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Relating centrality to impact parameter in nucleus-nucleus collisions

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In ultrarelativistic heavy-ion experiments, one estimates the centrality of a collision using a single observable, say n, typically given by the transverse energy or the number of tracks observed in a dedicated detector. The correlation between n and the impact parameter, b, of the collision is then inferred by fitting a specific model of the collision dynamics, such as the Glauber model, to experimental data. The goal of this paper is to assess precisely which information about impact parameter can be extracted from data without any specific model of the collision. Under the sole assumption that the probability distribution of n is Gaussian for a fixed b, we show that the probability distribution of impact parameter in a narrow centrality bin can be accurately reconstructed up to 5% centrality. We apply our methodology to Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) data. We propose a simple measure of the precision of the centrality determination, which can be used to compare different experiments.

I. INTRODUCTION

The impact parameter, b, of an ultrarelativistic nucleus-nucleus collision is a crucial quantity. It determines the size and transverse shape of the quark-gluon matter formed in the collision. Central collisions, at small b, yield large and round interaction regions, while peripheral collisions, characterized by larger values of impact parameter, yield smaller interaction regions with a pronounced elliptical anisotropy. The centrality dependence of various observables provides, then, insight into their dependence on the global geometry. The energy loss of high-momentum particles [1,2] or jets [3] is larger in central collisions, as it increases with the length of the path traversed by the particles inside the quark-gluon plasma. By contrast, elliptic flow [4,5] originates from the elliptical shape of the nuclear overlap region, and is larger in peripheral events [6].

The impact parameter of a single collision, even though it is a perfectly well-defined quantity at ultrarelativistic energies (in the sense that the quantum uncertainty is negligible), is not directly measurable. In experiments, impact parameter is estimated using a single observable, which we denote generically by n [7]. Depending on the experiment, n is either the number of particles (multiplicity) in a given detector [8,11] or the transverse energy deposited in a calorimeter [12,13]. The idea is that collisions with a small impact parameter produce on average larger values of n. However, the relation between n and b is not one-to-one, and the variation of n with b is not known a priori. This relation is usually inferred from a microscopic model of the collision, such as HIJING [14], or a two-component Glauber model [15] coupled with a simple model of particle production. The parameters of these models are tuned to reproduce the observed probability distribution of n. While these models offer a convenient parameterization, they may not describe the actual dynamics of a collision. This is suggested by the fact that different sets of parameters must be used for different colliding systems, and by the observation that the two-component Glauber model is disfavored by analyses of U+U collisions [16,17].

The goal of this article is to assess which information about the actual values of impact parameter can be extracted from the measured distributions of n, with as little theoretical bias as possible. In particular, as we shall see in the following sections, we do not need to introduce the concept of “participant nucleon”, which is a key ingredient of many microscopic models, but not a measurable quantity.

The term centrality originally refers to a classification according to impact parameter. In nowadays experiments, however, it refers to the classification of the collisions in terms of the parameter n. To avoid confusion, we call b-centrality the centrality determined with respect to impact parameter. The corresponding definitions are recalled in Sec. [I] In Sec. [II] we show that a correspondence between n and b can be drawn under the sole assumption that fluctuations of n for a given impact parameter are Gaussian. This Gaussian is characterized by a mean \( \bar{n} \) and a width \( \sigma \), which both depend on impact parameter. We test the validity of this assumption in Sec. [III] using model calculations. We argue that data allow to reconstruct unambiguously the full impact parameter dependence of the mean \( \bar{n} \) and the width \( \sigma \) for central collisions, and we explain how this can be done in practice. In Sec. [IV] we validate the proposed procedure using model calculations, where the impact parameter is known. We show that the fluctuations of impact parameter at a fixed centrality can be unambiguously reconstructed, and we apply this method to experimental data in Sec. [V].
II. TWO DEFINITIONS OF CENTRALITY

Collisions can be classified according to their impact parameter, $b$. We define the centrality, $c_b$, as the cumulative probability distribution of $b$:

$$c_b \equiv \frac{1}{\sigma_{inel}} \int_0^b P_{inel}(b')2\pi b'db',$$

where $\sigma_{inel}$ is the inelastic nucleus-nucleus cross section and $P_{inel}(b)$ is the probability that an inelastic collision occurs at impact parameter $b$. We name $c_b$ the $b$-centrality of the collision, in order to distinguish it from the usual centrality defined in heavy-ion experiments, to be discussed below. The probability distribution of $c_b$ is flat by construction: $P(c_b) = 1$ for $0 < c_b < 1$. Neither $b$ nor $c_b$ can be measured experimentally. They are known only in model calculations.\(^3\)

![Graph of probability distribution of $n/n_{knee}$ for different experiments](image)

**FIG. 1.** (Color online) Histograms of the probability distribution of $n$ measured by different experiments. Dashed line: STAR data on Au+Au collisions at $\sqrt{s} = 130$ GeV. Dot-dashed line: ALICE data on Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV. Solid line: ATLAS data on Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV. The horizontal axis of each histogram has been rescaled by the value of $n$ at the knee (see text).

