ell}\right) s_{\rm d\ell} \Omega_{\rm dm0} \delta_{\rm dm} \end{aligned} \tag{71}$$

for the $(0,0)$ component, and

$$\begin{aligned} -a_\ell \frac{k_i k_j - \delta_{ij} k^2}{H_0^2} (\phi_\ell - \psi_\ell) &= \gamma^{\phi_\ell} s_\ell \Omega_{\rm b0} \delta \\ &\quad + \gamma_{\rm dm}^{\phi_\ell} s_{\rm d\ell} \Omega_{\rm dm0} \delta_{\rm dm} \end{aligned} \tag{72}$$

for the (i,j) components. The coefficients γ_*^* arise from the fluctuations of the scalar field φ, which generate fluctuations δs_* of the vierbein coefficients s_* that relate the matter and gravitational metrics. They are given by

$$\begin{aligned} \gamma^{\psi_\ell} &= -\frac{\mathcal{H}}{H_0 r_{\rm d} a^3 s_\ell} \sqrt{\frac{3a_{\rm d}}{2\xi}} \frac{\beta H_0^2}{m^2 + k^2/a_{\rm d}^2} \Upsilon_{\psi_\ell}, \\ \gamma_{\rm dm}^{\psi_\ell} &= -\frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}^3 s_{\rm d\ell}} \sqrt{\frac{3a_{\rm d}}{2\xi}} \frac{\beta_{\rm dm} H_0^2}{m^2 + k^2/a_{\rm d}^2} \Upsilon_{\psi_\ell}, \\ \gamma^{\phi_\ell} &= \frac{\mathcal{H}}{H_0 r_{\rm d} a^3 r_\ell s_\ell} \sqrt{\frac{3a_{\rm d}}{2\xi}} \frac{\beta H_0^2}{m^2 + k^2/a_{\rm d}^2} \Upsilon_{\phi_\ell}, \\ \gamma_{\rm dm}^{\phi_\ell} &= \frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}^3 r_\ell s_{\rm d\ell}} \sqrt{\frac{3a_{\rm d}}{2\xi}} \frac{\beta_{\rm dm} H_0^2}{m^2 + k^2/a_{\rm d}^2} \Upsilon_{\phi_\ell}, \end{aligned} \tag{73}$$

where the factors Υ_{ψ_ℓ} and Υ_{ϕ_ℓ} are given in Eqs.(A31) and (A33). The contribution from the fifth force to the gravitational potentials ψ_ℓ and ϕ_ℓ is negligible if the coefficients γ_*^* are much smaller than unity. Then, we recover Einstein equations for these gravitational potentials that are close to their standard form,

$$|\gamma_*^*| \ll 1: \quad \begin{array}{l} \phi_\ell \simeq \psi_\ell \\ -\frac{2}{3} a_\ell \frac{k^2}{H_0^2} \psi_\ell \simeq s_\ell \Omega_{\rm b0} \delta + s_{\rm d\ell} \Omega_{\rm dm0} \delta_{\rm dm} \end{array} \tag{74}$$

We show in Fig. 6 the coefficients γ_*^* for the symmetric solution of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$. At $z=0$, we expect from Eqs.(73) that $|\gamma_*^*| \simeq (H_0/k)^2$ on small scales. Indeed, we can see in the figure that for $k = 10\mathcal{H}$ we have $|\gamma_*^*| \lesssim 10^{-2}$. Moreover,

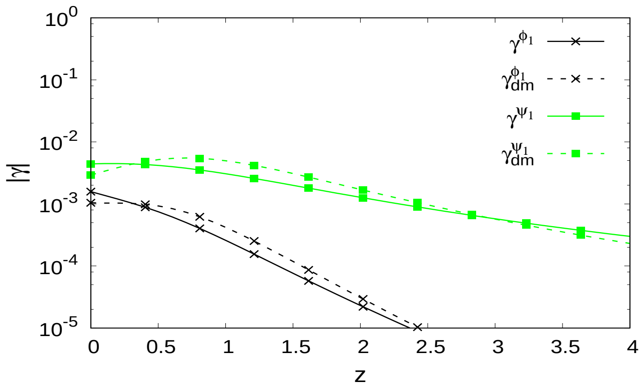

FIG. 6: Absolute value of the coefficients γ^*_* for the symmetric model of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$.

the amplitude shows a fast decrease at higher z. Therefore, on subhorizon scales the coefficients γ^*_* are much smaller than unity at all redshifts and we can always use the approximations (74).

The other solutions considered in Sections III F and III G give similar results to those found in Fig. 6.

2. Baryonic gravitational potentials ϕ and ψ

In the following, we assume that the properties (74) are satisfied. However, this is not sufficient to remove the fifth force because the dynamics of dark matter and baryons are set by their own metric potentials $\phi_{\rm d}$ and ϕ. Their relationship with the potentials ϕ_ℓ involves the scalar field and will give rise to a fifth force. Indeed, from the vierbeins (70) and their relations (8), we obtain at linear order

$$\begin{aligned} a\phi &= \sum_\ell b_\ell \left(s_\ell \phi_\ell + \delta s_\ell\right), \\ a\psi = &= \sum_\ell a_\ell \left(s_\ell \psi_\ell - \delta s_\ell\right). \end{aligned} \tag{75}$$

As for the gravitational potentials ϕ_ℓ and ψ_ℓ, the fluctuations of the coefficients s_ℓ and $s_{{\rm d}\ell}$, due to the perturbations of the scalar field $\delta\varphi$, give rise to non-standard terms. Using Eq.(74), we obtain

$$\begin{aligned} -\frac{2}{3} a \frac{k^2}{H_0^2} \phi &= \mu^\phi \Omega_{\rm b0} \delta + \mu^\phi_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm}, \\ -\frac{2}{3} a \frac{k^2}{H_0^2} \psi &= \mu^\psi \Omega_{\rm b0} \delta + \mu^\psi_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm}, \end{aligned} \tag{76}$$

with

$$\begin{aligned} \mu^\phi &= \sum_\ell \left[s_\ell^2 r_\ell + \frac{\mathcal{H} a_{\rm d}^2}{H_0 r_{\rm d} a^3} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_\ell}{d\ln a} b_\ell \right] \\ \mu^\phi_{\rm dm} &= \sum_\ell \left[s_\ell s_{{\rm d}\ell} r_\ell + \frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta_{\rm dm} k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_\ell}{d\ln a} b_\ell \right] \\ \mu^\psi &= \sum_\ell \left[s_\ell^2 - \frac{\mathcal{H} a_{\rm d}^2}{H_0 r_{\rm d} a^3} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_\ell}{d\ln a} a_\ell \right] \\ \mu^\psi_{\rm dm} &= \sum_\ell \left[s_\ell s_{{\rm d}\ell} - \frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta_{\rm dm} k^2}{k^2 + a_{\rm d}^2 m^2} \frac{ds_\ell}{d\ln a} a_\ell \right]. \end{aligned} \tag{77}$$

We recover the standard Poisson equations if $\mu^*_* = 1$.

We can split the coefficients μ^*_* into two parts. The first term, of the form $s_\ell^2 r_\ell$, is similar to a scale-independent renormalized Newton's constant and arises from the coefficients s_ℓ that relate the various metric potentials. The second part, of the form $ds_\ell/d\ln a$, arises from the fluctuations of the scalar field through δs_ℓ, and corresponds to a fifth force. It is scale dependent. Thus, we may define the renormalized Newton's constants (in units of the natural Newton's constant, $\mathcal{G}_{\rm N} = 1/8\pi M_{\rm Pl}^2$),

$$\begin{aligned} &\mathcal{G}^\phi = \sum_\ell s_\ell^2 r_\ell, \quad \mathcal{G}^\phi_{\rm dm} = \sum_\ell s_\ell s_{{\rm d}\ell} r_\ell, \quad \mathcal{G}^\psi = \sum_\ell s_\ell^2, \\ &\mathcal{G}^\psi_{\rm dm} = \sum_\ell s_\ell s_{{\rm d}\ell}, \end{aligned} \tag{78}$$

which are all positive.

The two baryonic metric potentials ϕ and ψ are generically different. First, if $r_\ell \neq 1$ the associated effective Newton's constants $\mathcal{G}^\phi$ and $\mathcal{G}^\psi$ are different. Second, the fifth-force contributions that enter ϕ and ψ have the same amplitude but opposite signs.

We show in Fig. 7 the coefficients μ^ϕ_* and μ^ψ_* for the symmetric solution of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$, as well as the effective Newton constants. At early times, when the scalar field has no effect and we converge to the Einstein-de Sitter cosmology, we recover General Relativity with $\mu^*_* \to 1$ and $\mathcal{G}^*_* \to 1$. At late times these coefficients show deviations of order unity. In this regime, the comparison of the two panels shows that the coefficients μ^*_* are dominated by the fifth-force contributions. This means that the fifth force is greater than Newtonian gravity. Moreover, the coefficients $\mu^\phi_{\rm dm}$ and μ^ψ become negative, which would give rise to very non-standard behaviors. Thus, the dark matter overdensities repel the baryonic matter at late times.

3. Dark matter gravitational potentials $\phi_{\rm d}$ and $\psi_{\rm d}$

In a similar fashion, the dark sector gravitational potentials $\phi_{\rm d}$ and $\psi_{\rm d}$ obey Poisson equations of the form

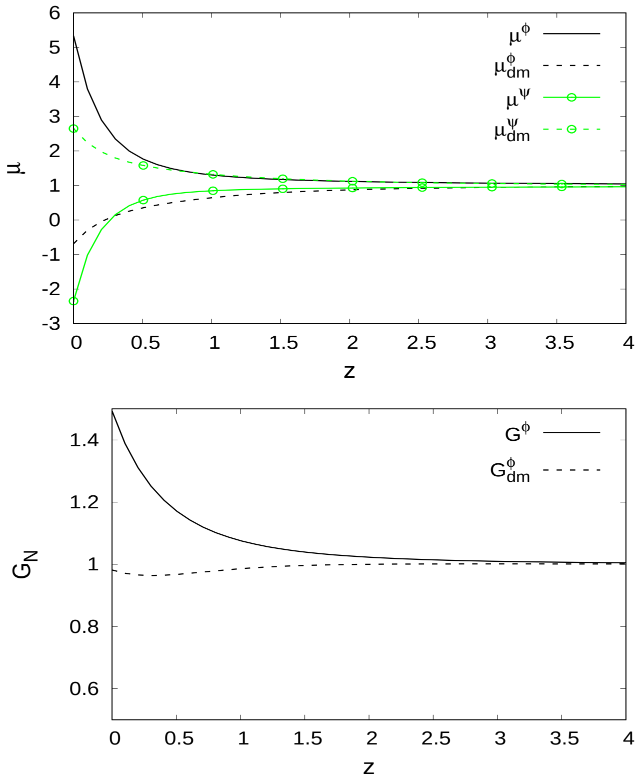

FIG. 7: *Upper panel:* coefficients μ_*^{ϕ} and μ_*^{ψ} for the symmetric model of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$. *Lower panel:* effective Newton constants $\mathcal{G}_*^{\phi}$. For this model, $\mathcal{G}_*^{\phi} = \mathcal{G}_*^{\psi}$.

