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Fluid dynamics of out of equilibrium boost invariant plasmas

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By solving a simple kinetic equation, in the relaxation time approximation, and for a particular set of moments of the distribution function, we establish a set of equations which, on the one hand, capture exactly the dynamics of the kinetic equation, and, on the other hand, coincide with the hierarchy of equations of viscous hydrodynamics, to arbitrary order in the viscous corrections. This correspondence sheds light on the underlying mechanism responsible for the apparent success of hydrodynamics in regimes that are far from local equilibrium.

Introduction. The observation that the evolution of the quark-gluon plasma produced in ultra-relativistic heavy ion collisions is well described by viscous hydrodynamic equations raises a number of interesting questions that are very much debated presently [1]. Traditional understanding of hydrodynamics would imply that the system has reached local equilibrium, and the small viscosity extracted from the analysis of the data is suggestive of short mean free paths. However, works on strongly coupled plasmas, using in particular holography techniques, indicate that viscous hydrodynamics works even when large anisotropies, that signal departure from local equilibrium, are still present [2]. At the same time, there is evidence that hydrodynamics is capable of describing small colliding systems, for which no clear separation a priori exists between microscopic and macroscopic scales (see e.g. the recent discussion in [1, 3] and references therein).

Recently, it has been argued that part of the success of hydrodynamics could be due to the existence of a stable attractor, to which the solution of the dynamical equations quickly converge before eventually reaching the viscous hydrodynamic regime [4]. This suggestion has triggered many studies, some of which involve sophisticated mathematical developments [5–9]. In this paper, we would like to offer an alternative perspective on the issue, based on the simple, and physically motivated observation, that the main features of the dynamics of expanding plasmas are determined by the competition between the expansion itself, which is dictated by the external conditions of the collisions, and the collisions among the plasma constituents which generically tend to isotropize the particle momentum distribution functions. These two competing effects give rise to two independent fixed points of a suitably defined dynamical quantity. Many recent results find a natural interpretation in the interplay between these two fixed points.

As in many works on this issue, we focus on the paradigmatic example of the Bjorken flow [10], and consider an expanding system of massless particles characterized by a distribution function f whose time evolution is given by a kinetic equation. Symmetry allows us to reconstruct the full space-time history of the system from the knowledge of what happens in a slice centered around the plane $z = 0$ where the collision takes place. The distribution function in that slice depends solely on the momentum of the particle and the (proper) time τ, i.e., $f = f(\mathbf{p}, \tau)$. Using a relaxation time approximation for the collision kernel, we can then write the following simple kinetic equation [11]

$$\left[\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right] f(\mathbf{p},\tau) = -\frac{f(\mathbf{p},\tau) - f_{\rm eq}(p/T)}{\tau_R}. \qquad (1)$$

Here $f_{\rm eq}(p/T)$ is a function that depends only on $p = |\mathbf{p}|$ and an effective temperature $T(\tau)$ which is determined by requiring that the energy density calculated with $f_{\rm eq}(p/T)$ and $f(\mathbf{p}, \tau)$ takes the same value, $\varepsilon \propto T^4$, at all times. The kinetic equation (1) makes transparent the competition alluded to above, between the expansion and the collisions. In the absence of the collision term, the expansion, controlled by the term $-p_z/\tau$ in the left hand side, leads to a flattened distribution, $f(\mathbf{p}, \tau) \to f_0(p_z\tau, \mathbf{p}_\perp)$, where f_0 is the initial distribution and $\mathbf{p}_\perp$ is the component of the momentum orthogonal to the z-axis. On the other hand, the collision term in the right hand side drives the distribution towards isotropy, at a rate controlled by the relaxation time τ_R.

Kinetics in terms of $\mathcal{L}$-moments. Although Eq. (1) can be easily solved numerically, more insight can be gained by using an alternative, albeit approximate, approach that eliminates from the description as much of irrelevant information as possible. Thus, in this paper, instead of considering the full distribution $f(\mathbf{p}, \tau)$, we focus on some of its moments, introduced in Ref. [12]:

$$\mathcal{L}_n = \int \frac{d^3\mathbf{p}}{(2\pi)^3 p^0} |\mathbf{p}|^2 P_{2n}(p_z/|\mathbf{p}|) f(\mathbf{p},\tau), \qquad (2)$$