In experiments, collisions are instead classified according to a single observable, $n$. The STAR Collaboration\(^2\) defines $n$ as the number of tracks of charged particles detected in the pseudorapidity window $-1 < \eta < 1$. The ALICE Collaboration\(^3\) uses the number of hits in two scintillators covering the windows $-3.7 < \eta < -1.7$ and $2.8 < \eta < 5.1$. The ATLAS Collaboration\(^4\) uses the energy deposited in two calorimeters in the symmetric windows $-4.9 < \eta < -3.2$ and $3.2 < \eta < 4.9$. Figure 1 displays the probability distribution of $n$, $P(n)$, measured by these three experiments.\(^5\) Since different detectors have different acceptance and efficiency, and $n$ can refer to a multiplicity or an energy, we rescale the value of $n$ by its value at the knee, to be defined precisely in Sec. III. Once rescaled, ATLAS and ALICE data are almost identical. STAR data differ in the tail, which is twice as broad\(^6\).

The cumulative distribution of $n$ defines the experimental measure of centrality, which we denote by $c$. It is defined by

$$c \equiv \int_n^\infty P(n')dn'.$$

Note that the centrality classification is in ascending order for $b$ and in descending order for $n$, which explains the different integration limits in Eqs. (1) and (2). The probability distribution of $c$ is also flat by construction: $P(c) = 1$ for $0 < c < 1$.

We have thus defined two measures of the centrality: $c_b$ and $c$, depending on whether one sorts events according to $b$ or to $n$. If the relation between $n$ and $b$ is one-to-one, both measures coincide, $c = c_b$. In practice, one observes a range of value of $n$ at a given value of $b$. The joint distribution of $n$ and $b$ is usually inferred from a specific model of the collision\(^11\)\(^15\).

III. RELATING CENTRALITY TO $b$-CENTRALITY

Here we simply assume that the probability of $n$ for fixed $b$ is Gaussian\(^7\):

$$P(n|c_b) = \frac{1}{\sigma(c_b)/\sqrt{2\pi}} \exp\left(-\frac{(n - \bar{n}(c_b))^2}{2\sigma(c_b)^2}\right),$$

where both the mean $\bar{n}$ and the width $\sigma$ depend on the impact parameter or, equivalently, on $c_b$. We expect this to be a good approximation because of the central limit theorem: $n$ is a multiplicity, or transverse energy, which gets contributions from many collision processes which are located at different points in the transverse plane, and, therefore, causally disconnected and independent.

It is useful to check this Gaussian approximation on a model\(^8\). We take $n$ as the total entropy deposited in

---

1 The results in this paper use the variable $c_b$, but one can easily express them in terms of $b$ using the change of variables $c_b = \pi b^2/\sigma_{inel}$. The value of $\sigma_{inel}$ needs to be taken from either data or some collision model.

2 Data on $P(n)$ collected by the CMS and PHENIX Collaborations are shown in\(^12\) and\(^10\), respectively.

3 We use uncalibrated 130 GeV STAR data\(^9\) rather than calibrated 200 GeV data\(^16\). We have checked that, once rescaled, the two distributions are very similar. The advantage of 130 GeV data is that errors are provided, so that we are able to assess the quality of our fits, as we shall see in Sec.\(^7\).

4 We stress, though, that our subsequent analysis of the experimental results does not rely on whether or not the chosen model provides a good description of data.
FIG. 2. (Color online) Shaded areas: Histograms of the probability distribution of \( n \) for fixed impact parameter in Pb+Pb collisions at \( \sqrt{s} = 2.76 \text{ TeV} \) in the TRENTo model. The values \( b = 0, 5, 8, 12 \text{ fm} \) were used, which correspond to \( c_b = 0, 10\%, 26\%, 58\% \), respectively. We have generated \( 5 \times 10^5 \) events for each value of \( b \). Solid lines are Gaussian fits. The quantity \( n \) is in arbitrary units.