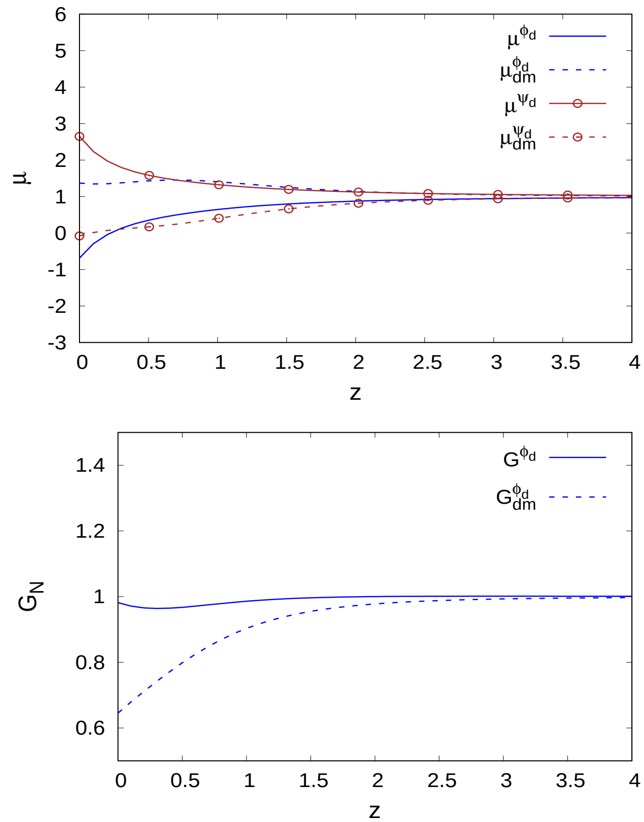

FIG. 8: *Upper panel:* coefficients $\mu_*^{\phi_{\rm d}}$ and $\mu_*^{\psi_{\rm d}}$ for the symmetric model of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$. *Lower panel:* effective Newton constants $\mathcal{G}_*^{\phi_{\rm d}}$ and $\mathcal{G}_*^{\psi_{\rm d}}$.

(76),

$$\begin{aligned} -\frac{2}{3} a_{\rm d} \frac{k^2}{H_0^2} \phi_{\rm d} &= \mu^{\phi_{\rm d}} \Omega_{\rm b0} \delta + \mu^{\phi_{\rm d}}_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm}, \\ -\frac{2}{3} a_{\rm d} \frac{k^2}{H_0^2} \psi_{\rm d} &= \mu^{\psi_{\rm d}} \Omega_{\rm b0} \delta + \mu^{\psi_{\rm d}}_{\rm dm} \Omega_{\rm dm0} \delta_{\rm dm}, \end{aligned} \tag{79}$$

with

$$\mu^{\phi_{\rm d}} = \sum_\ell \left[s_{{\rm d}\ell} s_\ell \frac{r_\ell}{r_{\rm d}} + \frac{\mathcal{H} a_{\rm d}^2}{H_0 r_{\rm d}^2 a^3} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta k^2}{k^2 + a_{\rm d}^2 m^2} \frac{d s_{{\rm d}\ell}}{d\ln a} b_\ell \right]$$

$$\mu^{\phi_{\rm d}}_{\rm dm} = \sum_\ell \left[s_{{\rm d}\ell}^2 \frac{r_\ell}{r_{\rm d}} + \frac{\mathcal{H}}{H_0 r_{\rm d}^2 a_{\rm d}} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta_{\rm dm} k^2}{k^2 + a_{\rm d}^2 m^2} \frac{d s_{{\rm d}\ell}}{d\ln a} b_\ell \right]$$

$$\mu^{\psi_{\rm d}} = \sum_\ell \left[s_{{\rm d}\ell} s_\ell - \frac{\mathcal{H} a_{\rm d}^2}{H_0 r_{\rm d} a^3} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta k^2}{k^2 + a_{\rm d}^2 m^2} \frac{d s_{{\rm d}\ell}}{d\ln a} a_\ell \right]$$

$$\mu^{\psi_{\rm d}}_{\rm dm} = \sum_\ell \left[s_{{\rm d}\ell}^2 - \frac{\mathcal{H}}{H_0 r_{\rm d} a_{\rm d}} \sqrt{\frac{2a_{\rm d}}{3\xi}} \frac{\beta_{\rm dm} k^2}{k^2 + a_{\rm d}^2 m^2} \frac{d s_{{\rm d}\ell}}{d\ln a} a_\ell \right]. \tag{80}$$

The renormalized Newton's constants are now

$$\begin{aligned} \mathcal{G}^{\phi_{\rm d}} &= \sum_\ell s_{{\rm d}\ell} s_\ell \frac{r_\ell}{r_{\rm d}}, \quad \mathcal{G}^{\phi_{\rm d}}_{\rm dm} = \sum_\ell s_{{\rm d}\ell}^2 \frac{r_\ell}{r_{\rm d}}, \\ \mathcal{G}^{\psi_{\rm d}} &= \sum_\ell s_{{\rm d}\ell} s_\ell, \quad \mathcal{G}^{\psi_{\rm d}}_{\rm dm} = \sum_\ell s_{{\rm d}\ell}^2, \end{aligned} \tag{81}$$

which are again positive. The comparison with Eq.(78) shows that the cross-terms are related by

$$\mathcal{G}^{\phi}_{\rm dm} = r_{\rm d} \mathcal{G}^{\phi_{\rm d}}, \quad \mathcal{G}^{\psi}_{\rm dm} = \mathcal{G}^{\psi_{\rm d}}. \tag{82}$$

We show in Fig. 8 the coefficients $\mu_*^{\phi_{\rm d}}$ and $\mu_*^{\psi_{\rm d}}$ for the symmetric solution of Fig. 1, at comoving wave number $k(z) = 10\mathcal{H}(z)$, as well as the effective Newton constants. We obtain behaviors that are similar to those found in Fig. 7 for the baryonic metric potentials. At late times the fifth force is again greater than Newtonian gravity and can lead to repulsive effects between baryons and dark matter.

C. Density and velocity fields

In their Jordan frame, associated with the metric $g_{\mu\nu}$, the baryons follow the usual equation of motion $\nabla_\mu T^\mu_\nu = 0$. This gives the standard continuity and Euler equations

$$\begin{aligned} &\frac{\partial \rho}{\partial \tau} + \nabla \cdot (\rho \mathbf{v}) + 3\mathcal{H}\rho = 0, \\ &\frac{\partial \mathbf{v}}{\partial \tau} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \mathcal{H}\mathbf{v} = -\nabla\phi. \end{aligned} \tag{83}$$

Using the Poisson equation (76), we obtain the evolution equation of the linear baryonic matter density contrast,

$$\frac{\partial^2\delta}{(\partial\ln a)^2} + \left[1+\frac{d\ln\mathcal{H}}{d\ln a}\right]\frac{\partial\delta}{\partial\ln a} = \frac{3H_0^2}{2a\mathcal{H}^2}\left[\mu^{\phi}\Omega_{\mathrm{b}0}\delta + \mu^{\phi}_{\mathrm{dm}}\Omega_{\mathrm{dm}0}\delta_{\mathrm{dm}}\right]. \quad (84)$$

The dark matter also follows its usual equation of motion, $\nabla_{\mathrm{d}\mu}T^{\mu}_{\nu}=0$, where $\nabla_{\mathrm{d}\mu}$ is now the covariant derivative associated with the dark sector metric $g_{\mathrm{d}\mu\nu}$. This gives the continuity and Euler equations

$$\frac{\partial\rho_{\mathrm{dm}}}{\partial\tau} + \nabla\cdot(\rho_{\mathrm{dm}}\mathbf{v}_{\mathrm{dm}}) + 3\mathcal{H}_{a_{\mathrm{dm}}}\rho_{\mathrm{dm}} = 0,$$
$$\frac{\partial\mathbf{v}_{\mathrm{dm}}}{\partial\tau} + (\mathbf{v}_{\mathrm{dm}}\cdot\nabla)\mathbf{v}_{\mathrm{dm}} + (2\mathcal{H}_{a_{\mathrm{d}}} - \mathcal{H}_{b_{\mathrm{d}}})\mathbf{v}_{\mathrm{dm}} = -r_{\mathrm{d}}^2\nabla\phi_{\mathrm{d}}, \quad (85)$$

where τ is still the conformal time of the baryonic metric. Using the Poisson equation, the evolution equation of the linear dark matter density contrast reads as

$$\frac{\partial^2\delta_{\mathrm{dm}}}{(\partial\ln a)^2} + \left[\frac{2\mathcal{H}_{a_{\mathrm{d}}}-\mathcal{H}_{b_{\mathrm{d}}}}{\mathcal{H}} + \frac{d\ln\mathcal{H}}{d\ln a}\right]\frac{\partial\delta_{\mathrm{dm}}}{\partial\ln a} = \frac{3r_{\mathrm{d}}^2H_0^2}{2a_{\mathrm{d}}\mathcal{H}^2} \times\left[\mu^{\phi_{\mathrm{d}}}\Omega_{\mathrm{b}0}\delta + \mu^{\phi_{\mathrm{d}}}_{\mathrm{dm}}\Omega_{\mathrm{dm}0}\delta_{\mathrm{dm}}\right]. \quad (86)$$

The baryonic and dark matter linear growing modes are coupled and given by the system of equations (84) and (86). We show in Fig. 9 their behavior as a function of redshift for the comoving wave number $k=0.1h/\mathrm{Mpc}$. At high redshift they follow the Λ-CDM reference, but at low redshift the dark matter perturbations grow faster than in the Λ-CDM cosmology whereas the baryonic perturbations grow more slowly. This is more clearly seen in the lower panel, as the growth rate $f_*=d\ln D_*^+/d\ln a$ amplifies the deviations from the Λ-CDM cosmology because of the time derivative.

The data points in Fig. 9 are only given to compare the magnitude of the deviation of the growth factor with observational error bars, but do not provide a meaningful test. Indeed, the Newton constant obtained in this scenario is amplified at $z=0$, as seen in Fig.7. This means that to compare with data we would need to run again this model by normalizing Newton's constant to its value at $z=0$ instead of $z\to\infty$, as we have done so far. We do not go further in this direction in this paper, because this model is already ruled out by the large time derivative $d\ln\mathcal{G}/dt\sim 0.7H_0$ at $z=0$, as we discuss in the next section.

Nevertheless, it is interesting to note that this model leads to a slower growth for the baryonic density perturbations than in the Λ-CDM cosmology. This is due to the decrease of the gravitational attraction of dark matter onto baryonic matter, shown by the coefficient μ^{ϕ}_{dm} in Fig. 7, which even turns negative at $z\lesssim 0.1$ (i.e., the fifth force between dark matter and baryons becomes repulsive). This is a distinctive feature of this model, as most modified-gravity scenarios amplify the growth of large-scale structures.

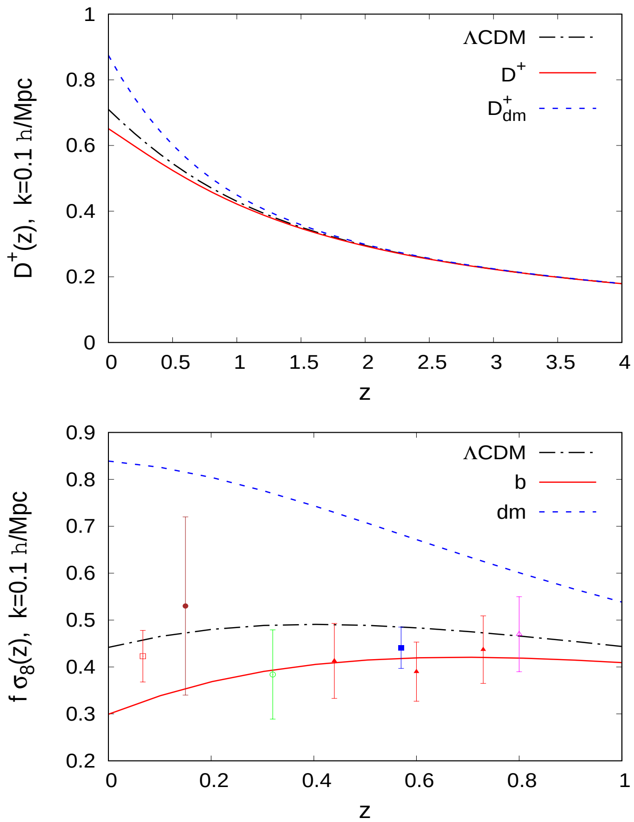

FIG. 9: *Upper panel:* linear growing modes $D^+(k,a)$ and $D^+_{\mathrm{dm}}(k,a)$, for the symmetric model of Fig. 1, at comoving wave number $k=0.1h/\mathrm{Mpc}$. *Lower panel:* growth factors $f\sigma_8$ and $f_{\mathrm{dm}}\sigma_{\mathrm{dm}8}$.

