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Scale-Chiral Symmetry, Proton Mass and Sound Velocity in Compact-Star Matter

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With a light dilaton σ and the light-quark vector mesons $V = (\rho, \omega)$ incorporated into an effective scale-invariant hidden local symmetric Lagrangian, scale-chiral symmetry – hidden in QCD – arises at a high density, $n_{1/2}$, as an “emergent” symmetry, a phenomenon absent in standard chiral perturbative approaches but highly relevant for massive compact stars. What takes place as the density increases beyond $n_{1/2} \sim 2n_0$ in compressed baryonic matter is (1) a topology change from skyrmions to half-skyrmions, (2) parity doubling in the nucleon structure, (3) the maximum neutron star mass $M \simeq 2.05M_\odot$ and the radius $R \simeq 12.19$ km and (4) the sound velocity $v_s^2/c^2 \simeq 1/3$ due to the “vector manifestation (VM)” fixed point of ρ and a “walking” dilaton condensate, which is intricately connected to the source of the proton mass.

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Introduction.— We construct an effective field theory in which scale-invariant hidden local symmetry (HLS) is implemented and apply it to compressed baryonic matter. We show that with the “bare” parameters of the effective Lagrangian matched to QCD at a suitable matching scale Λ_M , we can describe fairly reliably the bulk properties of nuclear matter and exploiting topological information obtained from skyrmion description of the Lagrangian – that we shall refer to as *sHLS*, extrapolate the theory to high density appropriate for compact stars reaching densities $\sim (5-6)$ times the normal nuclear matter density $n_0 \approx 0.16 \text{ fm}^{-3}$. We find that for densities in the vicinity of that of the interior of ~ 2 -solar mass star, there is a strong hint for an emergent scale symmetry, which leads to the sound velocity $v_s/c \approx 1/\sqrt{3}$ for massive compact stars and also accounts for *most* of the proton mass, with a mass-generation mechanism basically different from the standard paradigm anchored on spontaneously broken chiral symmetry. Approaching, at high density, the dilaton-limit (DL) fixed point for the σ and the vector manifestation (VM) fixed point for the ρ , scale-invariant hidden local symmetry manifests, giving rise to “mended symmetries” together with the massless π and a_1 .

Essential Ingredients and Assumptions.— Our strategy consists of combining what has been obtained in our previous works with new observations to arrive at new predictions. They are anchored on the following.

- (A) Our effective Lagrangian combines scale symmetry with chiral symmetry, putting a scalar Nambu-Goldstone, dilaton (σ), and the pseudo-scalar pseudo-NG, pions (π) on the same putting. Our approach does not rely on a precise scheme in implementing the dilaton but we find the scheme proposed by Crewther and Tunstall [1] the most versatile for our purpose, as explained in [2, 3]. The resulting scale-chiral Lagrangian is then made hidden local symmetric by elevating the energy/mass

scale to that of the lowest vector mesons $V = (\rho, \omega)$ following the argument of [4, 5]. We assume that the low-energy theorems are applicable in medium.

- (B) The scale-chiral Lagrangian – denoted *sHLS* in what follows – is Wilsonian-matched at the scale Λ_M to QCD in terms of the well-known current correlators. This procedure renders the “bare” parameters of *sHLS* dependent on the QCD condensates $\mathcal{C} = (\langle \bar{q}q \rangle, \langle G^2 \rangle, \dots)$ where q and G are, respectively, quark and gluon fields. The *sHLS*, when embedded in (nuclear) medium with density n , depends implicitly on density via its dependence on \mathcal{C} . We call this density dependence “*intrinsic density (ID) dependence*.” As defined, this ID dependence is Lorentz invariant.
- (C) Baryons are brought into the theory in two ways. One is to put the baryon fields explicitly into *sHLS* preserving scale-chiral symmetry. We call this Lagrangian *bsHLS*. Alternatively baryons can be generated as skyrmions from *sHLS* Lagrangian. This approach plays a key role for determining some of the important parameters of *bsHLS* that are unattainable otherwise.
- (D) Up to nuclear matter density n_0 , the available experimental data, accurately known up to lab momentum ~ 300 MeV, can be used to determine the ID dependence (IDD) of the bare parameters of *bsHLS*. This is feasible thanks to the low-energy theorems encoded in the Lagrangian in the vector, axial-vector and tensor channels. The in-medium pion decay constant, f_π^* , measured in deeply bound nuclear systems at $\sim n_0$ [6] controls the density dependence of the parameters [7, 8]. This procedure is equivalent to the standard chiral perturbative approach up to n_0 [9]. However going above n_0 is problematic because of the absence of guidance from both experiments and theory. We

