Axial-Vector Coupling Constant in Nuclei and Dense Matter

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(Dated: June 1, 2017)

The in-medium property of the axial-vector coupling constant g_A in nuclei and dense baryonic matter is reformulated in terms of the recently constructed scale-invariant hidden local symmetric (bsHLS) Lagrangian. It is shown that unlike the pion decay constant that slides with the vacuum change induced by density, the axial-current constant g_A remains unmodified up to high density relevant to compact stars in nuclear Gamow-Teller transitions (involving the space component of the axial current) whereas it gets strongly enhanced in axial-charge transitions (involving the time component of the axial current) as density nears nuclear matter density n_0 and stays more or less constant up to $\sim 6n_0$. The implications of these predictions on giant Gamow-Teller resonances in nuclei and on first-forbidden beta transitions (relevant to nuclear astrophysical processes) are discussed.

arXiv:1705.10864v1 [nucl-th] 30 May 2017

I. INTRODUCTION

How the weak axial-vector coupling constant g_A behaves in nuclei has a very long history. A fundamental quantity, it has impacts on nuclear structure as well as nuclear astrophysical processes. The basic issue raised is associated with how chiral symmetry, a fundamental property of QCD, is manifested in nuclei where the presence of strongly interacting nucleons is expected to modify the vacuum and hence the modified vacuum will affect the quark condensate $\Sigma \equiv |\langle\bar{q}q\rangle|$ as density is increased. It is now well established that the pion decay constant f_π decreases, reflecting the decrease of the condensate Σ at increasing density, which is an *intrinsic* property of QCD. It seems natural then to expect that the axial coupling constant will undergo a similar *intrinsic* decrease in nuclear matter.

There has been a suggestion since 1970's [1] that the axial coupling constant, $g_A \approx 1.27$ determined in the matter-free space, quenches to $g_A \approx 1$ in nuclei [2], which has invited an interpretation that it, in consistency with the dropping of f_π, signals a precursor to chiral restoration. This has led to extensive studies accompanied by controversies in nuclear Gamow-Teller transitions, most notably giant Gamow-Teller resonances [3]. The key question asked is whether giant GT resonances signal a quenched axial coupling constant.

In this paper, I revisit this problem and give an extremely simple and unambiguous answer with an effective field theory in which scale symmetry and chiral symmetry of QCD are incorporated. I predict that g_A for Gamow-Teller transitions remains *unaffected* by the quark condensate that slides with density whereas it gets strongly *enhanced* in axial-charge (first-forbidden) transitions.

II. SCALE-INVARIANT HLS LAGRANGIAN

My reasoning exploits the recently formulated effective field theory Lagrangian that purports to be valid for phenomena at low density as well as for high density $\sim 6n_0$ where n_0 is the normal nuclear matter density. This formulation probes nuclear matter as well as dense matter relevant to the recently discovered $\sim$ 2-solar mass compact stars.

It is constructed by implementing scale symmetry and hidden local symmetry (HLS) to baryonic chiral Lagrangian consisting of the pseudo-Nambu-Goldstone bosons, pions (π), and baryons, focusing specifically on nucleons N. The basic assumption that underlies the construction of the effective Lagrangian, dubbed bsHLS, with b standing for baryons and s standing for the scalar meson, is that there are two hidden symmetries in QCD: one, scale symmetry broken both explicitly by the QCD trace anomaly and spontaneously with the excitation of a scalar pseudo-Nambu-Goldstone boson, i.e., "dilaton" σ; two, a local flavor symmetry higgsed to give massive ρ and ω. Neither is visible in QCD in the matter-free vacuum, but the possibility, argued in [4], is that both can appear as emergent symmetries in dense matter and control the equation of state (EoS) relevant to compact stars. I will use the same Lagrangian for calculating nuclear responses to the electro-weak current. There are no unknown parameters in the calculation.