We show in Fig. 10 the growth factors obtained for the case (61) of Fig. 4, where the different metrics have different conformal times. This actually gives similar results for the linear growth of large-scale structures.

D. Gravitational slip

Because the fifth force enters with opposite signs in the ϕ and ψ gravitational potentials, see Eq.(77), the lensing potential $\phi_{\mathrm{lens}}=(\phi+\psi)/2$, which deflects light rays, and the dynamical potential ϕ, which determines the trajectory of massive bodies, are different. This means that the lensing mass of clusters of galaxies (deduced from lensing observations) and the dynamical mass (deduced from the galaxy velocity dispersion or the pressure profile of the hot gas in hydrostatic equilibrium) are also different. This is measured by the ratio η, which we define as

$$\eta = \frac{\phi+\psi}{2\phi} = \frac{1}{2} + \frac{\mu^{\psi}\Omega_{\mathrm{b}0}\delta + \mu^{\psi}_{\mathrm{dm}}\Omega_{\mathrm{dm}0}\delta_{\mathrm{dm}}}{2[\mu^{\phi}\Omega_{\mathrm{b}0}\delta + \mu^{\phi}_{\mathrm{dm}}\Omega_{\mathrm{dm}0}\delta_{\mathrm{dm}}]}. \quad (87)$$

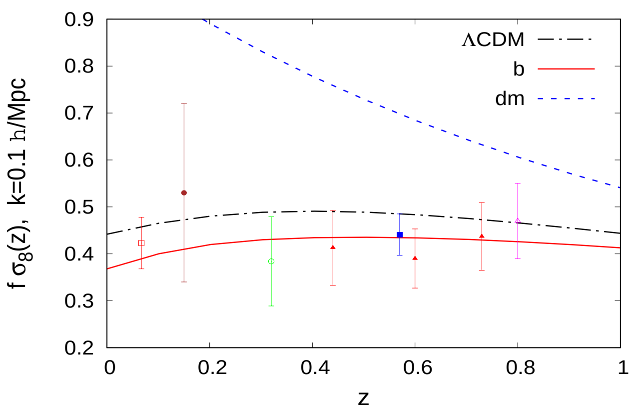

FIG. 10: Growth factors $f\sigma_8$ and $f_{\rm dm}\sigma_{\rm dm8}$, for the model of Fig. 4 with different conformal times.

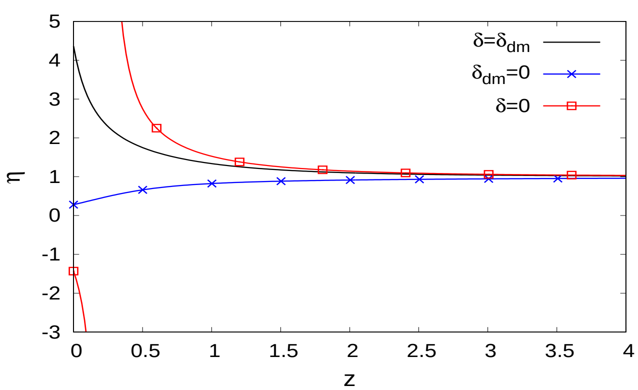

FIG. 11: Gravitational slip η of Eq.(87) for several values of the baryon to dark matter ratio $\delta/\delta_{\rm dm}$.

We show in Fig. 11 the gravitational slip η on sub-horizon scales, for $\delta = \delta_{\rm dm}$, $\delta_{\rm dm} = 0$ (which corresponds to cases where $\rho \gg \rho_{\rm dm}$), and $\delta = 0$ (for $\rho_{\rm dm} \gg \rho$). In agreement with Fig. 7, the three curves converge to the General Relativity value $\eta = 1$ at high redshift and show deviations of order unity at low z. Because the couplings to baryons and dark matter are different, the gravitational slip η depends on the relative amount of baryons and dark matter in the lens. On cosmological scales down to clusters of galaxies, which are the largest collapsed structures, we expect $\delta \simeq \delta_{\rm dm}$. This gives $\eta > 1$ at low z, hence the lensing mass would be greater than the dynamical mass. This ratio can reach a factor three at $z < 0.1$, but in practice, most cosmological lenses are at redshifts $z \gtrsim 0.5$, as the lensing efficiency goes to zero as the source redshift vanishes. This gives $1 < \eta \lesssim 1.7$. On the other hand, on subgalactic scales where baryons dominate, the gravitational slip is smaller than unity so that the lensing mass is smaller than the dynamical mass by a factor three at $z = 0$. In the case where dark matter dominates, η goes to infinity at $z \sim 0.3$ and becomes negative at lower redshift. This is because ϕ goes through zero and changes sign. This follows from $\mu_{\rm dm}^{\phi} < 0$, as seen in Fig. 7. This implies a repulsive fifth force from dark matter onto baryons, which dominates when the lens is mostly made of dark matter. This regime should not be reached in practice, as we have $\delta_{\rm dm} \sim \delta$ on large scales, where the separation of baryons from dark matter due to the fifth force has not yet had time to be efficient, as seen by the small impact on the linear growing modes in Fig. 9, whereas we typically have $\rho \gg \rho_{\rm dm}$ on subgalactic scales because radiative cooling processes make baryons collapse further and eventually form stars.

V. TENSOR AND VECTOR PERTURBATIONS

A. Tensor modes

We derive the tensor perturbations in Appendix A 2. Starting from the nonlinear Einstein equations (18) at the level of the vierbeins, we recover the results obtained from the quadratic action at the level of the metrics in [11–13] for a constant scalar field φ. After our choices of gauge, described in Appendix A 1, there remains four degrees of freedom in the tensor sector, associated with the two gravitons h_{1ij} and h_{2ij} of the gravitational metrics $g_{1\mu\nu}$ and $g_{2\mu\nu}$. The equation of motion of the first graviton h_{1ij} is given by

$$M_{\rm Pl}^2 \frac{a_1^2}{b_1} \left[h''_{1ij} + (3\mathcal{H}_{a_1} - \mathcal{H}_{b_1}) h'_{1ij} - \frac{b_1^2}{a_1^2} \nabla^2 h_{1ij} \right] - a_2 (\bar{p} s_1 s_2 a^2 + \bar{p}_\varphi s_{\rm d1} s_{\rm d2} b_{\rm d} a_{\rm d})(h_{1ij} - h_{2ij}) = 0, \quad (88)$$

and the equation of motion of the second graviton h_{2ij} is given by the permutation $1 \leftrightarrow 2$. Here we note $\bar{p}$ the total pressure of the baryonic sector fluids. In the radiation and matter eras, this is simply the radiation pressure, $\bar{p} = \bar{p}_\gamma = \bar{\rho}_\gamma/3$, while during the inflationary era it is the pressure $\bar{p}_\chi = -\bar{\rho}_\chi$ of the inflaton χ. We can see that the speed of the two gravitons is given by $c_{g\ell} = b_\ell/a_\ell$, which differs from the speed of light when $r_\ell \neq 1$. As compared with General Relativity, there is also an additional mass term that couples the two gravitons.

Until $z \sim 2$, before the dark energy era, the scalar field is almost constant and the propagation equations of the two gravitons simplify as Eq.(A14), where we omitted the indices ij. This system can be decoupled by introducing the graviton $h_{ij} = h_{{\rm d}ij}$ seen by the matter, which in this regime reads from Eqs.(A13) as

$$h_{ij} = h_{{\rm d}ij} = s_1^2 h_{1ij} + s_2^2 h_{2ij}, \quad (89)$$

and a second "hidden" graviton given by the combination

$$h_{-ij} = h_{1ij} - h_{2ij}. \quad (90)$$

This gives

$$h'' + 2\mathcal{H} h' - \nabla^2 h = 0, \quad (91)$$

$$h''_- + 2\mathcal{H} h'_- - \nabla^2 h_- - \frac{a^2 \bar{p}}{M_{\rm Pl}^2} h_- = 0. \quad (92)$$

Thus, we obtain a massless graviton h for the baryonic and dark matter metrics, which evolves as in General Relativity. On subhorizon scales it propagates with the speed of light. On scales greater than the horizon it contains a constant mode and a decaying mode that evolves as $h' \propto a^{-2}$. This physical mode (in the sense that it is the one seen by the matter metric) is governed by Eq.(91) throughout all cosmological eras and does not mix with the hidden graviton h_-.

The second hidden graviton h_- has a negative squared mass in the radiation era, as $\bar{p} = \bar{p}_\gamma > 0$, which becomes negligible in the matter era. In the radiation era, we have $\mathcal{H} = 1/\tau$ and $a = \sqrt{\Omega_{\gamma 0}} H_0 \tau$. The hidden massive graviton h_- obeys the equation of motion $h_-'' + \frac{2}{\tau} h_-' - \nabla^2 h_- - \frac{1}{\tau^2} h_- = 0$. It oscillates on subhorizon scales. On superhorizon scales it contains both a decaying mode and a growing mode

$$k \ll \mathcal{H}: \quad h_-^- \propto a^{-(1+\sqrt{5})/2}, \quad h_-^+ \propto a^{(\sqrt{5}-1)/2}, \qquad (93)$$

associated with the tachyonic instability. In the matter era, we have $\mathcal{H} = 2/\tau$ and $a \propto \tau^2$. The mass of the second graviton h_- becomes negligible and it behaves like the massless graviton, with a constant mode and a decaying mode $\propto a^{-3/2}$.

Although h_- is not seen by the matter, it should remain small at all epochs so that the perturbative approach applies. This implies that the initial tensor fluctuations at the onset of the radiation era must be sufficiently small. This is easily satisfied because the squared mass turns positive during the inflation era and the graviton decays [11]. During the inflationary stage, the tensor evolution equation is still given by Eq.(92), where $\bar{p}$ is now the pressure $\bar{p}_\chi = -\bar{\rho}_\chi$ of the inflaton χ. Because we now have $\bar{p}_\chi < 0$ the squared mass becomes positive and there is no tachyonic instability, and on superhorizon scales there are only two decaying modes

$$k \ll \mathcal{H}: \quad h_-^c \propto a^{-3/2} \cos\left(\frac{\sqrt{3}}{2} \ln a\right),$$
$$h_-^s \propto a^{-3/2} \sin\left(\frac{\sqrt{3}}{2} \ln a\right). \qquad (94)$$

Let us consider a mode k that remains above the horizon until the end of the radiation era, $k \leq a_{\rm eq} H_{\rm eq}$. It crosses the horizon during the inflationary stage at the time $a_k = k/H_I$, where H_I is the constant Hubble expansion rate of the inflationary de Sitter era. Then, the amplitude of the tensor mode h_- at the end of the radiation era reads as

$$h_-(a_{\rm eq}) = h_-(a_k) \left(\frac{a_f}{a_k}\right)^{-3/2} \left(\frac{a_{\rm eq}}{a_f}\right)^{(\sqrt{5}-1)/2}, \qquad (95)$$

where a_f is the scale factor at the end of the inflationary era. For $H_I \sim 10^{-5} M_{\rm Pl} \sim 10^{13}$GeV, $a_f \sim 10^{-28}$, $a_{\rm eq} \sim 10^{-3}$, we find that all modes with $k \leq a_{\rm eq} H_{\rm eq}$ remain in the perturbative regime, $h_-(a_{\rm eq}) \ll 1$, provided $h_-(a_k) \ll 10^{24}$. As we expect $h_-(a_k) \sim H_I/M_{\rm Pl} \sim 10^{-5}$, if the tensor fluctuations are generated by the quantum fluctuations, all modes remain far in the perturbative regime until the end of the radiation era. This is due to their decay during the inflationary stage on superhorizon scales, and to their small initial values associated with quantum fluctuations.