where P_{2n} is a Legendre polynomial of order $2n$. The moments $\mathcal{L}_n$ with $n \geq 1$ describe the momentum anisotropy of the system. In particular $\mathcal{L}_1 = \mathcal{P}_L - \mathcal{P}_T$ reflects the asymmetry between longitudinal (P_L) and transverse (P_T) pressures. The moment $\mathcal{L}_0$ coincides with the energy density, $\mathcal{L}_0 = \varepsilon = P_L + 2P_T$. Observe that the momentum weight of the integration in Eq. (2) is always $|\mathbf{p}|^2$, instead of being an increasing power of $|\mathbf{p}|$ as is the case in more standard approaches (see e.g. [13]). Thus, the $\mathcal{L}_n$'s contain little information on the radial shape of the momentum distribution, preventing us for instance to reconstruct from them the full distribution. However, this radial shape plays a marginal role in the isotropization of the momentum distribution, which is our main concern here. Note that all the $\mathcal{L}_n$ have the same dimension.

By using the recursion relations among the Legendre polynomials, we can recast Eq. (1) into the following (infinite) set of coupled equations

$$\begin{aligned}\frac{\partial \mathcal{L}_n}{\partial \tau} &= -\frac{1}{\tau}\left[a_n\mathcal{L}_n + b_n\mathcal{L}_{n-1} + c_n\mathcal{L}_{n+1}\right] - \frac{\mathcal{L}_n}{\tau_R} \quad (n\geq 1)\\ \frac{\partial \mathcal{L}_0}{\partial \tau} &= -\frac{1}{\tau}\left[a_0\mathcal{L}_0 + c_0\mathcal{L}_1\right], \end{aligned} \tag{3}$$

where the coefficients a_n, b_n, c_n are pure numbers

$$\begin{aligned} a_n &= \frac{2(14n^2+7n-2)}{(4n-1)(4n+3)}, \quad b_n = \frac{(2n-1)2n(2n+2)}{(4n-1)(4n+1)},\\ c_n &= \frac{(1-2n)(2n+1)(2n+2)}{(4n+1)(4n+3)}, \end{aligned} \tag{4a}$$

entirely determined by the free streaming part of the kinetic equation. Note that the collision term does not affect directly the energy density, but only the moments with $n \geq 1$. In fact, if one ignores the expansion, i.e., set $a_n = b_n = c_n = 0$, the moments evolve according to

$$\mathcal{L}_0(\tau) = \mathcal{L}_0(0), \quad \mathcal{L}_n(\tau) = \mathcal{L}_n(0)\,\mathrm{e}^{-\tau/\tau_R}. \tag{5}$$

This solution illustrates the role of the collisions in erasing the anisotropy of the momentum distribution as the system approaches equilibrium. Of course, the expansion prevents the system to ever reach this trivial equilibrium fixed point: instead, the system goes into an hydrodynamical regime, as we shall discuss later.

The system of Eqs. (3) lends itself to simple truncations. Thus by ignoring all moments of order higher than n, one obtains a finite set of $n+1$ equations that can be easily solved. The accuracy of such a procedure can be judged from Fig. 1, where the moments obtained from various truncations are compared with those of the numerical solution of Eq. (1) for an initial distribution typical of a heavy ion collision: $f(\tau_0, p_T, p_z) = f_0\Theta\left(Q_s - \sqrt{\xi^2 p_z^2 + p_T^2}\right)$ with $f_0 = 0.1$, $\xi = 1.5$, corresponding to an initial momentum anisotropy $\mathcal{P}_L/\mathcal{P}_T \approx 0.5$, and $\tau_0 = Q_s^{-1}$ [12]. Already the lowest order truncation at $n = 1$ captures the qualitative behaviour of the full solution. Note that the approach to the exact solution is alternating, which offers an estimate of the truncation error. The energy density approaches smoothly the hydrodynamic regime as $\tau \gtrsim \tau_R$, while the non monotonous behaviour of the ratio $\mathcal{L}_1/\mathcal{L}_0$ reflects the competition between expansion and collisional effects that we now analyze in more detail, starting with the free streaming regime.

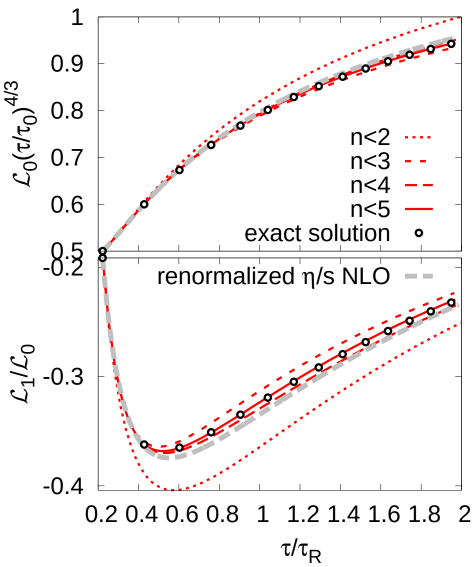

FIG. 1. (Color online) Comparison of the $\mathcal{L}$-moment equations obtained from various truncation of Eqs. (3) (lines), with those of the numerical solution of the kinetic equation (1) (symbols).