Since the arguments are quite involved and given in great detail elsewhere, I summarize as concisely as possible the essential points that figure in the formulation. In addition to the nucleon and the pion, the Nambu-Goldstone boson of chiral symmetry, there are two additional – massive – degrees of freedom essential for the bsHLS Lagrangian: The vector mesons $V = (\rho, \omega)$ and the scalar meson denoted χ. By now very well-known procedure, the vectors V are incorporated by hidden gauge symmetry (HLS) [5], which is gauge-equivalent to non-linear sigma model, that elevates the energy scale to the scale of vector mass ~ 770 MeV. The scalar χ is incorporated by using the "conformal compensator field" transforming under scale transformation with scale dimension 1, $\chi = f_\chi e^{\sigma/f_\chi}$. Here σ is the dilaton field, a pseudo NG boson of scale symmetry,

The underlying approach to nuclear EFT with bsHLS is the Landau Fermi-liquid theory based on Wilsonian renromalization group (RG). For this, the "bare" parameters of the EFT Lagrangian are determined at a "matching scale" Λ_M from which the RG decimation is to be made for quantum theory. The matching is performed with the current correlators between the EFT and QCD, the former at the tree-order and the latter in OPE. The QCD correlators contain, in addition to perturbative quantities, the nonperturbative ones, i.e., the quark condensate $\langle\bar{q}q\rangle$, the dilaton condensate $\langle\chi\rangle$, the gluon condensate $\langle G^2_{\mu\nu}\rangle$ and mixed condensates. The matching renders the "bare" parameters of the EFT Lagrangian dependent on those condensates. Since the condensates are characteristic of the vacuum, as the vacuum changes, the condensates slide with the change. Here we are concerned with density, so those condensates must depend on density. This density dependence, inherited from QCD, is an "intrinsic" quantity to be distinguished from mundane density dependence coming from baryonic interactions. It is referred to as "intrinsic density dependence" or IDD for short.

There are two scales to consider in determining how the IDDs enter in the EFT Lagrangian.

One is the energy scale. The initial energy scale is the matching scale from which the initial (or first) RG decimation is performed. In principle it could be the chiral scale $\Lambda_\chi \sim 4\pi f_\pi \sim 1$ GeV. In practice it could be lower, say, slightly above the vector meson mass. The scale to which the first decimation is to be made could be taken typically to be the top of the Fermi sea.

The other scale is the baryon density. The density relevant for massive compact stars can reach up to as high as $\sim 6n_0$. To be able to describe reliably the properties of both normal nuclear matter and massive stars, a changeover from the known hadronic matter to a different form of matter at a density $\sim 2n_0$ is required. In [4], it is a topology change from a skyrmion matter to a half-skyrmion matter. Being topological this property is most likely robust. In quark-model approaches, it could be the hadron-quark continuity that implements continuous transitions from hadrons to strongly-coupled quark matter or quarkyonic matter [6]. I believe, as conjectured in [4], that the two approaches are in some sense equivalent.

The changeover is not a bona-fide phase transition. However it impacts importantly on the EoS, making, for instance, the nuclear symmetry energy transform from soft to hard at that density, accommodating the observed $\sim$ 2-solar mass star. Of crucial importance for the process being considered is that when the matter is treated in terms of skyrmions, complementary to the bsHLS approach, the topology change at $n_{1/2} \sim 2n_0$ makes the IDDs differ drastically from below to above that density.

It turns out that up to $n \sim n_{1/2}$, the IDD is entirely given by the dilaton condensate $\langle\chi\rangle$. The χ field is a chiral scalar whereas $\bar{q}q$ is the fourth component of the chiral four vector. Therefore the dilaton condensate is not directly connected to the quark condensate, but as mentioned below, this dilaton condensate gets locked to the pion decay constant which is related to the quark condensate. While the quark condensate does not figure explicitly in the IDD at low densities, it controls the behavior of vector-meson masses at compact-star densities, $n \gtrsim n_{1/2}$ [4].