Therefore, the main constraint from the tensor sector is the measurement of the speed of gravitational waves from the binary neutron star merger GW170817 [25], which implies that at least one of the lapse factors r_ℓ is unity at $z = 0$, as discussed in section III G.

B. Vector modes

We derive the vector perturbations in Appendix A 3. We recover the results obtained in [11–13] for a constant scalar field φ. After our choices of gauge there remains six degrees of freedom in the vector sector, associated with the transverse vectors $\{C_{1i}, V_{1i}, V_{2i}\}$ of the two gravitational metrics. We also have the matter velocity fields v^i and $v^i_{\rm dm}$ in the baryonic and dark matter energy momentum tensors.

As in General Relativity, the gauge invariant combinations U^i and $U^i_{\rm dm}$ of the matter velocities and the $(0,i)$ vector component of the matter metrics, given by Eq.(A18), decouple. Thus, $U^i_{\rm dm}$ decays at all redshifts as a^{-1} whereas U^i is constant during the radiation era and decays as a^{-1} in the matter era.

From Eq.(A23) the vector mode $\mathcal{V}_i$ of the matter metrics decays as a^{-2} as in General Relativity. In addition, there is another "hidden" vector mode $\mathcal{V}_-$, associated with a different combination of the gravitational metric vectors, that shows a gradient instability in the radiation and matter eras. On subhorizon scales, it obeys the exponential growth (A28) in the radiation era and the power-law growth (A29) in the matter era.

Let us estimate the magnitude of this unstable vector mode at $z = 0$, for a wave number k that goes beyond the horizon at a_k during the inflationary stage and goes below the horizon at a'_k during the radiation era. Collecting the results of Appendix A 3, we obtain

$$k\mathcal{V}_- \sim k\mathcal{V}_-(a_k) \left(\frac{H_I}{H_{\rm eq}}\right)^{-3/2} \left(\frac{a_{\rm eq}}{a_f}\right)^{(\sqrt{5}+2)/2}$$
$$\times \left(\frac{k}{a_{\rm eq} H_{\rm eq}}\right)^{(4-\sqrt{5})/2} e^{\frac{k}{a_{\rm eq} H_{\rm eq}} \left(\frac{1}{\sqrt{5}} - \frac{\sqrt{2}}{3} \ln a_{\rm eq}\right)}. \qquad (96)$$

After horizon exit during the inflationary era, this mode first decays as $a^{-3/2}$ until the end of the inflationary era at a_f. Next, it grows as $a^{(\sqrt{5}-1)/2}$ during the radiation era, until it enters the horizon. Then, its subhorizon behavior deviates from the one of the tensor mode h_- as it shows the exponential growth (A28) until the matter era starts where it shows the power-law growth (A29). These last two stages give the exponential factor in Eq.(96), which is actually dominated by the matter era growth factor. If we assume that at horizon exit during the inflationary stage we have $C_1 \sim \mathcal{V}'_- \sim k\mathcal{V}_- \sim H_I/M_{\rm Pl}$, we obtain for $H_I \sim 10^{-5}M_{\rm Pl}$ that $k\mathcal{V}_- \ll 1$ at $z=0$ for $k \ll 0.3h{\rm Mpc}^{-1}$. Therefore, on weakly non-linear scales and below, the growth of the "hidden" vector jeopardises the perturbativity of the model and the gravitational metrics g_1 and g_2 become non-linear in this regime. This implies that the initial vector seeds at horizon exit during the inflationary era should be suppressed or that the scenario must be supplemented by additional mechanisms that damp the growth of this vector mode on small scales at high redshift.

VI. LINKS WITH DOUBLY COUPLED BIGRAVITY

The models that we have constructed have similarities with doubly coupled bigravity [11–13]. In doubly coupled bigravity there is no scalar field hence the Jordan frame vierbein couplings s_* are constant, such as

$$s_\ell = s_{{\rm d}\ell} = s_\ell^{(0)}, \tag{97}$$

with a universal coupling to all types of matter, i.e. baryons, CDM and radiation. In both the matter and radiation eras the scale factors are in the symmetric case

$$a_\ell = b_\ell, \quad \mathcal{H}_\ell = \mathcal{H}, \tag{98}$$

implying that the two metrics are proportional. The late time acceleration of the expansion of the Universe is obtained by adding a potential term

$$S_V = \Lambda^4 \int d^4x \sum_{ijk\ell} m^{ijk\ell} \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} e^a_{i\mu} e^b_{j\nu} e^c_{k\rho} e^d_{\ell\sigma} \tag{99}$$

comprising one scale and a completely symmetric tensor $m^{ijk\ell}$ which, up to rescaling, is associated to four coupling constants. This term is responsible for the late time acceleration where Λ^4 plays the role of the vacuum energy. Moreover, the potential term gives rise to a mass matrix for the gravitons whose order of magnitude corresponds to $\Lambda^4/M_{\rm Pl}^2 \sim H_0^2$, i.e very light gravitons.

At the background level, and as long as the scalar field is negligible, the bi-metric models considered here coincide with the bigravity theories. They differ when it comes to the phase of acceleration. In bigravity, this is simply realised as Λ^4 plays the role of dark energy. In scalar-bimetric models, there is no vacuum energy and the acceleration is simply due to the rapid variation of the scalar factors $s_*(\varphi)$, which imply that the baryonic and dark matter metrics do not mimic the ones of the Einstein-de Sitter space-time. In the acceleration phase in doubly coupled bigravity

$$r_2 \neq r_1, \tag{100}$$

that is, the two gravitational metrics do not have the same conformal time. For scalar bimetric models, we have seen that natural models obey $r_1 = r_2 = 1$ even at late times. In a similar fashion, in bigravity the consistency of the Friedmann equations gives a constraint equation that admits two branches of solutions [11–13], the interesting one for cosmology being $\mathcal{H}_{a_1}/r_1 = \mathcal{H}_{a_2}/r_2$ as noticed in Eq.(35). In our case, the scalar field provides an additional degree of freedom and there is no such constraint. As in General Relativity, the Friedmann equations and the equations of motion of the various fluids are automatically consistent. This follows from the fact that Eq.(34) is no longer a constraint equation, because of the scalar field dynamics. As we checked in section III C, the equation of motion of the scalar field is not independent of the Friedmann equations and of the equations of motion of the other fluids, as it can be derived from the latter.

When it comes to the scalar perturbations, bigravity in the doubly coupled case and scalar-bimetric models differ more drastically as the cosmological perturbations of the scalar field imply the existence of a scale, related to its effective mass, such that for large enough wavenumbers gravity is modified. This leads to a fifth force that is of order of the Newtonian force on cosmological scales at $z = 0$. Moreover, as the scalar field evolves in the late time Universe, the effective Newton constants (it is not unique anymore but depends on the species) drift with time. This has also an effect on cosmological perturbations.

Vector and tensor perturbations in the radiation era have similar behaviors in doubly coupled bigravity and scalar-bimetric models, with both tensor and vector instabilities. In the matter era, the non-trivial mass matrix for the two gravitons in doubly coupled bigravity implies that the two gravitons oscillate leading to birefringence [27]. Moreover, in doubly coupled bigravity, the speed of the gravitational waves differs from unity in the late time Universe as the ratio between the two lapse functions of the two metrics is not equal to one anymore. This is severely constrained by the LIGO/VIRGO observations. In contrast, in scalar-bimetric models we have shown that symmetric solutions where $a_\ell = b_\ell$ can be obtained even during the acceleration phase. In this case, the speed of the gravitational waves is always unity. Moreover, at the linear level there is no mixing between the tensor and vector instabilities that affect the "hidden" modes and the matter metrics.

As we are now going to analyze, the time variation of the scalar field in scalar-bimetric models poses new problems which are late time issues, i.e. not only restricted to the radiation era.

VII. RECOVERING GENERAL RELATIVITY ON SMALL SCALES ?

As shown in Fig. 7, the scenarios obtained so far are not consistent with small-scale tests of General Relativity. First, the fifth force is too large, being about twice stronger than Newtonian gravity at $z = 0$, as measured by the ratio $\mu^\phi/\mathcal{G}^\phi - 1$. Second, the time-derivative of the effective Newton constant is too high at $z = 0$, with $d\ln\mathcal{G}/dt \sim 0.7H_0$ whereas the Lunar Laser Ranging experiment gives the upper bound $0.02H_0$ ($d\ln\mathcal{G}/dt < 1.3\times10^{-12}\,\mathrm{yr}^{-1}$) [16]. Third, the change of the Newton constant from its large-redshift value to its current value is too large. Indeed, we obtain an increase of $\mathcal{G}$ of about 50% from its high-z asymptote to its value at $z = 0$. Here we normalized the Planck mass at $z = \infty$ to its measured value in the Solar System today, and defined the cosmological parameters in terms of the same Planck mass in Eq.(29). Instead, we should normalize both the Newton constant at $z = 0$ and the cosmological parameters (i.e., the matter densities) to the measured value of $\mathcal{G}_{\mathrm{N}0}$. However, we would face the same problem. Because we have no dark energy, to recover the Λ-CDM expansion at high z with the same background densities, we need the effective Newton constant at high z to be the same as in the Λ-CDM scenario, which is also the measured value today. Thus, we need $\mathcal{G}_{\mathrm{N}}$ at $z = 0$ to be equal to $\mathcal{G}_{\mathrm{N}}$ at $z \gg 1$, unless we modify the dark matter and radiation densities by a similar amount (with respect to the Λ-CDM reference). However, it is not possible to change the background densities by 50% while keeping a good agreement with the CMB and BBN constraints.

These three problems are not necessarily connected. In modified-gravity models, the fifth force is assumed to be damped in the local environment by non-linear screening mechanisms (which use the fact that the Solar System length scale is much smaller than cosmological distances and/or the local density is much higher than the cosmological background densities). However, it is usually assumed that the time-dependence of the Newton constant, and often its value, remain set by the cosmological background, which acts as a boundary condition. In particular, derivative screening such as the Vainshtein screening, where the non-linear terms are invariant under $\varphi \to \varphi + \alpha t$ with arbitrary α, does not seem to prevent a slow drift of Newton constant. Then, unless the local Newton constant can be significantly decoupled from the cosmological background solution (e.g., through a more efficient screening that remains to be devised), we need to modify the background solution itself to decrease both $d\ln\mathcal{G}/dt(z = 0)$ and $\Delta\mathcal{G} = \mathcal{G}(z = 0) - \mathcal{G}(z = \infty)$.

