The Mistral base case to validate kinetic and fluid turbulence transport codes of the edge and SOL plasmas

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ABSTRACT

Experimental data from the Tore Supra experiments are extrapolated in the SOL and edge to investigate the Kelvin–Helmholtz instability. The linear analysis indicates that a large part of the SOL is rather unstable. The effort is part of the set-up of the Mistral base case that is organised to validate the codes and address new issues on turbulent edges, including the comparison of kinetic and fluid modelling in the edge plasma.

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1. Introduction

The issue of transport in the edge region of fusion plasmas is among the most difficult and most crucial issue in understanding the operation of present devices and hence to predict that of ITER. Recent simulations with the code TOKAM-3D [1] have shown that when self-consistent turbulent transport was simulated one could recover the large-scale parallel flows that are observed in the experiments. Various global 3D simulation codes are presently in operation including fluid representations and gyrokinetic representations. As has been achieved for core turbulence [2], there is now a growing need to determine a reference base case in order to provide a means to both compare the codes (benchmarking) but also validate them. In particular, it is important to assess the role of the kinetic effects in the physics of the edge, SOL and divertor. It is clear that a divertor configuration will have to be considered for such a purpose. However, such simulations are demanding for the present state of the art modelling effort since the cold plasma is understood to be a key player of the barrier formation [5]. The latter point is all the more important that rotation shear of the plasma is understood to be a key player of the barrier formation [5], the H-mode remaining to be consistently modelled.

2. The physics of the Mistral base case

The Tore Supra experiments that we consider in the Mistral base case have the following common characteristics. Magnetic surfaces are circular with typical major radius \( R \sim 2.4 \) m, minor radius \( a \sim 0.7 \) m, elevation with respect to the midplane \( z \sim 0 \) m, toroidal magnetic field \( B_\phi = 2–4 \) T and plasma current \( I_p = 0.6–1 \) MA.

2.1. Magnetic equilibrium

The toroidal field is generated by \( N_{coil} = 18 \) identical circular coils \( R_{coil} = 2.2 \) m, inner minor radius \( a_{coil} = 1.154 \) m (radial thickness \( \Delta a_{coil} = 0.226 \) m, each coil has \( N_c = 2028 \) turns, The ripple is approximated by \( \delta B_{\phi,\text{ripple}} = (B_\phi) \sin(N_{coil} \theta) \left( -0.5 \Delta a \right) \exp[-(a_{coil} - r)/(2 \Delta a_{ripple}) - (\theta/\theta_{\text{ripple}})^2] \). Where \( \delta B_{\phi,\text{ripple}} \) is aligned along...
the radial coordinate in the toroidal frame with major radius $R_{\text{coil}}$ at the position $(r, \theta, \varphi_r)$ with $\varphi_r = 0$ at the midplane low field, $\varphi_r = 0$ between two coils and $A_{\text{ripple}} = 0.182$ m, $\theta_{\text{ripple}} = 5\pi/6$, and the maximum radial displacement governed by the ripple at the coil location is $\Delta_0 = 0.066$ m. The averaged toroidal field is $2\pi R(B_0) = \mu_0 N_{\text{coil}} I_{\text{coil}}$, where $I_{\text{coil}}$ is the coil current (ranging from 500 A to 1000 A). Within the precision of the measurements, the magnetic surfaces are circular with shifted magnetic surfaces depending on the Shafranov shift parameter $A = \beta_0 + l/2$, with major radius $R_s = (R_0^2 + (a^2 - r^2)(1 + A))/2$, $R_s$ being the major radius of the last closed magnetic surface with radius $a$. In this framework, the poloidal magnetic field can conveniently be approximated by $B_t = (B_0)(1 + A r \cos(\theta)/R_s)$ where $2\pi R(B_0) = \mu_0 I_p$. Analytical expression for the safety factor, holding for the outer plasma with vanishing plasma current, and intrinsic magnetic coordinates can then be calculated [6].