III. AXIAL CURRENT WITH IDD

That the IDD could be entirely given by the dilaton condensate was conjectured in 1991 [7], and it is confirmed to hold up to the density $n \lesssim n_{1/2}$ [4]. What is new in the new development is that in the Wilsonian renormalization-group formulation of nuclear effective field theory adopted [4], this IDD-scaling is all that figures up to $n_{1/2}$. However it undergoes a drastic change at $n \gtrsim n_{1/2}$ [4]. This change is important for compact-star matter but does not affect the axial-current problem.

The effect of the scale-symmetry explicit breaking at the leading order is embedded entirely in the dilaton potential, so it does not enter explicitly in the axial response functions in nuclei and nuclear matter that we are interested in. This makes the calculation of the "intrinsically modified" g_A^* in nuclear medium extremely simple. All we need is the part of the bsHLS Lagrangian, scale invariant and hidden local symmetric, that describes the coupling of the nucleon to the external axial field $\mathcal{A}_\mu$. Writing out explicitly the covariant derivatives involving vector fields, hidden local and external, and keeping only the external axial vector field $\mathcal{A}_\mu$, and to the leading order in the explicit scale symmetry breaking, the relevant Lagrangian takes the form

$$\mathcal{L} = i\overline{N}\gamma^\mu\partial\mu N - \frac{\chi}{f_\chi} m_N \overline{N}N + g_A\overline{N}\gamma^\mu\gamma_5 N\mathcal{A}_\mu + \cdots \quad (1)$$

Note that the kinetic energy term and the nucleon coupling to the axial field are scale-invariant by themselves and hence do not couple to the conformal compensator field. Put in the nuclear matter background, the bare parameters of the Lagrangian will pick up the medium VeV and scale as

$$m_N^*/m_N = \langle\chi\rangle^*/f_\chi \equiv \Phi, \quad g_A^*/g_A = 1 \quad (2)$$

where f_χ is the medium-free VeV $\langle\chi\rangle_0$ and the $*$ represents the medium quantities. The first is one of the scaling relations given in [7]. The second is new and says that the Lorentz-invariant axial coupling constant does not scale in density. Now in medium, Lorentz invariance is spontaneously broken, which means that the space component, $g_A^{\rm s}$, could be different from the time component $g_A^{\rm t}$. Writing out the space and time components of the nuclear axial current operators, one obtains

$$\vec{J}_A^{\pm}(\vec{x}) = g_A^{\rm s} \sum_i \tau_i^{\pm}\vec{\sigma}_i\delta(\vec{x}-\vec{x}_i), \quad (3)$$

$$J_5^{0\pm}(\vec{x}) = -g_A^{\rm t} \sum_i \tau_i^{\pm}\vec{\sigma}_i\cdot(\vec{p}_i-\vec{k}/2)/m_N\delta(\vec{x}-\vec{x}_i) \quad (4)$$

where $\vec{p}$ is the initial momentum of the nucleon making the transition and $\vec{k}$ is the momentum carried by the axial current. In writing (3) and (4), the nonrelativistic approximation is made for the nucleon. This approximation is valid not only near n_0 but also in the density regime $n \gtrsim n_{1/2} \sim 2n_0$. This is because the nucleon mass never decreases much after the parity-doubling sets in at $n \sim n_{1/2}$ at which $m_N^* \to m_0 \approx (0.6-0.9)m_N$ [4].

A simple calculation gives

$$g_A^{\rm s} = g_A, \quad g_A^{\rm t} = g_A/\Phi \tag{5}$$

with Φ given by (2). This is the unequivocal prediction of the IDD with bsHLS.

IV. GAMOW-TELLER TRANSITIONS

One can now look at what IDD scaling (5) gives for nuclear axial transitions. First consider the Gamow-Teller transition which is dominated by the Gamow-Teller operator (3).