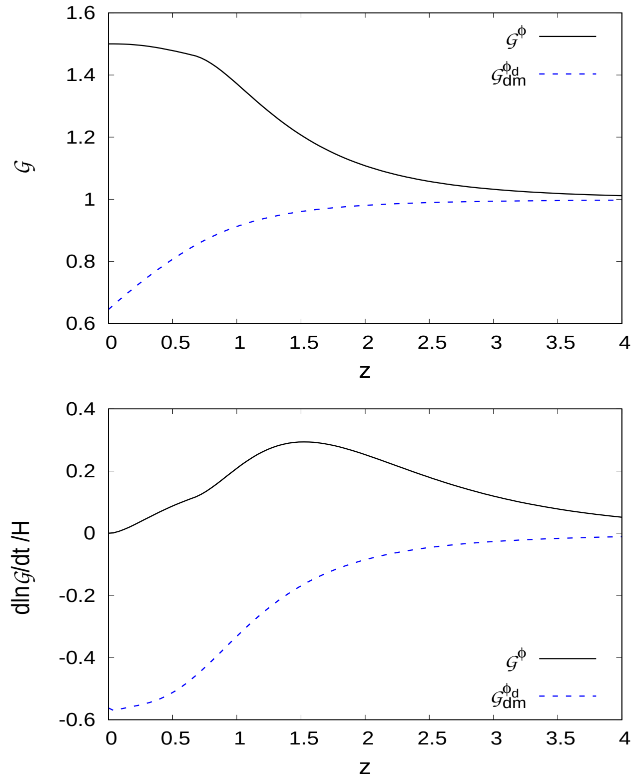

FIG. 12: *Upper panel:* baryonic sector and dark sector Newton constants, normalized to $\mathcal{G}_{\mathrm{N}}$. *Middel panel:* time derivatives $d\ln\mathcal{G}/dt$ normalized to H. *Lower panel:* growth factors $f\sigma_8$ and $f_{\mathrm{dm}}\sigma_{\mathrm{dm}8}$.

A. Reducing $d\ln\mathcal{G}/dt$

1. *Constant $\mathcal{G}$?*

The most elegant way to reduce $d\ln\mathcal{G}/dt$ below the Hubble time scale would be to keep it (almost) constant, so that one would not need any tuning to decrease the time derivative precisely at $z = 0$. Moreover, this would ensure that $\mathcal{G}$ would be about the same at $z = 0$ and $z \gg 1$.

a. Scenarios with common conformal time Let us first consider the case of the scenarios with $r_\ell = 1$, described in section III F. Then, from Eq.(78) a constant $\mathcal{G}$ corresponds to a constant λ in Eq.(52). Unfortunately, the solution (53) does not exist for any $\lambda(a)$, as the argument of the square root needs to remain positive. Numerically, we found that it is not possible to keep a constant $\lambda(a) = 1$, at all times. This can be understood from the behavior of the scales factors a_ℓ. As noticed in Fig. 3 and explained below Eq.(55), the behavior of the scale factors a_* and Hubble expansion rates $\mathcal{H}_*$ are almost independent of the evolution of the coefficients s_ℓ, because we impose a Λ-CDM-like expansion for the baryonic metric. This implies that the ratios a_ℓ/a decrease with time, as the gravitational metrics $g_{\ell\mu\nu}$ follow an expansion close to the Einstein-de Sitter prediction (because we do not put any cosmological constant or dark energy component that would play the same role). Then, to keep the square root real in Eq.(53), $\lambda(a)$ must typically increase with time. In any case, its value at $z=0$ must be greater than unity. From the values of a_ℓ/a read in Fig. 3, we find $\lambda(z=0) \gtrsim 1.5$. This means that Newton's constant $\mathcal{G}$ at $z=0$ must be about 50% greater than its value at high redshift.

We show in Fig. 12 the Newton constants for the baryonic and dark sectors obtained in this manner, with the function λ used for Fig. 3 such that $d\lambda/da = 0$ at $z=0$. This allows us to reduce $d\ln\mathcal{G}^\phi/Hdt$ at all redshifts below 0.3, and make it smaller than the Lunar Laser Ranging upper bound at $z=0$. On the other hand, for the dark sector we still have the generic feature $d\ln\mathcal{G}^{\phi_{\rm d}}_{\rm dm}/Hdt$ of order unity at $z=0$. Making $\lambda(a)$ almost constant at low z is not so artificial, in the sense that it is a simple constraint on the coefficients s_ℓ, which are likely to be correlated in any case. Moreover, the plateau for $\mathcal{G}^\phi$ can be reached at $z \gtrsim 1$, and does not need to be tuned at $z=0$ precisely. However, a few numerical tests suggest that it is difficult, or impossible, to make the transition for $\mathcal{G}^\phi$ occur at much higher redshifts, such as $z=10$. This tends to make s_2 negative at intermediate redshifts, amplifying the dip already seen in Fig. 3, and we prefer to keep the coefficients s_ℓ positive (but this requirement may be unnecessary).

From the arguments discussed above, if the sum $s_1^2+s_2^2$ reaches a constant value at late times, or satisfies a finite upper bound, the decrease of the ratios a_ℓ/a must eventually stop in the future (a simple case is where each coefficient s_ℓ eventually becomes constant). Then, as the gravitational metrics, the baryonic metric must recover an Einstein-de Sitter expansion, unless the energy density and pressure of the scalar field become dominant. Therefore, in this framework where the acceleration of the expansion is not due to an additional dark energy fluid, the self-acceleration is only a transient phenomenon. An alternative would be that the Newton constant resumes its growth in the future, but this would introduce an additional tuning as the slow-down of $d\ln\mathcal{G}^\phi/Hdt$ would be a transient phenomenon that must be set to occur precisely around $z \simeq 0$.

It is interesting to note that the non-symmetric solutions, such as (52), give rise to behaviors beyond those obtained in models where the baryonic and dark matter metrics are simply given by different conformal rescalings of a single Einstein-frame metric. There, we only have two free functions, $A(\varphi)$ and $A_{\rm d}(\varphi)$, with $g_{\mu\nu} = A^2\tilde{g}_{\mu\nu}$ and $g_{{\rm d}\mu\nu} = A_{\rm d}^2\tilde{g}_{\mu\nu}$. This would correspond for instance to $s_1 = A$ and $s_2 = 0$, that is, there is no second gravitational metric. As there is only one coupling A, both the baryonic scale factor a and the baryonic Newton constant $\mathcal{G}^\phi$ depend on $A(\varphi)$ and run at the same rate. This means that it is not possible to have a self-accelerated expansion, driven by $A(\varphi)$, while keeping $\mathcal{G}^\phi$ constant. In the bi-metric scenario, even in the common conformal time case, we can take advantage of the two free functions $s_1(\varphi)$ and $s_2(\varphi)$ to keep a constant Newton strength $\mathcal{G}^\phi$ while having self-acceleration. However, as explained above, this can only happen for a finite time (if we require $s_\ell > 0$) and we cannot reduce the gap $\Delta\mathcal{G} = \mathcal{G}(z=0) - \mathcal{G}(z=\infty)$. Therefore, this scenario is not sufficient to make the model agree with observational constraints. Presumably, increasing the number of metrics, hence of degrees of freedom and free functions of the model, would make it increasingly easy to reconcile a constant Newton strength with self-acceleration.

b. Scenarios with different conformal times In the case of the scenario (61), with $r_\ell \neq 1$, we explicitly checked that we can build solutions such that $\mathcal{G}^\phi$ remains constant at all times, by tuning the factors r_ℓ. More precisely, from Eq.(78) a constant $\mathcal{G}^\phi$ corresponds to

$$\frac{d\mathcal{G}^\phi}{d\ln a} = 0 : \quad \sum_\ell 2 s_\ell r_\ell \frac{ds_\ell}{d\ln a} + s_\ell^2 \frac{dr_\ell}{d\ln a} = 0. \qquad (101)$$

Using the expressions (59), we can write $\{ds_\ell/d\ln a\}$ in terms of $\{dr_\ell/d\ln a\}$. This determines for instance the derivative $dr_2/d\ln a$ while keeping r_1 free, so that this family of solution is still parameterized by a free function $r_1(a)$. However, this usually gives $\mathcal{G}^\psi \neq \mathcal{G}^\phi$, see Eq.(78), with a relative deviation of order unity. To be consistent with Solar System data, in particular with the Shapiro time delay that measures the travel time of light rays in gravitational potentials, we must have $|\psi/\phi - 1| \leq 5\times 10^{-5}$ [28]. On the other hand, as explained in section III G we need $r_1 = 1$ (or $r_2 = 1$) at $z=0$ to comply with the multi-messenger gravitational waves event GW170817. This would give both $s_2 = 0$ and $\mathcal{G}^\psi = \mathcal{G}^\phi$ at $z=0$. However, when we try to combine Eq.(101) with $r_1 \to 1$ at $z=0$ in a few numerical tests, we find singular behaviors with b_2 becoming negative before $z=0$ and $a_2 \to 0$ at $z=0$. This is somewhat reminiscent of the impossibility to achieve a constant $\mathcal{G}^\phi$ in the simpler case $r_1 = r_2 = 1$ shown in Fig. 12. Because the scenarios $r_\ell \neq 1$ already require some tuning, with $|r_1 - 1| < 3\times 10^{-15}$ at $z=0$, we do not investigate further this family of solutions.

2. Constant s_ℓ at late times

A natural solution to obtain a small $d\ln\mathcal{G}/dt$ at low z is to consider models where the coefficients s_ℓ reach a constant at late times. This also removes any fifth force on baryons, as $\beta = 0$ from Eq.(64). However, this also makes the baryonic metric expansion rate converge again to an Einstein-de Sitter behavior, in agreement with the simple solution of section III D. The deviation of $s_1^2 + s_2^2$ from unity in this late-time asymptote again corresponds to a different value for the associated Newton's constant, as compared with the one obtained at high redshift.

We show in Fig. 13 our results for the symmetric solution of Fig. 1, which is modified at late times so that

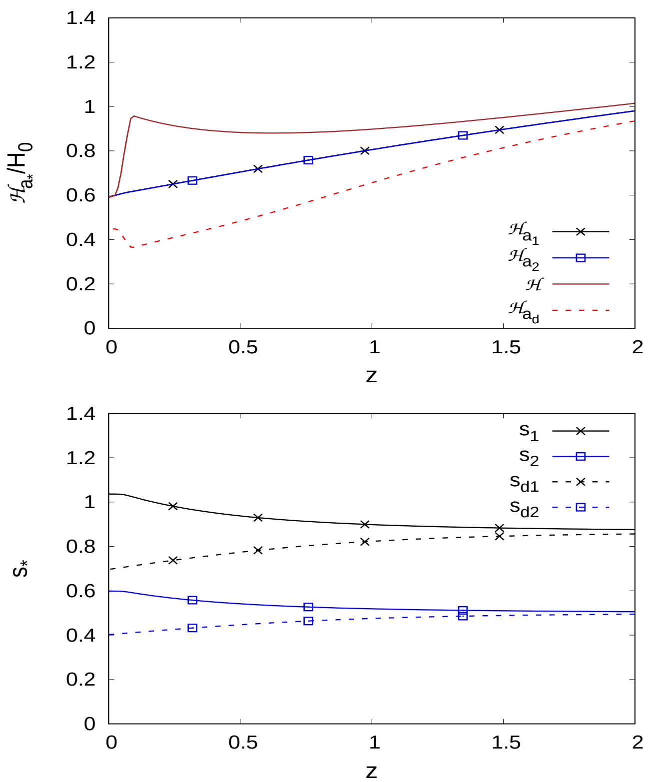

FIG. 13: Conformal Hubble expansion rates (upper panel) and coefficients s_* (lower panel) as a function of redshift, for a solution where the baryonic coefficients s_ℓ are constant at late times.

the baryonic coefficients are constant for $a > 0.9$. In terms of these coefficients, this model is rather simple as the accelerated expansion of the Universe is a transient phenomenon, due to the transition of the coefficients s_i between two constant asymptotes. By requiring the Hubble expansion rate to follow the Λ-CDM history until $z \gtrsim 0.1$, we make the transition to the final Einstein-de Sitter behavior occur in a very small redshift interval. This leads to a sharp decrease for the baryonic expansion rate $\mathcal{H}(z)$, which suddenly drops to the expansion rate $\mathcal{H}_{a_1} = \mathcal{H}_{a_2}$ of the gravitational metrics. This also leads to a sudden increase in the growth rate of large scale structures, which resumes the faster growth associated with Einstein-de Sitter cosmologies.