The free streaming fixed point. The free streaming regime is described by Eq. (3) where one ignores the collision term. It is not hard to see that the resulting equation possesses a stable solution at large time, in which all moments decay as $1/\tau$ and are proportional to each other: $\mathcal{L}_n(\tau) = A_n\mathcal{L}_0(\tau)$, where the dimensionless constants A_n characterize the moments of a distribution that is flat in the p_z direction [12]

$$A_n = P_{2n}(0) = (-1)^n\,\frac{(2n-1)!!}{(2n)!!}. \tag{6}$$

Note that $A_1 = -1/2$, corresponding to a vanishing longitudinal pressure. As for the factor $1/\tau$ it reflects the conservation of the energy in the increasing comoving volume ($\tau\varepsilon(\tau) = \text{cste}$). Defining

$$g_n(\tau) = \tau\partial_\tau \ln \mathcal{L}_n, \tag{7}$$

we get from Eq. (3)

$$g_n(\tau) = -a_n - b_n\frac{\mathcal{L}_{n-1}}{\mathcal{L}_n} - c_n\frac{\mathcal{L}_{n+1}}{\mathcal{L}_n} - (1-\delta_{n0})\frac{\tau}{\tau_R}. \tag{8}$$

The solution above corresponds to a fixed point for the g_n's. Dropping the last term, and using the expression (6) for the ratio of moments, one indeed verifies easily that for all n, $g_n(\tau) = -1$. If the initial ratios of moments are chosen according to Eq. (6), the g_n's remain constant in time (all equal to -1), whereas for arbitrary initial conditions, they will reach the fixed point at late time. Note that the fixed point obtained from a truncation at a finite order differs slightly from -1: for instance, in the simplest truncation at $n < 2$, $g_0 = g_1 = -0.92937$ instead of -1, and $A_1 \approx -0.6$ instead of -0.5.

The hydrodynamic fixed point. We know from our previous study [12] that, at late times, $\mathcal{L}_n(\tau)$ admits the following expansion, analogous to a gradient expansion[1]

$$\mathcal{L}_n(\tau) = \frac{1}{\tau^n}\sum_{m=0}^{\infty}\frac{\alpha_n^{(m)}}{\tau^m}\,. \tag{9}$$

The coefficients in Eq. (9) are nothing but transport coefficients, except for the first moment, equal to the energy density, i.e., $\alpha_0^{(m)} = \varepsilon\delta_{m0}$. The behavior of $\varepsilon(\tau)$ at large time is obtained from Eq. (3), ignoring the contribution of $\mathcal{L}_1$. Since $a_0 = 4/3$, this behavior is that of ideal hydrodynamics, $\varepsilon(\tau) \sim \tau^{-4/3}$, and hence $T(\tau) \sim \tau^{-1/3}$. The leading and sub-leading transport coefficients in Eq. (9) can be determined analytically. To do so, we return to Eq. (8) and note that a cancellation of the relaxation term has to occur in order to eliminate the exponential decaying contributions to the moments. This cancellation determines the leading order coefficient, viz. $\alpha_n^{(0)} = (-\tau_R)^n\varepsilon\prod_{i=1}^n b_i$. In particular, $\alpha_1^{(0)} = -b_1\tau_R\varepsilon = -2\eta$, with η the shear viscosity. In a conformal invariant setting [14], we allow τ_R to depend on the temperature, with $\tau_R T(\tau)$ kept constant[2]. Then, one gets $\alpha_n^{(0)} \sim \tau^{-(4-n)/3}$ which implies that in leading order, $\mathcal{L}_n(\tau) \sim \tau^{-(4+2n)/3}$. This defines the hydrodynamic fixed point, $g_n(\tau) = -(4+2n)/3$.[3] The subleading coefficients in Eq. (9) are then fixed by imposing this asymptotic power law, which yields

$$\frac{\alpha_n^{(1)}}{\alpha_n^{(0)}} = \tau_R b_n\left[\frac{1}{b_n}\left(\frac{4+2n}{3} - a_n\right) - \frac{\alpha_{n-1}^{(1)}}{\alpha_n^{(0)}}\right]. \tag{10}$$

The first few coefficients reproduce the values of known transport coefficients [12, 15], for instance $\alpha_2^{(0)} = \frac{64}{105}\varepsilon\tau_R^2 = \frac{4}{3}(\lambda_1 + \eta\tau_\pi)$, $\alpha_1^{(1)} = -\frac{32}{315}\varepsilon\tau_R^2 = \frac{4}{3}(\lambda_1 - \eta\tau_\pi)$, with λ_1 and τ_π as defined in [14].