assume that the IDD so determined can be extrapolated, at least, slightly above n_0 , say, $\sim 2n_0$. This density regime $0 \lesssim n \lesssim 2n_0$ is referred to as Region-I (R-I for short).

- (E) The density regime $n \gtrsim 2n_0$ is totally unknown, both experimentally and theoretically. The most important IDD in the parameters of $bsHLS$ for $n \gtrsim 2n_0$, referred to as Region-II (R-II), can however be extracted by skyrmions simulated on crystal lattice. It involves a topology change from skyrmions to half-skyrmions at a density denoted $n_{1/2}$. The existence of the half-skyrmion phase is predicted on the basis of symmetry [10] and is actually seen in simulations on crystal [11]. This prediction is robust and implementable in continuum treatments. Being topological, it involves only the pion field and is unaffected by other degrees of freedom. The half-skyrmion phase is characterized by that the quark condensate averaged over the unit cell, denoted Σ , vanishes, although the condensate is non-zero locally, hence supporting chiral density wave. The pion decay constant is non-zero there, hence it is not in the Wigner-Weyl phase. The precise value for $n_{1/2}$ cannot be pinned down theoretically, but based on phenomenology, we estimate it to lie at $2 \lesssim n/n_0 \lesssim 3$.
- (F) With the $bsHLS$ with the bare parameters with IDD determined in both regions, the energy density, $\epsilon(n)$, of the system is calculated using the V_{lowk} renormalization-group (RG) technique as employed in [7, 8]. The EoS, the key element for compact-star physics, is gotten from ϵ . Now doing the relativistic mean-field calculation with $bsHLS$ is equivalent to performing the double-decimation Landau Fermi-liquid calculation in the V_{lowk} RG [12].
- (G) Treating nuclear matter in R-I is straightforward. Both RMF of $bsHLS$ and the V_{lowk} derived from $bsHLS$ will do equally well. They are equivalent in the sense first suggested by Matsui [13]. There is no need to resort to the skyrmion crystal model, which is not realistic, in low-density regime. However in R-II as density increases, the half-skyrmion description becomes more realistic and predictive. As described below, this allows one to map the results of the crystal simulation that capture mesonic and baryonic excitations in the *background* of the multi-soliton configuration to the IDD of the parameters of $bsHLS$.

Role of Topology Change.— The most important ingredient in our approach is the topology change from skyrmions to half-skyrmions in the crystal approach with $sHLS$ at the density $n_{1/2}$. There are two outstanding observations associated with this change. One is that it makes the nuclear tensor forces change drastically from below to above $n_{1/2}$. The other is that the matter above

$n_{1/2}$ behaves like a Fermi liquid. Both seem to be totally at variance with standard nuclear models.

Consider the symmetry energy E_{sym} that figures in the energy per particle of the system given as

$$E(n, \alpha) = E_0(n) + \alpha^2 E_{sym}(n) + O(\alpha^4) \quad (1)$$

where $\alpha = (N - Z)/(N + Z)$ with $N(Z)$ standing for the neutron (proton) number. When calculated with skyrmions put on crystal lattice to simulate density effects, the E_{sym} calculated is found to have a cusp at $n_{1/2}$ [14]. This cusp looks non-normal and may be taken as an artifact of the crystal model. However it can be neatly explained by the behavior of the nuclear tensor forces at the cross-over density [14]. In fact this feature gives a strong support for the role of topology in R-II as argued in [14].