Even though the change of the coefficients s_ℓ is very small, as compared with the solution of Fig. 1, this leads to a change for the Hubble expansion rate of order unity. Indeed, by making the coefficients s_i constant at late times, we change their time-derivative $ds_i/d\tau$ from a quantity of order $1/H_0$ to zero over a small time $\Delta\tau$. This yields a divergent second derivative $d^2 s_\ell/d\tau^2 \propto 1/(\Delta\tau)$. However, from Eq.(25) we can see that $d\ln\mathcal{H}/d\ln a$, being a second derivative of the scale factor, contains term such as $d^2 s_\ell/d\tau^2$ and also grows as $1/(\Delta\tau)$. Then, even if we let the transition time $\Delta\tau$ go to zero the change of $\mathcal{H}$ remains finite and of order unity, in agreement with Fig. 13. The drop of $H(z)$ at low z to about 60% of the Λ-CDM extrapolation H_0 implies a deviation of the distance modulus, $\mu = 5\log_{10}(d_L/10\text{pc})$, of $\Delta\mu = -5\log_{10}(0.6) \simeq 1.1$. However, the dispersion of the distance modulus of observed type Ia supernovae in the range $0.01 < z < 1$ is of order 0.3, before binning [29], and does not show such a steep step. Therefore, the Hubble diagram shown in Fig. 13 is ruled out by low redshift supernovae.

In addition, we still have a total increase of $\mathcal{G}$ of about 50% between the high-z and low-z values of the effective Newton constant. Therefore, this scenario would not solve this third problem in any case.

B. Need for screening beyond quasi-static chameleon mechanisms

We have seen in the previous section that the coefficients s_ℓ are unlikely to have reached constant values by $z = 0$, to be consistent with the low-z Hubble diagram. This yields a fifth force that is of the same order as the Newtonian force on cosmological scales. All scenarios also imply a decrease of order 50% of the effective Newton constant at higher redshifts, which makes it impossible to recover the reference Λ-CDM expansion rate unless the matter and radiation densities are also modified. This means that such scalar-bimetric models can only satisfy observational constraints if gravity in the Solar System is decoupled from its behavior on cosmological scales.

Within modified-gravity scenarios, the recovery of General Relativity on small scales is often achieved by introducing non-linear screening mechanisms that damp the effect of the fifth force. For instance, chameleon screening makes the scalar field short-ranged in high-density environments, because its effective potential and its mass depend on the matter density. In a similar fashion, dilaton and symmetron scenarios damp the fifth force by making its coupling vanish in high-density environments, following the Damour-Polyakov screening.

It is interesting to note that these screening mechanisms cannot appear in the models considered in this paper, because the scalar field always remains in the linear regime. A first way to see this is from Eq.(65), which yields $\delta\varphi/M_{\rm Pl} \sim v^2$ for a structure of virial velocity $v^2 \sim \mathcal{G}M/r$, mass M and radius r. Then, in non-relativistic environments, from clusters of galaxies to the Solar System, where $v^2 \ll 1$, we have $\delta\varphi \ll \bar{\varphi}$ as we found in Fig. 1 that $\bar{\varphi} \sim M_{\rm Pl}$. This also implies that $\delta s_\ell \ll \bar{s}_\ell$. Thus, from clusters of galaxies to the Solar System the fluctuations of the scalar field remain small and are not sufficient to significantly modify the coefficients s_ℓ. This means that the effective Poisson equation (i.e., the coefficients μ_*^*) keeps the same deviation from General Relativity on all these scales.

This configuration can be compared with the usual chameleon or Damour-Polyakov screenings, shown by $f(R)$ or Dilaton and Symmetron models. There, the Jordan frame metric is typically related to the Einstein-frame metric by a conformal coupling, $g_{\mu\nu} = A^2(\varphi)\tilde{g}_{\mu\nu}$. The fifth force $c^2\nabla \ln A$ again arises from the fluctuations of this metric coefficient A, through the fluctuations of the scalar field. However, in these models which typically include a cosmological constant, either explicitly or as the non-zero minimum of some potential, the conformal coupling always remains very close to unity, $|A-1| \lesssim 10^{-5}$. This ensures that one follows the Λ-CDM background while having effects on cosmological structures that can be of order unity, with $\delta A \sim \phi$. The very small variation of the background value of $\bar{A}$ also means that it is easy to introduce a screening mechanism, because the spatial perturbations of $\delta\varphi$ and δA can be of the same order as those of the cosmological background over $\delta z \sim 1$, so that the non-linear regime is easily reached (this may be more easily understood from a tomographic point of view). In the model considered in this paper, the difficulty arises from the fact that we require background variations of order unity for the coefficients s_ℓ, which play a role similar to $A^2(\varphi)$ in the conformal coupling models, whereas spatial variations should remain of order 10^{-5} of the same order as the standard Newtonian potential. This implies that spatial fluctuations of the scalar field value are not sufficient to reach the non-linear regime. This analysis agrees with the "no-go" theorem of Ref.[5], which concludes from the same arguments that usual chameleon models cannot provide a self-acceleration of the Universe, and must rely on a form of dark energy (typically a hidden cosmological constant, written as the non-zero minimum of some potential).

A way out of this difficulty is to introduce screening mechanisms that do not rely on the scalar field value, but on its derivatives. Then, even though $\delta\varphi$ remains small, its spatial derivatives $\partial^n\delta\varphi$ can be large on small enough scales. This corresponds to K-mouflage and Vainshtein mechanisms. This can be achieved by adding terms in $(\partial\varphi)^4/M^4$ or $\Box\varphi(\partial\varphi)^2/M^3$. In this case, these non-linear terms dominate over the simple kinetic terms at short distance depending on the value of M. As a result, the coupling of the scalar field to the baryons (and incidentally the one to dark matter) is reduced and local tests of gravity are satisfied. However, this only solves the fifth-force problem and it does not solve the problems associated with the value of Newton constant and its time drift. (In these screening scenarios, they are usually assumed to be set by the cosmological background, which acts as a boundary condition.)

The analysis above implicitly assume the quasi-static approximation, where the scalar field relaxes to its environment-dependent equilibrium and screening appears through the spatial variations of its mass, coupling or inertia. If the quasi-static approximation is violated, the configuration may be more complex. In fact, from the analysis of section VII A, we can see that we need a local value of Newton constant that is decoupled from the one on cosmological scales. More precisely, we need its local value to remain equal to its background value at high z, before the dark energy era. This calls for a new screening mechanism, or a more efficient implementation of K-mouflage or Vainshtein screening, that goes beyond the quasi-static approximation and decouples the small-scale Newton constant from its current large-scale cosmological value. For instance, the local Newton constant should remain equal to the one at the formation of the Solar System.

VIII. CONCLUSION

We have seen in this paper that the scalar-bimetric model allows one to recover an accelerated expansion without introducing a cosmological constant or an almost constant dark energy density. This relies on the time-dependent mapping between the gravitational metrics g_1 and g_2 and the baryonic and dark matter metrics g and $g_{\rm d}$. Because at late times the deviation between the Λ-CDM and Einstein-de Sitter backgrounds is of order unity, the coefficients s_ℓ that define this mapping must show variations of order unity.

When all metrics have the same conformal time, the expansion rates of the gravitational and dark matter metrics are almost independent of the details of the model [e.g., the shape of the functions $s_\ell(\varphi)$], once we require a Λ-CDM expansion for the baryonic metric. Then, the gravitational metrics remain close to an Einstein-de Sitter expansion (because there is no dark energy), while the dark matter metric behaves in a way opposite to the baryonic metric, with a stronger deceleration than in the Einstein-de Sitter case. When the conformal times are different, the scale factors a_* can show slightly different behaviors, and even more so the lapse factors b_*. This scenario is very strongly constrained by the multi-messenger event GW170817, which requires that at least one of the two massless gravitons propagates at the speed of light at $z < 0.01$. This implies that at least one of the ratios b_ℓ/a_ℓ must be unity at low z. This also implies that the baryonic metric becomes independent at low z of the gravitational metric where $c_g \neq 1$, but the dark matter metric still remains sensitive to both gravitational metrics.

As the coefficients s_ℓ must show variations of order unity to provide a self-acceleration, we generically have deviations of order unity for the effective Newton constants and for the contribution from the fifth force to the dynamical potential seen by particles. The dynamics of baryonic and dark matter perturbation show distinctive features, due to the fact that they couple to different metrics and that their mappings evolve in opposite fashion. While the total force (Newtonian gravity and fifth force) from baryons onto baryons, and from dark matter onto dark matter, is typically amplified at low redshift, the cross-force between baryons and dark matter is damped and even turns negative. This means that dark matter and baryons would tend to segregate (although this does not have the time to happen by $z = 0$ on large scales). Then, the growth of dark matter density fluctuations is amplified (because of the stronger self-gravity) while the growth of baryonic density fluctuations is decreased on cosmological scales (because of the lower cross-gravity, as dark matter is dominant on large scales). This could provide interesting features; for instance, most modified-gravity models predict instead an amplification of baryonic density perturbations.

However, before a detailed comparison with cosmological observations, these models present major difficulties with small-scale tests of gravity. First, the fifth force is of the same order as Newtonian gravity. Second, the baryonic effective Newton constant generically evolves on Hubble time scales. Third, it is greater than its high-z value by about 50%. These features are related to the self-acceleration, which implies modifications of order unity on Hubble time scales.

Thanks to the two couplings associated with the two gravitational metrics, it is possible to keep the baryonic effective Newton strength almost constant at low z. (By keeping the sum $s_1^2+s_2^2$ constant while the two coefficients vary.) This is beyond the reach of simpler models where the baryonic metric would be given by a conformal rescaling of a single Einstein-frame metric [which provides a single coupling $A(\varphi)$]. However, this can only work for a finite time. Either the baryonic and dark matter metrics eventually recover an Einstein-de Sitter expansion in the future, or the Newton coupling resumes its growth in the future. In this framework, it is more natural to make the self-acceleration only a transient phenomenon, associated with the running of the couplings $s_\ell(\varphi)$ between two constant asymptotes (where the fifth force and the running of Newton constants disappear). (The alternative scenario, where the coefficients $s_\ell(\varphi)$ have already reached their constant asymptote at low z, is rejected by measurements of the Hubble expansion rate, by low-z supernoave or local standard candles such as cepheids.) However, this cannot reduce the gap between the high-z and low-z values of Newton constant.

On small scales, Solar System tests of gravity imply that we must recover General Relativity. In modified-gravity scenarios, this is often achieved by introducing non-linear screening mechanisms that damp the effect of the fifth force. As in the case of single-metric and single-field models, we explain that a chameleon mechanism cannot work. It cannot efficiently screen the fifth force in a self-accelerated model. This leaves derivative screening mechanisms, such as K-mouflage and Vainshtein screenings. Therefore, the scalar field Lagrangian must be supplemented by higher-order derivative terms, that become dominant on small scales and provide the convergence to General Relativity. on small scales by damping the fifth force. However, we need to go beyond usual implementations, as we also require the local Newton constant to be decoupled from its cosmological value and to remain equal to its high-redshift value. Then, the sum $s_1^2 + s_2^2$ is no longer required to be almost constant at low z and this extends the family of realistic models to all solutions with common conformal time.