We now apply the fitting procedure described in the previous section to \( P(n) \). For scenario (A), we vary the lower cutoff from \( n_{\text{min}} = 1.8 \) to \( n_{\text{min}} = 9.1 \), corresponding to centralities \( c = 80\% \) and \( c = 62\% \), and we find that the problem is underconstrained, in the sense that one cannot determine two unknown functions \( \bar{n}(c_b) \) and \( \sigma(c_b) \) from a single function \( P(n) \). We shall argue that one can only constrain the mean \( \bar{n}(c_b) \), and the value of the width for central collisions, \( \sigma(0) \). Since the variation of \( \sigma \) with \( c_b \) cannot be determined from data alone, we test two different scenarios:

- (A) \( \sigma(c_b) = \sigma(0) \sqrt{\bar{n}(c_b)/\bar{n}(0)} \)
- (B) \( \sigma(c_b) = \sigma(0) \).

The first scenario, (A), assumes that the variance is proportional to the mean, which would be true if \( n \) were the sum of contributions from independent nucleon-nucleon collisions. Scenario (B) is motivated by the observation that the width of the histograms observed in Fig. 2 varies little between \( b = 0 \) and \( b = 8 \text{ fm} \). We state that this is an artifact of the Monte Carlo model, where particle production is essentially determined by the participant nucleons. Since the number of participant nucleons is bounded by the total number of nucleons, fluctuations of \( n \) are consequently reduced by the presence of this upper cutoff. There is, however, no deep theoretical reason to believe that this particular feature of the Monte Carlo models is realistic.

For each scenario, (A) or (B), we need a smooth function \( \bar{n}(c_b) \), and a constant, \( \sigma(0) \), such that \( P(n) \) defined by Eqs. (3) and (5) fit experimental data. We use the following functional form of \( \bar{n}(c_b) \), which guarantees positivity:

\[
\bar{n}(c_b) = n_{\text{knee}} \exp \left(-a_1 c_b - a_2 c_b^2 - a_3 c_b^3\right) \quad (6)
\]

One could as well use other functional forms, requiring the final fitting function to be a smooth, positive, monotonically decreasing function of \( c_b \), with no singularities in the interval \( 0 \leq c_b < 1 \). We carry out a 5-parameter fit to \( P(n) \) using Eqs. (3) and (5), with parameters given by \( n_{\text{knee}}, a_1, a_2, a_3 \) and \( \sigma(0) \). In order to eliminate peripheral collisions from the fit, we only use values of \( n \) above a cutoff \( n_{\text{min}} \), which we specify in each case.

IV. VALIDATION OF THE METHOD

We now validate this procedure of relating \( n \) to \( c_b \) using Monte Carlo simulations, where both \( b \) and \( n \) are known in each event. We simulate Pb+Pb collisions at \( \sqrt{s} = 2.76 \text{ TeV} \) using the same setup of TRENTo as in Fig. 2 and again we use the entropy of each event to construct the probability distribution \( P(n) \). We generate \( 10^7 \) events. We determine \( c_b \) of each event by sorting events according to \( b \), and \( c \) by sorting them according to \( n \). Symbols in Fig. 3 correspond to the distribution \( P(n) \) from this Monte Carlo calculation.

A crucial quantity which we will use throughout this work is the position of the knee of the distribution of \( n \). We define it as the mean value of \( n \) at zero impact parameter:

\[
n_{\text{knee}} \equiv \bar{n}(0) \quad (4)
\]

The observed distribution of \( n \), \( P(n) \), is eventually obtained by integrating Eq. (3) over \( c_b \), i.e.,

\[
P(n) = \int_0^1 P(n|c_b) dc_b \quad (5)
\]

In this paper, we determine smooth functions \( \bar{n}(c_b) \) and \( \sigma(c_b) \) such that \( P(n) \) matches experimental data. This
the results of the fit are stable. The solid line in Fig. 3 displays the fit corresponding to the larger value of $n_{\text{min}}$. The fit provides an excellent description of $P(n)$ in the model. For case (B), fluctuations are larger and we use a larger cutoff, $n_{\text{min}} = 36$. Above the cutoff, the two fits are indistinguishable in Fig. 3. The fit parameters are listed in Table I for both scenarios. The error on the width $\sigma$ is in arbitrary units.