Following the RG procedure, one performs the double decimations as in [4]. Although the IDD does not affect the g_A^s, one has to consider what other degrees of freedom can contribute in the RG decimations. The first energy scale one encounters in the Gamow-Teller channel as one makes the first decimation from the matching scale is the Δ-hole excitation $E_{\Delta-h}$ lying ~ 300 MeV above the Fermi sea. Other channels could be safely ignored. This Δ-hole effect has to be taken into account in calculating the Gamow-Teller response functions. Since the weak current acts only once, this effect can be included in the modification of the Gamow-Teller coupling constant g_A^s. This was worked out a long time ago, which could be phrased in terms of Landau-Migdal's g_0' parameter in the Δ-N channel [8, 9] when treated in Landau's Fermi liquid theory in the space of N and Δ. This can also be phrased in terms of the Ericson-Ericson-Lorenz-Lorentz (EELL) effect in the pion-nuclear interaction [10]. If one takes g_0' equal in the NN, $N\Delta$ and Δ-Δ channels, that is, universal, then one finds that g_A^* for Gamow-Teller transitions in nuclear matter is renormalized to $g_A^* \approx 1$. There is also a QCD sum-rule result arriving at the same result [11]. This seems to indicate that this effective g_A is consistent with what was arrived at in light nuclei [2].

In order to understand what this "renormalization" means, imagine doing a large-scale, or preferably full-scale, shell-model calculation within the nucleon configuration space only. This corresponds to doing the RG decimation from the energy scale $E_{\Delta-h} \approx 300$ MeV. Hence using this *renormalized* $g_A^\star \approx 1$ in calculating the GT matrix element, one would find $(g_A^*/g_A)^2 \approx 0.6$ quenching in the strength. Indeed an observation of the famous $\sim 40\%$ quenching in Gamow-Teller strength has baffled experimentalists for many years. However, more recent experiments on giant Gamow-Teller resonances find that this quenching has more or less disappeared [3]. In terms of g_0', this means that $g_0'|_{\Delta N} \ll g_0'|_{NN}$.

This result raises several questions: First, it is a well-known fact that $g_0'|_{NN}$ is by far the strongest quasiparticle interaction controlling spin-isospin excitations in nuclei. So why the ΔN channel is so suppressed compared with the NN channel is difficult to understand; second, at least a part of the Landau parameters can be associated with an IDD. Take for example the in-medium ρ mass with its IDD given by the dialton condensate Φ. In the mean-field treatment of the anomalous orbital gyromagnetic ratio of the proton δg_l in heavy nuclei [12], it has been shown that the Landau parameter F_1' is also related to Φ when treated in the single-decimation RG using the bsHLS Lagrangian. Thus at least a part of the Landau parameters can be associated with IDD. Which part of the Landau parameters is concerned depends on the precise way the RG decimations are defined.

The message from what we have learned is this: It is a fundamental task to pin down what the Δ-hole modification of g_A is by doing a precision calculation of giant Gamow-Teller resonances in heavy nuclei and determine the deviation of g_A, if any, from the free-space value. There is no known reason why the deviation should be zero even though as shown in this paper there is no *intrinsic* QCD correction *directly associated* with the vacuum change in the chiral condensate.

V. AXIAL-CHARGE TRANSITIONS

The first forbidden β-decay process $0^- \leftrightarrow 0^+$ with $\Delta I = 1$ is governed by the axial charge operator (4). This process has an axial-vector coupling constant enhanced by $1/\Phi$ (for $\Phi < 1$) and furthermore, more importantly, receives very important one-pion exchange-current contribution with a vertex $\mathcal{A}_0\pi NN$. See Fig. 1. This vertex is of the form of current algebra with two soft pions and gives an $\mathcal{O}(1)$ correction to the single-particle operator [13]. The two-body operator is an exactly known pionic-ranged two-body operator, so it can be calculated very accurately if the accurate wave function is known. In fact the ratio R of the two-body matrix element over the one-body matrix element, surprisingly large as a meson exchange-current effect, is highly insensitive to nuclear density. It ranges $R = 0.5 \pm 0.1$ over the wide range of nuclei from light to heavy or in terms of density, $n \sim (0.5-1.0)n_0$ [14]. With the two-body effect