In the V_{lowk} approach (with $bsHLS$), the tensor forces – which play the dominant role in E_{sym} – are given by the exchange of a pion and a ρ with the parameters subject to IDD. Surprisingly they seem to be renormalization-group (RG) invariant, i.e., $(d/d\bar{\Lambda})V_{lowk}^{tensor} = \beta([V_{lowk}^{tensor}], \bar{\Lambda}) = 0$ both in the vacuum and in medium [15]. This observation is made numerically, and remains to be proven rigorously. It resembles closely the flow equation for Landau Fermi-liquid fixed point interactions [16] although the tensor force is non-local unlike the Landau parameters which are local. If correct, then the E_{sym} could be accurately given by the fixed-point property both in R-I and in R-II. We shall take this as an assumption to be confirmed. As observed in [7, 8], the cusp in the skyrmion crystal description of E_{sym} then suggests that the hidden gauge coupling g_ρ goes over, at $n_{1/2}$, from a constant to one that drops rapidly toward the vector manifestation (VM) fixed point $g_\rho \rightarrow 0$ in consistency with the vector manifestation of the ρ meson at which $g_\rho \rightarrow 0$. This feature will be elaborated below. This changeover is responsible for E_{sym} to go from soft to hard at $n_{1/2}$ that accounts for the massive stars [7, 8].

The second observation is the indication that in R-II, the half-skyrmion matter resembles Fermi-liquid. When the averaged quark condensate Σ vanishes, there remains a large amount of non-vanishing mass, $m_{skyrmion} = m'_0 + \Delta(\Sigma) \rightarrow m'_0 \neq 0$ as $\Sigma \rightarrow 0$ with the property [17],

$$m'_0 \propto f_\pi^* \propto \langle \chi \rangle^* \quad \text{with} \quad \frac{\partial \langle \chi \rangle^*}{\partial n} = 0, \quad n > n_{1/2} \quad (2)$$

which is reproduced in the mean field calculation of $bsHLS$ [18]. This m'_0 resembles the chiral-invariant nucleon mass m_0 in the parity-doublet model [19]. In fact in heavy-light systems in medium, it is found that $m_0 = m'_0$ [20]. The major difference is that here m'_0 “emerges” from strongly correlated system instead of put in *ab initio* by hand.

There are two points to stress here. First the part of the nucleon mass implied by $m'_0 \approx m_0$ is substantial, $\sim (0.7 - 0.9)m_N$ and points to the source of the proton mass, which is at variance with the standard paradigm for hadron mass based on spontaneously broken chiral

symmetry [21]. The standard paradigm would suggest that the ratio of in-medium masses $\mathcal{R} = m_\rho^*/m_N^* \rightarrow$ constant $\sim 2/3$ as chiral restoration is approached. The prediction here is that $\mathcal{R} \rightarrow 0$. Second the half-skyrmion crystal, when identified to be equivalent to the mean field of $bsHLS$, provides the IDD of $bsHLS$ Lagrangian in R-II through Eq.(2).

It has remained mysterious since many years [12] that the hidden gauge coupling g_ρ should be constant up to near n_0 and then drop rapidly to arrive at the VM fixed point (VMFP for short) $g_\rho = 0$. This puzzle can be resolved with the IDD described above.

The nuclear matter at equilibrium can be described in terms of Wilsonian RG [16]. There the quasiparticle interactions have vanishing β functions in the limit $N \equiv k_F/(\Lambda - k_F) \rightarrow \infty$ (where Λ is the cutoff for decimation). With the vector mesons and the dilaton of $bsHLS$ Lagrangian, considered heavy compared with the Fermi sea scale, integrated out to give the marginal four-point quasiparticle interactions, nuclear matter can be considered to be at its Fermi-liquid fixed point [22].