In Fig. 4, we show $\bar{n}(c_b)$ returned by the fit, i.e., we calculate Eq. 4 using the parameters of Table I for scenario (A). We display the comparison between this analytical estimate and the results directly obtained by binning TRENTo results in $c_b$, and computing the mean value of $n$ in each bin. Agreement is within 0.5%, all the way up to $c_b = 70\%$, which corresponds to a value of $n$ smaller than the lower cutoff applied to $P(n)$. We also calculate $\sigma(0)$ directly, by generating $10^5$ collisions with $b = 0$ fm, and computing the standard deviation of $n$. The resulting value of $\sigma(0)$ agrees with the corresponding fit parameter.

It is useful to note that approximate values of $\sigma(0)$ and $\bar{n}(c_b)$ can be read off directly from the shape of $P(n)$. Specifically, $\sigma(0)$ can be inferred from the width of the tail of the distribution on the right of the knee. $\bar{n}(c_b)$ is instead related to the shape of $P(n)$ left of the knee. If $\sigma(c_b)$ is very small, the distribution of $n$ for fixed $b$ is very narrow, so that $c$ and $c_b$ tend to coincide. In this limit, $\bar{n}(c_b)$ defined by Eq. (2) is equal to the inverse function of $\bar{n}(c)$.

$$P(n = \bar{n}(c_b)) \simeq -\left(\frac{d\bar{n}(c_b)}{dc_b}\right)^{-1}.$$  

In the inset of Fig. 4, we check the validity of this approximation in our TRENTo calculation by direct comparison of $\bar{n}(c_b)$ to $n(c)$. We find that $\bar{n}(c)$ deviates from $n(c)$ only above $n \simeq 170$, corresponding to 1.5% centrality. This means that $\bar{n}(c_b)$ carries an information which can not be inferred from data alone only in the vicinity of $n_{\text{knee}}$.

To complete our procedure, we show how one can reconstruct the distribution of impact parameter for a given

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FIG. 5. (Color online) Probability distribution of $b$-centrality in a very narrow centrality bin. Histograms: direct calculation using TRENTo. Lines: Probability distributions of $c_b$ reconstructed through the Bayes’ theorem, Eq. (8). Solid and dashed lines correspond to scenarios (A) and (B), respectively. The second panel corresponds to $n = n_{\text{knee}}$. Note that for $n \geq n_{\text{knee}}$ (two uppermost panels), the most probable value of $c_b$ is $c_b = 0\%$. Left of the knee, on the other hand, the most probable value of $c_b$ is $c$ \textsuperscript{2}, as clearly visible in the two lowermost panels. All curves displayed in this figure have area normalized to unity.

value of $n$, i.e., at a given value of centrality. In the TRENTo model, this can done directly by sorting events into very narrow centrality bins, and then looking at the distribution of $c_b$ in each bin. We show such histograms of $c_b$ for a few selected centrality bins in Fig. 5. Each panel corresponds to a bin of width 0.1%, centered around the displayed value of $c$ (for instance, the panel with $c = 1\%$

shows the distribution of $c_b$ for $0.95\% < c < 1.05\%$).

In an experimental situation where the impact parameter is not known, these distributions can be reconstructed from $P(n)$ using Bayes’ theorem:

$$P(c_b|c) = \frac{P(c|c_b)}{P(n|c_b)} = \frac{P(n|c_b)}{P(n)},$$

where, in the first line, we have used the property that the distribution of $c_b$ and $c$ are uniform, i.e., $P(c_b) = P(c) = 1$. The distribution $P(n|c_b)$ is given by Eq. (3), where $\bar{n}(c_b)$ and $\sigma(c_b)$ can be obtained from the fitting procedure.

One first needs to determine the value of $n$ corresponding to a given centrality, $c$. This can be done straightforwardly using the fitting function, which offers a smooth interpolation of $P(cb|c)$ and $P(n|c_b)$.

Eventually, once $n$ is determined, $P(cb|c)$ is given by application of Bayes’ theorem, Eq. (8), to $P(n|cb)$ in Eq. (3). The reconstructed distributions are shown as lines in Fig. 5. Scenarios (A) and (B) are represented by solid and dashed lines, respectively. Both are in perfect agreement with the direct calculation up to 4\% centrality. Discrepancies between both scenarios and the direct calculation start to appear around $c = 8\%$, meaning that our approximated formulas for $\sigma(cb)$ starts to break down around $c \sim 10\%$ in this model calculation. In a sense, this is a consequence of the fact that the variation of $\sigma$ with $c_b$ can not be inferred from the sole $P(n)$.