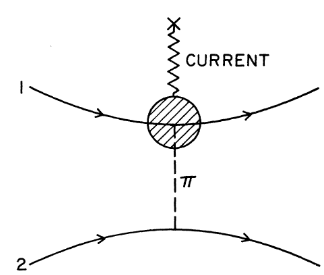

FIG. 1. Two-body exchange current. The upper vertex involves two soft pions for the axial charge transition.

taken into account, the effective axial-charge operator is obtained by making the replacement in (4) by

$$g_A^t \to g_A^{s*} = \epsilon g_A \tag{6}$$

with

$$\epsilon = \Phi^{-1}(1 + R/\Phi). \tag{7}$$

To make an estimate of ϵ, we need the scaling factor Φ. It is an easy calculation to find that in medium, the dilaton condensate is locked to the quark condensate, leading to [4]

$$\Phi = f_\chi^*/f_\chi \approx f_\pi^*/f_\pi. \tag{8}$$

The pion decay constant f_π^* is measured from deeply bound pionic nuclear systems [15]. It can be parameterized in density (up to n_0) as

$$\Phi \approx 1/(1 + 0.25n/n_0). \tag{9}$$

The prediction (7) has been well confirmed in experiments ranging from $A = 12$ to $A = 205 - 212$ [16, 17]. Here for illustrative purpose, let me quote the result in Pb nuclei. Taking the density to be near the nuclear matter density, one has $\Phi(n_0) \approx 0.8$ in accordance with Eq. (9) – which is fit to the experiment [15]. Substituting $R = 0.5 \pm 0.1$, one prediocts the enhancement factor

$$\epsilon(n_0) \approx 2.0 \pm 0.2. \tag{10}$$

This factor compares well with Warburton's result in Pb region [17]

$$\epsilon^{exp} = 2.01 \pm 0.05. \tag{11}$$

This is an old story. And there is nothing new as far as the numerical result is concerned. However it should be noted that given that nuclear structure techniques are vastly improved in the two decades and half since 1991, the ratio R could now be calculable extremely accurately for the ranges of nuclei involved, in particular in the $A = 12 - 16$ region. Furthermore the extraction of ϵ^{exp} that requires certain theoretical inputs, e.g., single-particle first-forbidden decay matrix elements, could be improved tremendously over the result of 1991 [17]. Given that the axial charge exchange-current operator is unambiguously calculable in the EFT framework, with higher-order corrections strongly suppressed, this enhanced process would constitute the most spectacular and pristine evidence for meson-exchange currents in nuclei.

What is perhaps a lot more notable is the role played by soft pions in the process. Considering the soft probe, A_0, as pionic, one can think of the upper vertex of Fig. 1 as involving two soft pions. This is the core of the current algebras of 1960, which is now fully captured in nonlinear sigma model with derivative coupling, and hence in currently successful effective quantum field theory. This can be considered as a striking case of Weinberg's "folk theorem" on effective quantum field theory "proven" in nuclear physics [18]. In fact this matter, dating from 1970's, illustrates that "what nuclear physicists have been doing all along is the correct first step in a consistent approximate scheme."

Now when the energy scale probed in nuclear processes is much less than the pion mass ~ 140 MeV, the pion can also be integrated out in the spirit of the "folk theorem" and one obtains "pionless effective field theory" for nuclear physics, which is eminently a respectable effective field theory. With no pions present, however, there is no explicit footmark of chiral symmetry, i.e., no smoking gun for the spontaneously breaking of chiral symmetry. Yet the theory seems to work fairly well in various low-energy processes involving light nuclei including the solar proton fusion process that is dominated by the Gamow-Teller operator. The question is: What about the double-soft process that makes such a big effect in the axial-charge transitions? Could it be hidden in the pionless effective field theory, somewhat like the hidden symmetries discussed above?

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