Going beyond the equilibrium density, as argued, the Fermi-liquid structure should apply. Now consider the parameter space of $bsHLS$ on top of the Fermi-liquid fixed point. Approaching the IR fixed point with the scale parameter $\bar{\Lambda} \equiv \Lambda - k_F \rightarrow 0$, the parameters of the EFT Lagrangian should scale such that the β function for the quasiparticle interactions be zero at a given Fermi-momentum k_F . Suppose the density is changed from k_{F1} to k_{F2} . Then certain parameters should change, say the quasiparticle mass as an example, from $m^*(k_{F1}, \bar{\Lambda} = 0)$ to $m^*(k_{F2}, \bar{\Lambda} = 0)$ to preserve $\beta(k_{F1}, \bar{\Lambda} = 0) = \beta(k_{F2}, \bar{\Lambda} = 0) = 0$. This means that the Fermi-liquid fixed point quantities are closely related to each other at given density so that $g_V^*(k_F, \bar{\Lambda} = 0)$ as well as $m^*(k_F, \bar{\Lambda} = 0)$ should be dependent on $\langle\chi\rangle^*$ and k_F to have $\beta(k_F, \bar{\Lambda} = 0) = 0$. Thus in the density regime $n \lesssim n_{1/2}$ (R-I), the condensate $\langle\chi\rangle^*$ given in the mean field [18], locked to the quark condensate $\langle\bar{q}q\rangle$, decreases as observed in experiments [6].

Now going to $n > n_{1/2}$, the dilaton condensate $\langle\chi\rangle^*$ should stay constant as predicted by the theory, i.e., (2). This requires that $g_{\rho,\omega}^*(k_F, \bar{\Lambda} = 0)$ scale to preserve $\beta(k_F, \bar{\Lambda} = 0) = 0$ as density increases. This interplay between the dilaton condensate and the vector coupling for the Fermi-liquid fixed point structure turns out to bring about the change in density dependence of g_ρ^* from $g_\rho^* \approx g_\rho$ near $n = n_0$ to $g_\rho^* \rightarrow 0$ near the VM fixed point.

Dilaton-Limit Fixed Point (DLFP).—Given a Lagrangian of the form of $bsHLS$, one can gain further insight into the structure of dense matter in terms of scale-chiral symmetry without resorting to the input from the topology change discussed above. In [18, 23], it was shown in terms of Wilsonian renormalization group (RG) equations that there is a possible infrared fixed point in $bsHLS$

$$(m_N, g_{V\rho} - 1, g_A - g_{V\rho}) \rightarrow (0, 0, 0) \quad (3)$$

for the nucleon mass m_N , the ρNN coupling $g_{\rho NN} = (g_{V\rho} - 1)g_\rho$ – with g_ρ the hidden gauge coupling and $g_{V\rho} - 1$ a medium-renormalization – and g_A for the axial-vector current. The parameters of the Lagrangian satisfy a set of coupled renormalization-group equations and can flow in various different directions. We consider how the fixed point (3) can be reached in dense medium.

In the mean field, equivalent to Landau fixed-point theory, there are singular terms when $\langle\chi\rangle^*$ goes to zero arising from the non-linear terms of N , $\bar{\sigma} \equiv \langle\chi\rangle^*\sigma/f_\sigma$ and $\bar{\pi} \equiv \langle\chi\rangle^*\pi/f_\sigma$. Taking the limit $\langle\chi\rangle^* \rightarrow 0$ corresponds to the DLFP, where $f_\pi = f_\sigma$ – which also follows from the locking of two symmetries in the CT theory [1] – and $g_A = g_{V\rho}$ are required to avoid the singularity. In this limit, one obtains a Gell-Mann-Lévy (GML)-type linear sigma model with the degenerate $O(4)$ multiplet for the scalar and pseudo-scalar fields (massless in the chiral limit) [18, 24, 25]. There the ρ mesons are decoupled from the nucleons.