### V. APPLICATION TO DATA

Our method of relating $c$ to $c_b$ being validated on model calculations, we apply now the fitting procedure to experimental data. We fit the experimental curves of $P(n)$

<table>
<thead>
<tr>
<th>$n_{\text{knee}}$</th>
<th>STAR [9]</th>
<th>ALICE [11]</th>
<th>ATLAS [13]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(0)$</td>
<td>21.5</td>
<td>731</td>
<td>0.1131</td>
</tr>
<tr>
<td>$a_1$</td>
<td>3.55</td>
<td>4.11</td>
<td>4.05</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.8</td>
<td>-1.9</td>
<td>-1.5</td>
</tr>
<tr>
<td>$a_3$</td>
<td>1.6</td>
<td>4.4</td>
<td>4.1</td>
</tr>
</tbody>
</table>

TABLE II. Values of fit parameters for several experiments.

---

\textsuperscript{6} Experimental data typically present $P(n)$ in a bin center, and the centrality, $c$, at the boundary between two bins.
shown in Fig. 1 using Eq. (3). The results shown in this section correspond to scenario (A), with a lower centrality cutoff. We have checked that results are stable if one varies the cutoff, or if one uses scenario (B). The values of the fit parameters extracted from experimental data are given in Table III for all the analyzed experiments. The values of $n_{knee}$ and $\sigma(0)$ are in the same units as $n$, which are usually arbitrary, and vary from one experiment to the other. The other fit parameters are dimensionless.

\begin{equation}
\frac{\sigma(0)}{n_{knee} a_1 \sqrt{2\pi}}
\end{equation}

where, in the last equality, we have used Eq. (3). We have checked that, using this simple estimate, the values of $c_{knee}$ shown in Table III are reproduced to a good accuracy, within 1%.

Ultimately, using Bayes’ theorem, we can use the fits of experimental data to reconstruct the distribution of impact parameter for a fixed centrality. Figure 7 presents the distribution of $c_b$ for a few selected values of centrality percentile. The distributions obtained from ALICE and ATLAS data are very similar, therefore, we show only a comparison between the curves derived from ALICE and STAR data. The distribution extracted from STAR data is much broader than the one extracted from ALICE data. This is a direct consequence of the wider tail of $P(n)$ measured by the STAR Collaboration [see Fig. 1]. Note that, as expected, the distributions extracted from STAR and ALICE data are almost identical if one rescales both $c$ and $c_b$ by $c_{knee}$, as we show in Fig. 8.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Experiment & $c_{knee}$ \\
\hline
STAR & $0.81\% \pm 0.10\%$ \\
ALICE & $0.349\% \pm 0.023\%$ \\
ATLAS & $0.313\% \pm 0.011\%$ \\
\hline
\end{tabular}
\caption{Fraction of events above the knee for various heavy-ion experiments.}
\end{table}

\section{VI. Conclusion}

We have shown that even though the impact parameter of a nucleus-nucleus collision is not a measurable quantity, precise information about impact parameter is contained in available experimental data. We have delineated a procedure allowing to reconstruct accurately the probability distribution of impact parameter at a given centrality (as defined experimentally by a multiplicity or a transverse energy), up to $5 - 10\%$ centrality. This reconstruction does not involve the concept of participant

![Graph showing distribution of impact parameter](image-url)
We have shown that the mean value of $n$ at a fixed impact parameter can be reconstructed accurately up to 70% $b$-centrality. The standard deviation of $n$ around the mean can instead be reconstructed only for $b = 0$: Its centrality dependence cannot be inferred from $P(n)$ alone. The mean and the standard deviation of $n$ at fixed $b$ are more natural quantities from a theory point of view than $P(n)$, as they can be directly obtained in a model by fixing the impact parameter. While the standard deviation may depend on detector details (in particular, purely statistical fluctuations are larger in relative value if the acceptance is smaller), the mean, $\bar{n}(c_b)$, provides a robust quantity for model comparisons.

It would be useful to extend this study to proton-nucleus collisions. However, our assumption that $n$ has Gaussian fluctuations is not satisfied in model calculations, even for central collisions. We have checked that the fit procedure is less successful in describing p+Pb collisions at $\sqrt{s} = 5.02$ TeV [20]. In particular, the tail of the distribution of $P(n)$ is not as well reproduced, because it is not Gaussian. It will be worth investigating further the reasons of this failure of the Gaussian ansatz, to shed more light on the eventual mechanism of medium production in smaller collision systems.

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