The attractor. One may define an attractor solution as the particular solution of Eqs. (3) which, at short time, coincides with the free streaming fixed point $g_n = -1$, and at large time goes over to the hydrodynamic fixed point. It can be determined numerically, by solving Eqs. (3) with initial conditions specified by the constants (6). We have checked that g_0 obtained in this way is consistent with what was found by other methods in Ref. [3, 4]. The solution, obtained by truncating Eqs. (3) at $n < 20$, is displayed in Fig. 2 for the first few $g_n(\tau)$. The universal character of the curves is worth emphasizing. All the g_n's behave in the same way, interpolating between the two fixed point $g_n \approx -1$[4] and $g_n = -(4+2n)/3$, the transition occurring when $\tau \sim \tau_R$.

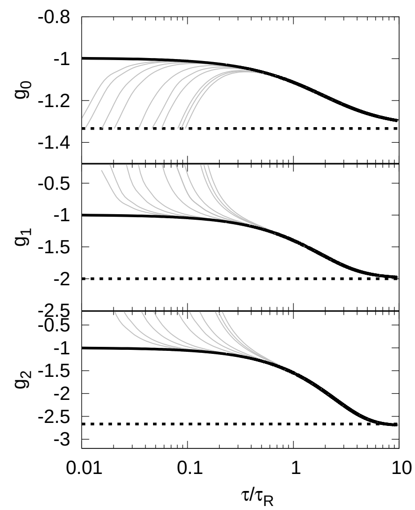

FIG. 2. Attractor solutions (black solid lines) to the $\mathcal{L}$-moment equations cut at $n < 20$, in terms of g_0, g_1 and g_2. Dotted lines correspond to the hydrodynamic fixed point. Solutions with random initial conditions are shown in grey.

Hydrodynamics. At this point, we note that the truncations of the equations (3) for the moments are closely related to successive viscous corrections to hydrodynamics. We have already seen that the lowest order truncation, i.e., with only $\mathcal{L}_0$ non vanishing, is identical to ideal hydrodynamics. The truncation at order $n = 1$ yields two coupled equations that can be cast in the form

$$\partial_\tau\epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{\Pi}{\tau}\,,\quad \partial_\tau\Pi = \frac{4}{3}\frac{\eta}{\tau\tau_R} - a_1\frac{\Pi}{\tau} - \frac{\Pi}{\tau_R}\,, \tag{11}$$

where $\Pi \equiv -c_0\mathcal{L}_1$, and we used the leading order relation $4\eta/(3\tau\tau_R) = c_0 b_1\varepsilon/\tau$. These are just the second order viscous hydrodynamic equations, in the version of Ref. [16] with $\beta_{\pi\pi} = a_1 = 38/21$. The first order viscous hydrodynamics uses the solution of the second equation (11) for small τ_R, viz. $\Pi \simeq 4\eta/(3\tau) = (16/45)\varepsilon(\tau_R/\tau)$. The much studied (lack of) convergence of the hydrodynamic gradient expansion in the context of Bjorken flow concerns the series of the coefficients $\alpha_1^{(n)}$ in Eq. (9) for $\mathcal{L}_1 \sim \Pi$, as can be deduced from the solution of the coupled equations (11) at large time [5].

Taking higher moments into account is tantamount to including higher order viscous corrections. For instance,

[1] For Bjorken flow, the gradient expansion coincides with an expansion in powers of τ_R/τ, which may also be viewed as an expansion in Knudsen number.

[2] The constant is given by $\tau_R T(\tau) = 5\eta/s$, with the entropy density given by $s = 4\varepsilon/(3T)$.

[3] In the conformal invariant setting, this result could also be obtained from a simple dimensional analysis. For a time-independent relaxation time, the hydrodynamic fixed point is instead $g_n(\tau) = -(4+3n)/3$.