“Walking” Dilaton Condensate.—The dilaton condensate in the half-skyrmion phase with the property of (2) can be reproduced in the mean-field of $bsHLS$ in terms of an interplay between the ω mass and the dilaton condensate, the former blocking the dropping of the latter $\propto f_\pi^*$ in R-I, as one goes beyond $n_{1/2}$. This is illustrated in [18].

Now the question is: How is the dilaton limit fixed point approached with the “walking” dilaton condensate “running” to zero? The answer is in the many-body correlations inducing $(g_{V\rho} - 1, g_{V\omega} - 1) \rightarrow (0, 0)$ as the DLFP is approached. It is not in the IDD in contrast to the VM fixed point which involves the hidden gauge coupling intrinsic to QCD. As the dilaton condensate goes to zero toward the DLFP, we have the scale symmetry restored, $\theta_\mu^\mu = \partial_\mu D^\mu \rightarrow 0$, but it is an “induced” symmetry restoration. This resembles the scenario of chiral symmetry restoration involving a soft scalar mode [26].

Mended Symmetries.— The DLFP brings π and σ together while the VMFP brings π and ρ together. The two fixed points could come together or separately. If they come separately, the former should come before the latter since DLFP is driven by correlated interactions without the gauge coupling going to zero. Furthermore it has been shown that the axial vector meson a_1 could also come together with the ρ at the VMFP [27]. This means that π , σ , ρ and a_1 could all come together at some density which may or may not coincide with chiral restoration. In the chiral limit, this would involve massless gauge bosons – ρ and a_1 – together the Nambu-Goldstone bosons π and σ . An attractive possibility is that they form the multiplet of Weinberg’s “mended symmetries” [28].

Predictions on Compact Stars.— We now apply the $bsHLS$ with the IDD fixed as above to the properties of compact stars.

Consider the in-medium vacuum expectation value of the energy-momentum tensor $\langle\theta_\mu^\mu\rangle = \epsilon(n) - 3P(n)$ expressed in terms of the thermodynamic quantities, the

energy density $\epsilon(n)$ and the pressure $P(n)$. The energy-momentum tensor and the sound velocity are related by $\frac{\partial}{\partial n}(\theta^\nu_\nu) = \frac{\partial \epsilon(n)}{\partial n} \left(1 - 3\frac{v_s^2}{c^2}\right)$ where we used $\frac{v_s^2}{c^2} = \frac{\partial P(n)/\partial n}{\partial \epsilon(n)/\partial n}$ for the sound velocity v_s . Now if $\langle\theta^\mu_\mu\rangle$ is a constant independent of density, given that the energy-density of compact-star matter has no known extremum, i.e., $\frac{\partial \epsilon(n)}{\partial n} \neq 0$, then it will follow that $v_s/c = \sqrt{1/3}$.

Let us first look at the mean-field approximation in R-II. The trace of the energy-momentum tensor in dense matter in the mean-field approximation with the $bsHLS$ Lagrangian is given by $\langle\theta^\mu_\mu\rangle \propto \langle\chi\rangle^*$ ⁴. This is because to the leading order in the scale-chiral counting, $O(p^2)$, the energy density is scale-invariant since $bsHLS$ is scale-invariant, the scale symmetry breaking being only in the dilaton potential. Therefore if $\langle\chi\rangle^*$ stays constant in $n \gtrsim n_{1/2}$ as seen in (2), the sound velocity will become $v_s/c = \sqrt{1/3}$.

Now what happens if one goes beyond the mean-field approximation? Given that the mean-field approximation is presumably equivalent to doing Fermi-liquid fixed theory, going beyond the mean-field must then be tantamount to calculating higher loop corrections to the Landau fixed point theory. Such a calculation using the V_{lowk} RG double decimation has been formulated and applied to compact-star matter in [8]. We have applied the same formalism with only a slight modification of the approach to the VM fixed point – which is of course unknown and hence a free parameter – in [8] (without affecting the known properties of nuclear matter well described therein).