3. Stepping from the local probe measurements to 2D background plasma

An important issue in the validation effort relies on using dimensionless parameters, such as $\rho^*$, as well as appropriate coordinates. Shaped plasmas have led to consider the poloidal magnetic flux as the appropriate radial coordinate. Regarding parallel transport, one finds that intrinsic magnetic coordinates, the angles $\vartheta^*$ and $\varphi^*$, are more appropriate since they are characterised by the following proportionality relations: $ds = q R_0 d\vartheta^*$ and $ds = q R_0 d\varphi^*$, where $R_0$ is a reference major radius constant on a magnetic surface and $s$ the curvilinear abscissa. Proportionality between the connection length (along the $s$ coordinate) and the angles is particularly powerful to address the effect of poloidally localised sources. Edge plasma physics are particularly difficult to diagnose in a comprehensive and coherent way. In the most optimistic case, one must consider a 2D description of the steady state plasma, steady state standing here for the time averaged plasma. Ripple and non-axisymmetric plasma facing components will govern 3D effects. However, measurements in the plasma boundary are localised poloidally and toroidally. Full torus, turbulence simulations of the edge and SOL plasma [1,11] will then have to match these local conditions. Similarly, transport simulations in the edge are being performed in the limiter configuration with the SOLEDGE-2D code [12,13] and aim at recovering these measurements by adjusting the transport coefficients for the transverse particle and momentum diffusion. It can be shown that for large enough temperatures, the plasma in limiter configurations tends to be isothermal in the parallel direction. However, ion and electron temperature physics are also presently included in this modelling effort [14]. We will consider here such background plasma together with an analytical description to analyse the Kelvin–Helmholtz stability of such plasmas. This specific interest is both governed by the need to gain insight in the physics of the edge flows and to investigate potential issues raised by instabilities generated in the vicinity of the limiters. The chosen approach adopted here is similar to the so-called onion skin modelling [15]. One assumes that the total plasma pressure $II = n(T_e + T_i)(1 + M_i^2)$ is approximately constant along the field lines [12,13], so that the measurement of the saturation current (in $n_c$) and that of the parallel Mach number yield $II/(m c_s)$ where $c_s$ is the sound speed $(m c_s^2 = T_e + T_i)$. Reasonable assumptions on the source location and magnitude then allow one to extrapolate the local measurements to the whole SOL. For the sake of simplicity, we consider a piecewise transverse particle flux and assume that the variation of the Mach number is governed by this cross-field transport. This assumption is rather specific of limiter configurations where SOL recycling will tend to be small. Given the input from the experimental analysis, we assume that the ballooned transport extends from $-\theta_{\text{lim}}$ to $\theta_{\text{lim}}$, hence with up–down symmetry with respect to the outboard midplane [4,8], $\theta_{\text{lim}} \approx 0.22 \pi$ in intrinsic coordinate $\theta^*$, see Fig. 1. The probe is located approximately at $\theta_{\text{probe}} \approx 0.45 \pi$ while the bottom limiter is located at $\theta_{\text{lim}} \approx -0.45 \pi$. Following the formalism presented in [16], one can relate the particle flux to the integral of the source function $\Gamma(\vartheta^*) = \int dS$. Using the appropriate angle $\theta^*$ then allows one to determine the parallel particle flux at the limiter, in both the co and counter direction and at the probe location:

$$
\Gamma_{\text{lim}}^+ = [S_{\text{bal}}(\theta_{\text{lim}} - \theta_{\text{stag}}) + S_{\text{amp}}(-\theta_{\text{lim}} + \theta_{\text{stag}})]q R_0
$$

$$
\Gamma_{\text{lim}}^- = -[S_{\text{bal}}(\theta_{\text{lim}} + \theta_{\text{stag}}) + S_{\text{amp}}(2\pi - \theta_{\text{lim}} - \theta_{\text{stag}})]q R_0
$$

$$
\Gamma_{\text{probe}} = -[S_{\text{bal}}(\theta_{\text{lim}} - \theta_{\text{stag}}) + S_{\text{amp}}(\theta_{\text{probe}} - \theta_{\text{stag}})]q R_0
$$

The choice of the signs of the fluxes is done in agreement with the experimental definition of the sign of the Mach number. In this
analysis $\theta^\ast\text{stag}$ is the poloidal location of the stagnation point while $S_{\text{unif}}$ is the uniform source and $S_{\text{bal}}$ is the ballooned step increase of the source on the low field side. We assume here that the total plasma pressure is nearly constant along the field lines and introduce the control parameter $A$ such that:

$$A = \frac{2M}{1 + M^2} = \frac{2Tc_s}{H/m_i}$$

where $c_s$ is the sound velocity and $H$ the total plasma pressure. This parameter weighs the relative contribution of the parallel variation of the Mach number $\partial_s M$ and that of the density $\partial_s \log(n)$ in the variation of the total plasma pressure $H$. At the limiter, one readily finds $A = \pm 1$ while at the probe one can compute $A$ given the Mach number measurements. The constant $H$ and constant $c_s$ approximation, supported in [13,14], then allows one to obtain two constraints and then determine the peaking factor of the ballooned transport $S_{\text{bal}}/S_{\text{unif}}$ as well as the location of the stagnation point $\theta^\ast\text{stag}$, Fig. 2. Given the shape and location of the ballooned source term as well as its peaking factor, it is then possible to map the probe measurement on the total SOL. A 2D plot of parameter $A$ together with profiles of $A$ and of the density at the probe location are shown on Figs. 3a and b. Two parts in the extension of the data are shown, the parallel extension and a radial extension into the edge plasma. An exponential decay of $A$ is chosen into the plasma edge that governs a rapid smoothing of the density in the poloidal direction as well as a decrease of the Mach number in the plasma edge. This radial extrapolation is not backed by experimental data, at this stage it is only a reasonable assumption.

4. Local stability analysis of Kelvin–Helmholtz instability

Several instability mechanisms have been analysed in the edge and SOL plasmas [17,18]. Of particular interest is the Kelvin–Helmholtz instability driven by gradients of the parallel flow velocity [19] in both experiments and in the modelling effort [11]. The extrapolation of the experimental data obtained in Section 3 is used here to determine a value of one of the critical parameters that drive the onset of the Kelvin–Helmholtz instability. The Schmidt parameter is defined as the ratio of the momentum diffusion coefficient $\eta$ and of the particle diffusion coefficient $D_\perp$ in the transverse direction, $Sc = \eta/\eta_\perp$. These parameters are effective coefficients governed by turbulence (in particular $D_\perp$ is undefined for collisional transport). In practise, these coefficients thus depend on the turbulent transport that will include Kelvin–Helmholtz turbulence. In the spirit of the linear analysis, one should consider only the subgrid transport and turbulent transport not stemming from the Kelvin–Helmholtz instability. This cannot be achieved when considering experimental data. Rather than seeking to extrapolate these coefficients from the present data, we evaluate the required Schmidt parameter to trigger the instability. The linear analysis, assuming scale separation between the unstable
modes and the gradients that drive the instability [11], then allows one to calculate the Schmidt parameter that will exhibit unstable Kelvin–Helmholtz turbulence. One then finds the following necessary condition:

\[ \text{Sc}^2 - 2(1 + G/2)\text{Sc} + 1 > 0 \] (3a)

\[ G = \left[ \left( \frac{d\log(n)}{dr} \right)^2 - \left( \frac{dM}{dr} \right)^2 \right] \left( \frac{d\Gamma}{ndr} \right)^2 \] (3b)

One readily finds that both \( \text{Sc} < 1 \) and \( \text{Sc} > 1 \) branches are possible. We consider here the situation such that the Schmidt parameter tends to be small, weak viscosity compared to particle diffusion. One finds that for moderate values of the Schmidt parameter, typically \( \text{Sc} < 0.1 \), most of the SOL away from the stagnation point is unstable. The edge layer close to the SOL appears to be the most unstable.

**5. Discussion and conclusion**

In this paper, we have presented a first analysis of the Mistral base case. As such, the Mistral base case represents a challenge to both fluid and gyrokinetic codes. First, quantitative information will be required from the fluid codes although low collisionality regions are included in the simulation domain. Second, it is mandatory to take into account electron transport since adiabatic electrons do not drive any electrostatic cross-field transport of particles. Two global codes will be used to investigate these experiments. The code SOLEDGE-3D [11] has started running with this kind of background plasma. The goal is to investigate the Kelvin–Helmholtz instability in these limiter shots. The code XGCP [20], a gyrokinetic PIC code aiming at core, edge and SOL simulations is being modified to address the physics of the Tore Supra shots. The experimental analysis performed in this paper in order to generate a 2D axisymmetric plasma background provides a first analysis of the turbulent properties. Using a linear analysis, one can investigate the unstable regions in the edge and SOL plasma. The difficulty in estimating the Schmidt parameter using experimental evidence has led us to determine the required value to generate Kelvin–Helmholtz unstable modes. Schmidt values in the 0.1 range indicate that Kelvin–Helmholtz is likely to be unstable in such experimental setting. The MISTRAL base case together with the gyrokinetic code XGC and fluid code SOLEDGE-3D is a first step in the effort to develop a new generation of plasma-wall interaction simulation codes. It includes benchmarking and validation activity, open to all codes. The possibility to enrich the MISTRAL base case together with the code development and validation procedure is crucial as one step towards first principal simulations of edge plasma simulations of ITER.

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