[4] Because of the truncation at $n < 20$, the fixed point does not lie exactly at -1, but at -1.00294

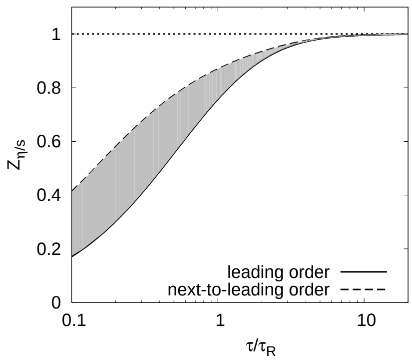

FIG. 3. Renormalization constant $Z_{\eta/s}$ as a function of τ/τ_R. The leading order corresponds to Eq. (14), the next-to-leading order include the correction due to $g_3(\tau)$.

the lowest order contribution of $\mathcal{L}_2$ to the equation for $\mathcal{L}_1$ reads

$$\frac{c_0 c_1 \mathcal{L}_2}{\tau} = \frac{c_1 b_2}{c_0 b_1 \varepsilon} \frac{\Pi^2}{\tau} \tag{12}$$

where we have used Eqs. (9) and (10) to write $\mathcal{L}_2 = \alpha_2(0)/\tau^2 = \alpha_2^{(0)}/(\alpha_1^{(0)} c_0)^2 \Pi^2$. It can be verified that the correction (12) coincides with the third order viscous correction derived in Ref. [17]. Obviously, it would be straightforward to obtain in this way higher order viscous corrections, if needed. Note that since $b_n \sim n$ at large n, $\alpha_n^{(0)} \propto n!$, and the series of the $\alpha_n^{(0)}$ suffers from the same lack of convergence as that of the $\alpha_1^{(n)}$ determining the viscous part of the energy momentum tensor.

Renormalization of η/s. Alternatively, the effects of the higher moments can be treated as a renormalization of the viscosity entering the equations for $\mathcal{L}_0$ and $\mathcal{L}_1$. To see that, rewrite the equation for $\mathcal{L}_1$ as

$$\partial_\tau \mathcal{L}_1 = -\frac{1}{\tau}\left(a_1 \mathcal{L}_1 + b_1 \mathcal{L}_0\right) - \left[1 + \frac{c_1 \tau_R}{\tau} \frac{\mathcal{L}_2}{\mathcal{L}_1}\right] \frac{\mathcal{L}_1}{\tau_R}, \tag{13}$$

with $Z_{\eta/s}^{-1} \equiv \left[1 + \frac{c_1 \tau_R}{\tau} \frac{\mathcal{L}_2}{\mathcal{L}_1}\right]$. The dimensionless ratio $\mathcal{L}_2/\mathcal{L}_1$ is analytically related to the attractor $g_2(\tau)$, the leading order result being

$$\frac{\mathcal{L}_2}{\mathcal{L}_1} = -\frac{b_2}{a_2 + \tau/\tau_R + g_2(\tau)} \, . \tag{14}$$

Sub-leading contributions involving higher g_n's can be obtained iteratively. The quantity $Z_{\eta/s}$ in Eq. (13) then defines a multiplicative renormalizaiton of η/s (or equivalently of τ_R: $\tau_R \to Z_{\eta/s}\tau_R$), whose variation with τ_R is displayed in Fig. 3. Since successive corrections alternate in sign, the grey band provides an estimate of the error. At large times, corresponding to a system in local thermal equilibrium, $Z_{\eta/s}$ is close to unity. For systems far-from-equilibrium, $Z_{\eta/s}$ tends to vanish. Thus, in systems out-of-equilibrium, higher order viscous corrections effectively reduce the value of η/s entering the second order viscous hydrodynamic equations, an effect first pointed out by Lublinsky and Shuryak [18]. As can be seen on Fig. 1 (grey dashed line), this simple renormalization brings the solution of the lowest non trivial truncation quite close to the exact solution. That is, with this correction, second order viscous hydrodynamics reproduces accurately the exact solution of the kinetic theory.

In summary, we have seen that it is possible for viscous hydrodynamics to describe accurately the evolution of boost invariant plasmas, even in regimes where the usual conditions of applicability of hydrodynamics are not satisfied. This is because the viscous hydrodynamic equations can be mapped into equations for moments of the momentum distribution that account exactly for the underlying kinetic theory. Although the present discussion relies on specific properties of Bjorken flow and the use of a simplified kinetic equation, we expect some general features to be robust, such as the existence of the free streaming and the hydrodynamic fixed points[5], joined by an attractor solution, or the renormalization of the effective viscosity. Clearly these results may have impact on the interpretation of heavy ion data and deserve further study.

ACKNOWLEDGEMENTS

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