A surprisingly simple way of parameterizing the EoS given by the V_{lowk} RG procedure is to express the energy per particle of the dense matter in R-II by

$$E/A = -m_N + B\left(\frac{n}{n_0}\right)^{1/3} + D\left(\frac{n}{n_0}\right)^{-1} \quad (4)$$

where m_N is the nucleon mass, B and D are constants of mass dimension. One can show that (4) is the solution of $\frac{dP}{dn} = \frac{1}{3}\frac{d\epsilon}{dn}$ which assumes the constant $\langle\theta^\mu_\mu\rangle$. The strategy then is to pick the coefficients B and D for $\alpha = (0, 1)$ to reproduce the E_0 and E_{sym} in (1) that reproduces accurately those obtained from the V_{lowk} RG with $bsHLS$ with the IDD that encode the “walking condensate” associated with the DLFP and VMFP in R-II. The induced effects that accompany the DLFP, namely in $g_{V\rho}$ and $g_{V\omega}$, are captured in the V_{lowk} approach in the RG decimations, extraneous to IDD. This strategy can

be made to work extremely well. Indeed very accurate fits can be obtained for both E_0 and E_{sym} in the density range $2 \lesssim n/n_0 \lesssim 6$ relevant for massive compact stars.¹ This means that the star matter can have $v_s/c \simeq 1/\sqrt{3}$ for density regime $n > n_{1/2}$. Given in Fig. 1 are the results predicted in the V_{lowk} RG with $bsHLS$ for the sound velocity for both neutron and symmetric nuclear matter and M vs. R . The result for v_s is totally unprecedented and intriguing: there is no other (known to us) trustful theory that gives $v_s/c < 1/\sqrt{2}$! Given that there is still

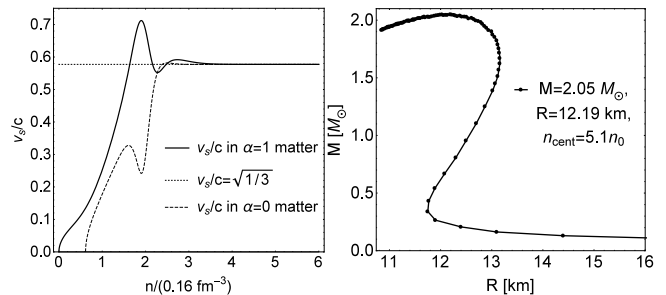


FIG. 1. Left panel: The sound velocity predicted in the V_{lowk} RG with $bsHLS$ for $\alpha = \frac{N-Z}{N+Z}$. The oscillation of v_s near $n_{1/2}$ is considered to be an artifact of the sharp changeover of the states before and after $n_{1/2}$. Right panel: (M)ass vs. (R)adius of the neutron star with the beta equilibrium taken into account.

quite a bit of room for adjustments in the IDD, quantitative comparison with observed data may not be very meaningful. Nonetheless the results obtained in this approach are satisfactory and encouraging: Maximum star mass $M \simeq 2.05M_\odot$, radius $R \simeq 12.19$ km, central density $n_{cent} \simeq 5.1n_0$, $E_{sym} \simeq 26$ MeV and $L \simeq 49$ MeV at equilibrium density. For completeness, we also list the predictions for normal nuclear matter which, as mentioned, should be comparable to those of the standard chiral perturbation approach: Equilibrium density $n_{eq} \simeq 0.15n_0$, B.E. $\simeq 16$ MeV, compression modulus $K \simeq 215$ MeV.

Details will be reported elsewhere.

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¹ The fit parameters are $B_{\alpha=(0,1)} = (570.031 \text{ MeV}, 686.4 \text{ MeV})$ and $D_{\alpha=(0,1)} = (439.676 \text{ MeV}, 252.888 \text{ MeV})$. For lack of space the detailed fits will be shown elsewhere.

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