This paper only provides a first study of such bi-metric models with self-acceleration. We have shown that basic requirements already strongly constrain these scenarios, and they are actually ruled out unless and efficient screening mechanism comes into play on small scales. We leave for future works a detailed study to determine whether such scenarios can be consistent with cosmological data at the perturbative level. However, the main challenge is to devise adequate screening mechanisms, if they exist. This would also have a great impact on other modified gravity models, by explaining why gravity on cosmological scales could be decoupled from Solar System tests. Another issue is to stabilize the hidden vector modes, so that they remain in the linear regime and do not mix with the matter metrics.

Appendix A: Linear perturbations

1. Gauge choices

Following the approach of sections II A and II B, we define the perturbations of the gravitational sector in terms of the vierbeins, $\delta e^a_{\ell\mu}$, instead of the metrics $\delta g_{\ell\mu\nu}$. In this appendix, to avoid ambiguities between the indices $a, b, ...$ associated with the Minkowski metric η_{ab} and the indices $\mu, \nu, ...$ associated with the curved spacetime metrics $g_{\mu\nu}$, all vierbeins are kept under the form $e^a_{\ell\mu}$, that is, the upper index refers to the Minkowski metric and the lower index to the curved metric [i.e., we do not introduce the inverse vierbeins such as e^μ_a as in Eq.(11)]. The first lower index always refers to the associated sector ($\ell = 1, 2$ for the two gravitational metrics or b, d for the baryonic and dark metrics).

Then, the symmetric constraint (9) reads

$$\eta_{ab} \left[\delta e^a_{1\mu} e^b_{2\nu} - \delta e^a_{1\nu} e^b_{2\mu}\right] = \eta_{ab} \left[\delta e^a_{2\mu} e^b_{1\nu} - \delta e^a_{2\nu} e^b_{1\mu}\right] .$$

As our background takes the diagonal forms (22)-(23), these constraints simplify to

$$\delta e^i_{10} a_2 + \delta e^0_{1i} b_2 = \delta e^i_{20} a_1 + \delta e^0_{2i} b_1, \tag{A1}$$

and

$$a_2(\delta e^j_{1i} - \delta e^i_{1j}) = a_1(\delta e^j_{2i} - \delta e^i_{2j}), \tag{A2}$$

for all i and (i, j) in $\{1, 2, 3\}$. The metric perturbation $\delta g_{\ell\mu\nu}$ is given from the definition (6) by

$$\delta g_{\ell\mu\nu} = (\delta e^a_{\ell\mu} e^b_{\ell\nu} + e^a_{\ell\mu} \delta e^b_{\ell\nu})\eta_{ab}. \tag{A3}$$

For the diagonal background (23) this simplifies to

$$\delta g_{\ell 00} = -2b_\ell \delta e^0_{\ell 0}, \quad \delta g_{\ell 0i} = \delta g_{\ell i0} = a_\ell \delta e^i_{\ell 0} - b_\ell \delta e^0_{\ell i},$$
$$\delta g_{\ell ij} = a_\ell(\delta e^i_{\ell j} + \delta e^j_{\ell i}). \tag{A4}$$

The perturbations of the matrices $X^{\mu}_{\ell\nu}$ defined in Eq.(11) also simplify as

$$\delta X^0_{20} = -\frac{\delta e^0_{20} b_1}{b_2^2} + \frac{\delta e^0_{10}}{b_2}, \quad \delta X^i_{20} = -\frac{\delta e^i_{20} b_1}{a_2 b_2} + \frac{\delta e^i_{10}}{a_2},$$
$$\delta X^0_{2i} = -\frac{\delta e^0_{2i} a_1}{b_2 a_2} + \frac{\delta e^0_{1i}}{b_2}, \quad \delta X^i_{2j} = -\frac{\delta e^i_{2j} a_1}{a_2^2} + \frac{\delta e^i_{1j}}{a_2}. \tag{A5}$$

The permutation $1 \leftrightarrow 2$ provides $\delta X^{\mu}_{1\nu}$.

As in General Relativity, we can split the gravitational perturbations in scalar, vector and tensor modes. As in [13], we can do so at the level of the vierbeins and we can write

$$\delta e^0_{\ell 0} = b_\ell \phi_\ell\,, \quad \delta e^0_{\ell i} = a_\ell[-\partial_i V_\ell + C_{\ell i}]\,,$$
$$\delta e^i_{\ell 0} = b_\ell[-\partial^i W_\ell + D^i_\ell]\,,$$
$$\delta e^i_{\ell j} = a_\ell[-\psi_\ell \delta^i_j + \partial^i\partial_j U_\ell + \partial_j V^i_\ell + \partial^i W_{\ell j} + h^i_{\ell j}]\,, \tag{A6}$$

where the spatial indexes are raised and lowered with δ^{ij} and δ_{ij}, so that $\partial^i = \partial_i$ and $V^i_\ell = V_{\ell i}$. The transversality conditions are

$$\partial^i C_{\ell i} = \partial_i D^i_\ell = \partial_i V^i_\ell = \partial^i W_{\ell i} = 0, \quad \partial_i h^i_{\ell j} = 0\,,$$

and tracelessness corresponds to

$$h^i_{\ell i} = 0.$$

This provides the perturbations of the gravitational metrics as

$$\delta g_{\ell 00} = -2 b_\ell^2 \phi_\ell, \quad \delta g_{\ell 0i} = a_\ell b_\ell[\partial_i(V_\ell - W_\ell) + D_{\ell i} - C_{\ell i}],$$
$$\delta g_{\ell ij} = a_\ell^2[-2\psi_\ell \delta_{ij} + 2\partial_i\partial_j U_\ell + \partial_i(V_{\ell j} + W_{\ell j})$$
$$+\partial_j(V_{\ell i} + W_{\ell i}) + h_{\ell ij} + h_{\ell ji}]. \tag{A7}$$

The baryonic and dark vierbeins and metrics obey the same decompositions, obtained from the combinations (8).

This gives 32 degrees of freedom for the two gravitational metrics: 10 scalars $\{\phi_\ell, V_\ell, W_\ell, \psi_\ell, U_\ell\}$, 8 vectors $\{C_{\ell i}, D^i_\ell, V^i_\ell, W_{\ell i}\}$, and 2 non-symmetric tensors $h^i_{\ell j}$. As explained in section II B, this can be reduced to 16 degrees of freedom when we use the invariance under the diagonal Lorentz transformations and diffeomorphisms, and the symmetry constraints (A1)-(A2). First, the diagonal Lorentz invariance allows us to set

$$V_2 + W_2 = 0, \;\; C_{2i} = 0, \;\; W_{2i} = 0, \;\; h_{2ij} - h_{2ji} = 0, \tag{A8}$$

which reduces the number of degrees of freedom to 26. Second, the symmetry constraints (A1)-(A2) simplify as

$$h_{1ij} - h_{1ji} = 0, \quad V_{1i} - W_{1i} = V_{2i},$$
$$a_2 b_1 D_{1i} + b_2 a_1 C_{1i} = a_1 b_2 D_{2i},$$
$$a_2 b_1 W_1 + b_2 a_1 V_1 = a_1 b_2 W_2 + b_1 a_2 V_2, \tag{A9}$$

which reduces the number of degrees of freedom to 20. Third, the invariance over diffeomorphisms allows us to set

$$D_{2i} = 0, \; U_2 = 0, \; V_2 = 0, \tag{A10}$$

by using the coordinate transformations $\{x^i \to x^i + \int \frac{b_2}{a_2} D^i_2 d\tau\}$, $\{x^i \to x^i + \partial^i U_2, \; x^0 \to x^0 - \frac{a_2^2}{b_2^2}\partial_\tau U_2\}$, and $\{x^0 \to x^0 - \frac{a_2}{b_2} V_2\}$. This reduces the number of degrees of freedom to 16.

Collecting (A8), (A9) and (A10), we obtain the gauge conditions

$$V_2 = W_2 = U_2 = 0, \; C_{2i} = D_{2i} = W_{2i} = 0, \tag{A11}$$

the two tensors $h_{\ell ij}$ are symmetric, and we have the relations

$$W_{1i} = V_{1i} - V_{2i}, \; D_{1i} = -\frac{b_2 a_1}{a_2 b_1} C_{1i}, \; W_1 = -\frac{b_2 a_1}{a_2 b_1} V_1. \tag{A12}$$

This leaves 6 scalars $\{\phi_\ell, \psi_\ell, V_1, U_1\}$, 3 vectors $\{C_{1i}, V_{1i}, V_{2i}\}$, and 2 symmetric tensors $\{h_{\ell ij}\}$, which correspond to 16 degrees of freedom.

2. Tensor perturbations

In the tensor sector, we have the two symmetric transverse and traceless tensors $h_{\ell ij}$ of the two gravitational metrics, and their counterparts in the baryonic and dark metrics, which are given from (8) by

$$a h_{ij} = s_1 a_1 h_{1ij} + s_2 a_2 h_{2ij},$$
$$a_{\rm d} h_{{\rm d}ij} = s_{\rm d1} a_1 h_{1ij} + s_{\rm d2} a_2 h_{2ij}. \tag{A13}$$

We focus on the propagation of gravitational waves in the cosmological background, without source terms from the matter distributions. The Einstein equations (18) give for $(\mu,\nu) = (i,j)$ the equation (88) in the main text.

Until $z \sim 2$, the coefficients s_ℓ and $s_{{\rm d}\ell}$ are almost constant and we follow the simple solution (40), with a negligible scalar field energy density. Then, Eq.(88) and its symmetric over $1 \leftrightarrow 2$ simplify as

$$h_1'' + 2\mathcal{H} h_1' - \nabla^2 h_1 - s_2^2 \frac{a^2 \bar{p}}{M_{\rm Pl}^2}(h_1 - h_2) = 0,$$
$$h_2'' + 2\mathcal{H} h_2' - \nabla^2 h_2 - s_1^2 \frac{a^2 \bar{p}}{M_{\rm Pl}^2}(h_2 - h_1) = 0, \tag{A14}$$

where we omit the indices ij and we consider non-relativistic dark matter. Here $\bar{p}$ is the total pressure in the baryonic sector. It is the radiation pressure in the radiation and matter eras and the inflaton pressure in the inflationary era.

3. Vector perturbations

In the vector sector, we have the three vectors C_{1i}, V_{1i}, V_{2i} for the gravitational part. The perturbations of the energy momentum tensors are

$$\delta T^{00} = 0, \; \delta T^{0i} = a^{-2}\left[\bar{\rho} v^i + \bar{p}(D^i - C^i)\right],$$
$$\delta T^{ij} = -a^{-2}\bar{p}[\partial^i(V^j + W^j) + \partial^j(V^i + W^i)], \tag{A15}$$

for the baryonic matter and radiation, and

$$\delta T_{\rm d}^{00} = 0, \; \delta T_{\rm d}^{0i} = a^{-2}\bar{\rho}_{\rm dm} v_{\rm dm}^i, \; \delta T_{\rm d}^{ij} = 0, \tag{A16}$$

for the dark matter and the scalar field, where v^i and $v_{\rm dm}^i$ are the matter velocity fields. We denote by $\bar{\rho}$ and $\bar{p}$ the total density and pressure in the baryonic sector. In particular, $\bar{\rho} = \bar{\rho}_{\rm b} + \bar{\rho}_\gamma$ and $\bar{p} = \bar{p}_\gamma = \bar{\rho}_\gamma/3$ in the radiation and matter eras, whereas $\bar{\rho} = \bar{\rho}_\chi$ and $\bar{p} = \bar{p}_\chi = \bar{\rho}_\chi$ in the inflationary era. Here and throughout this section we focus on redshifts $z \gtrsim 2$ before the dark energy era, where the scalar field is almost constant with a negligible energy density, and we follow the simple solution (40).

As in General Relativity, the equations of motion of matter, $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_{{\rm d}\mu} T_{\rm d}^{\mu\nu} = 0$, decouple from the Einstein equations and read as

$$\begin{aligned} & \frac{\partial}{\partial\tau}[(\bar{\rho}+\bar{p})U^i] + 4\mathcal{H}[(\bar{\rho}+\bar{p})U^i] = 0, \\ & \frac{\partial}{\partial\tau}[\bar{\rho}_{\rm dm} U_{\rm dm}^i] + 4\mathcal{H}\bar{\rho}_{\rm dm} U_{\rm dm}^i = 0, \end{aligned} \tag{A17}$$

where we introduced the usual gauge invariant velocities, which in our gauge write as

$$U^i = v^i - 2s_1^2 C_1^i, \quad U_{\rm dm}^i = v_{\rm dm}^i - 2s_1^2 C_1^i. \tag{A18}$$

Thus, $(\bar{\rho}+\bar{p})U^i \propto a^{-4}$ and $\bar{\rho}_{\rm dm} U_{\rm dm}^i \propto a^{-4}$. Then, $U_{\rm dm}^i$ decays as a^{-1} at all times, whereas U^i is constant during the radiation era and decays as a^{-1} during the matter era.

Taking $U^i = U_{\rm dm}^i = 0$, the Einstein equations (18) give for $(\mu,\nu) = (0,i)$:

$$\begin{aligned} & \frac{M_{\rm Pl}^2}{a^2}\nabla^2(2V_{1i}' - V_{2i}' + 2C_{1i}) = 3s_2^2(\bar{\rho}+\bar{\rho}_{\rm dm}+\frac{\bar{p}}{3})C_{1i}, \\ & \frac{M_{\rm Pl}^2}{a^2}\nabla^2 V_{2i}' = -3s_1^2(\bar{\rho}+\bar{\rho}_{\rm dm}+\frac{\bar{p}}{3})C_{1i}, \end{aligned} \tag{A19}$$

while the components $(\mu,\nu) = (i,j)$ give:

$$\begin{aligned} & M_{\rm Pl}^2[2V_{1i}'' - V_{2i}'' + 2C_{1i}' + 2\mathcal{H}(2V_{1i}' - V_{2i}' + 2C_{1i})] = \\ & \qquad \bar{p}a^2 s_2^2 2(V_{1i} - V_{2i}), \\ & M_{\rm Pl}^2[V_{2i}'' + 2\mathcal{H}V_{2i}'] = \bar{p}a^2 s_1^2 2(V_{2i} - V_{1i}). \end{aligned} \tag{A20}$$

We can check that these four equations for the three unknowns C_{1i}, V_{1i}, V_{2i} are compatible, as the last equation (A20) can be derived from the first three Einstein equations. As for the case of tensors, it is useful to introduce the two combinations

$$\mathcal{V} = s_1^2(2V_1' - V_{2i}' + 2C_1) + s_2^2 V_2', \tag{A21}$$

$$\mathcal{V}_- = V_1 - V_2, \tag{A22}$$

where we omit the spatial index i.

By combining the two Einstein equations of (A20), we obtain

$$\mathcal{V}' + 2\mathcal{H}\mathcal{V} = 0. \tag{A23}$$

Thus, $\mathcal{V}$ always decays as a^{-2}. It turns out that this is the gauge invariant vector that appears in the matter metrics. Indeed, in the regime where the scalar field is constant that we consider here, we obtain for the vector components of the matter vierbeins and metric

$$D_i - C_i = -2s_1^2 C_{1i}, \quad V_i + W_i = s_1^2(2V_{1i} - V_{2i}) + s_2^2 V_{2i}.$$

The gauge invariant vector of the matter metric is $(V_i' + W_i') - (D_i - C_i) = \mathcal{V}_i$. Therefore, matter only couples to the decaying mode $\mathcal{V}_i$.

From the Einstein equations (A19)-(A20) we obtain in Fourier space, for wave number k,

$$C_1 = \frac{\mathcal{V}_-}{1 + \frac{3a^2}{2M_{\rm Pl}^2 k^2}(\bar{\rho}+\bar{\rho}_{\rm dm}+\bar{p}/3)} \tag{A24}$$

and

$$\begin{aligned} & \mathcal{V}_-'' + \left[\frac{2\mathcal{H} + \frac{\bar{\rho}'+\bar{\rho}_{\rm dm}'+\bar{p}'/3}{\bar{\rho}+\bar{\rho}_{\rm dm}+\bar{p}/3}}{1 + \frac{3a^2}{2M_{\rm Pl}^2 k^2}(\bar{\rho}+\bar{\rho}_{\rm dm}+\bar{p}/3)} + 2\mathcal{H} \right] \mathcal{V}_-' \\ & - \frac{2k^2\bar{p}\left[1 + \frac{3a^2}{2M_{\rm Pl}^2 k^2}(\bar{\rho}+\bar{\rho}_{\rm dm}+\bar{p}/3)\right]}{3(\bar{\rho}+\bar{\rho}_{\rm dm}+\bar{p}/3)} \mathcal{V}_- = 0. \end{aligned} \tag{A25}$$

On subhorizon scales this simplifies as

$$\begin{aligned} k \gg \mathcal{H} \; : \quad & \mathcal{V}_-'' + \left[4\mathcal{H} + \frac{\bar{\rho}'+\bar{\rho}_{\rm dm}'+\bar{p}'/3}{\bar{\rho}+\bar{\rho}_{\rm dm}+\bar{p}/3} \right] \mathcal{V}_-' \\ & - \frac{2k^2\bar{p}}{3(\bar{\rho}+\bar{\rho}_{\rm dm}+\bar{p}/3)} \mathcal{V}_- = 0, \end{aligned} \tag{A26}$$

whereas on superhorizon scales this gives

$$k \ll \mathcal{H}: \quad \mathcal{V}_-'' + 2\mathcal{H}\mathcal{V}_-' - \frac{a^2\bar{p}}{M_{\rm Pl}^2}\mathcal{V}_- = 0. \tag{A27}$$

Thus, the mode $\mathcal{V}_-$ shows a gradient instability on subhorizon scales in the radiation and matter eras, where $\bar{p} = \bar{p}_\gamma > 0$.

Let us consider in turns the inflationary, radiation and matter eras. In the inflationary era, Eq.(A26) gives on subhorizon scales $\mathcal{V}_-'' + 4\mathcal{H}\mathcal{V}_-' + k^2\mathcal{V}_- = 0$, so that the vector mode $\mathcal{V}_-$ oscillates with frequency $\omega = \pm k$. On superhorizon scales, Eq.(A27) gives $\mathcal{V}_-'' - \frac{2}{\tau}\mathcal{V}_-' + \frac{3}{\tau^2}\mathcal{V}_- = 0$. This is the same evolution equation as for the tensor modes, and we obtain the same two decaying solutions as in Eq.(94).

In the radiation era, on subhorizon scales we obtain $\mathcal{V}_-'' - \frac{k^2}{5}\mathcal{V}_- = 0$. This gradient instability leads to the two exponential modes

$$k \gg \mathcal{H}: \quad \mathcal{V}_-^{\pm} \propto e^{\pm k\tau/\sqrt{5}}. \tag{A28}$$

On superhorizon scales we again recover the same behavior as for tensors, $\mathcal{V}_-'' + \frac{2}{\tau}\mathcal{V}_-' - \frac{1}{\tau^2}\mathcal{V}_- = 0$, with the power-law growing and decaying modes (93).

In the matter era, we obtain on subhorizon scales $\mathcal{V}_-''+\frac{2}{\tau}\mathcal{V}_-'-\frac{8k^2}{9\mathcal{H}_{\rm eq}^2\tau^2}\mathcal{V}_-=0$, which gives the power-law growing and decaying modes

$$k\gg\mathcal{H}:\quad \mathcal{V}_-^{\pm}\propto\tau^{\lambda_\pm}\quad\text{with}\quad \lambda_\pm=-\frac{1}{2}\pm\frac{1}{2}\sqrt{1+\frac{32k^2}{9\mathcal{H}_{\rm eq}^2}}. \tag{A29}$$

On superhorizon scales we have $\mathcal{V}_-''+\frac{4}{\tau}\mathcal{V}_-'-\frac{4\tau_{\rm eq}^2}{\tau^4}\mathcal{V}_-=0$. Long after the radiation-matter equality, $\tau\gg\tau_{\rm eq}$, this gives a constant mode and a decaying mode $\mathcal{V}_-\propto\tau^{-3}$.

4. Scalar perturbations

In the scalar sector, we have the six scalars $\{\phi_\ell,\psi_\ell,V_1,U_1\}$, whereas $W_1=-(r_2/r_1)V_1$. The extra two scalars added to the four Newtonian potentials are not dynamical [11] and there is no scalar instability. In this paper, we consider the quasi-static approximation, which applies to the formation of large-scale structures. Thus, we neglect time derivatives and only keep the highest order spatial derivatives. Then, only the four scalars $\{\phi_\ell,\psi_\ell\}$ remain. The fluctuation of the scalar field energy density $\delta\rho_\varphi$ also drops out as it is of second order over $\delta\varphi$. However, we take into account the fluctuations δs_ℓ and $\delta s_{{\rm d}\ell}$ of the vierbein coefficients s_ℓ and $s_{{\rm d}\ell}$ generated by the fluctuations of the scalar field.

For non-relativistic matter components, the (0,0) component of the Einstein equations (18) gives for the metric $g_{1\mu\nu}$

$$\frac{2a_1}{3H_0^2}\nabla^2\psi_1=s_1\Omega_{\rm b0}\delta+s_{\rm d1}\Omega_{\rm dm0}\delta_{\rm dm}+\Upsilon_{\psi_1}\frac{d\ln a}{d\bar\varphi}\delta\varphi, \tag{A30}$$

with

$$\Upsilon_{\psi_1}=\left(\Omega_{\rm b0}+\frac{\Omega_{\gamma0}}{a}\right)\left[\left(1+s_1\frac{b_1}{a}\right)\frac{ds_1}{d\ln a}+s_1\frac{b_2}{a}\frac{ds_2}{d\ln a}\right] +(\Omega_{\rm dm0}+\xi)\left[\left(1+s_{\rm d1}\frac{b_1}{b_{\rm d}}\right)\frac{ds_{\rm d1}}{d\ln a}+s_{\rm d1}\frac{b_2}{b_{\rm d}}\frac{ds_{\rm d2}}{d\ln a}\right]. \tag{A31}$$

The (i,j) components of the Einstein equations give

$$\frac{b_1}{H_0^2}\left[-\partial_i\partial_j(\phi_1-\psi_1)+\delta_{ij}\nabla^2(\phi_1-\psi_1)\right]=\Upsilon_{\phi_1}\frac{d\ln a}{d\bar\varphi}\delta\varphi, \tag{A32}$$

with

$$\Upsilon_{\phi_1}=\frac{\Omega_{\gamma0}}{a}\left[\left(1+s_1\frac{a_1}{a}\right)\frac{ds_1}{d\ln a}+s_1\frac{a_2}{a}\frac{ds_2}{d\ln a}\right] +3\xi r_{\rm d}\left[\left(1+s_{\rm d1}\frac{a_1}{a_{\rm d}}\right)\frac{ds_{\rm d1}}{d\ln a}+s_{\rm d1}\frac{a_2}{a_{\rm d}}\frac{ds_{\rm d2}}{d\ln a}\right]. \tag{A33}$$

We can use the Klein-Gordon equation (65) satisfied by the scalar field to eliminate $\delta\varphi$. In Fourier space, this gives Eqs.(71) and (72).

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