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LETTER TO THE EDITOR

Spiral-driven accretion in protoplanetary discs

I. 2D models

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ABSTRACT

We numerically investigate the dynamics of a 2D non-magnetised protoplanetary disc surrounded by an inflow coming from an external envelope. We find that the accretion shock between the disc and the inflow is unstable, leading to the generation of large-amplitude spiral density waves. These spiral waves propagate over long distances, down to radii at least ten times smaller than the accretion shock radius. We measure spiral-driven outward angular momentum transport with 10^{-8} M_\odot yr^{-1}. We conclude that the interaction of the disc with its envelope leads to long-lived spiral density waves and radial angular momentum transport with rates that cannot be neglected in young non-magnetised protostellar discs.

Key words. accretion, accretion disks – hydrodynamics – waves

1. Introduction

Accretion is an essential phenomenon in astrophysics as it is the way through which gravitational energy is transformed into heat and therefore observable radiation. However, the physical process responsible for accretion is still debated. Although the magnetorotational instability (MRI, Balbus & Hawley 1991) provides an efficient accretion mechanism, its applicability to protoplanetary discs is questionable since these objects are very weakly ionised and might quench the MRI through various non-ideal magnetohydrodynamical (MHD) processes (Turner et al. 2014).

The usual approach to protoplanetary disc dynamical modelling is to assume that these objects are isolated in space. In this context, most of the models rely on processes such as MRI-driven turbulence, self-gravity, or winds to drive accretion. However, these discs are not isolated systems, and the question is whether the surrounding material, albeit much less dense than the central object, which is assumed to be a solar-mass star, could perturb it sufficiently to drive accretion. This possibility was proposed by Padoan et al. (2005) to explain accretion rates that scale like the central protostar mass squared. This scenario was later refined by Throop & Bally (2008), Klessen & Hennebelle (2010), and Padoan et al. (2014) and interpreted as a consequence of Bondi-Hoyle accretion onto the protostar.

In this Letter, we explore the importance of external accretion using a very simplified model. We consider a 2D hydrodynamic Keplerian disc onto which falls gas coming from a surrounding envelope. The disc is inviscid and hydrodynamically stable so that no accretion can occur without this external inflow. We first present in detail the model and the numerical method used to solve the equations of motion. We then explore some of the results obtained using this model and finally discuss the limitations and implications of our findings.
where we choose $\Sigma_0 = 1700 \text{ g cm}^{-2}$ and $R_c = 40 \text{ au}$, which corresponds to typical protoplanetary discs observed in Ophiuchus (Andrews et al. 2009) with a mass of $4.8 \times 10^{-2} M_\odot$. The temperature profile is chosen to be locally isothermal so that $c_s/c_K = \varepsilon = 0.1$ or 0.05, where $c_s$ is the sound speed and $c_K \equiv \sqrt{GM_\odot/R}$ is the Keplerian velocity. The equation of state is then simply $P = c_s^2 \Sigma$. Assuming a vertically isothermal hydrostatic profile, this model leads to an accretion disc with a constant aspect ratio $H/R = \varepsilon$, where $H$ is the typical vertical disc scale height.

2.2. Boundary conditions

The outer radial boundary condition is located at $R_{out} = 400 \text{ au}$. To mimic matter falling onto the disc, we inject material at the outer radial boundary condition that then falls onto the disc. The material is injected over an azimuthal extent $0 < \theta < \theta_{inj}$ with a sonic radial velocity, $v_{inj} = -c_s$. We vary three parameters for the injected material: its specific angular momentum $L_{out} \equiv v_{inj}(R_{inj}) R_{inj}$, its accretion rate $M_{out} \equiv \int d\theta \Sigma v_{inj}$, and $\Sigma_{out}$ being deduced from the desired accretion rate, and the azimuthal width of the inflow $\theta_{inj}$.

The inner radial boundary is located at $R_{inj} = 1 \text{ au}$. The inner boundary condition is forced to the initial density profile on the same timescale. To avoid spurious reflection of spiral density waves, we add a damping zone in the region $1 \text{ au} < R < 3 \text{ au}$. In this region, we exponentially relax non-axisymmetric velocity fluctuations on a fixed timescale set to eight local orbital periods. To avoid mass accumulation in the damping zone, we also relax the density to the initial density profile on the same timescale.

2.3. Numerical algorithm

We use Pluto 4.0 (Mignone et al. 2007) to solve the equations of motion on a cylindrical grid, using log-spaced grid cells in the radial direction. Parabolic reconstruction is used in each cell to cope with the non-uniform radial grid, and a third-order Runge-Kutta algorithm is used to evolve the system in time. We use the HLL Riemann solver1 at cell boundaries combined with orbital advection (Mignone et al. 2012) to allow for long integration times. We have tested that starting with sonic white noise perturbation on $v$ and without any inflow, the disc was hydrodynamically stable with $\alpha \sim 10^{-2}$ after a few local orbits, as expected with our density profile (3). The resolution of each run was $(N_R, N_\theta) = (512, 512)$ for $\varepsilon = 0.1$ or $(N_R, N_\theta) = (1024, 1024)$ for $\varepsilon = 0.05$, which corresponds to a locally isotropic resolution of eight points per $H$. We checked that doubling the resolution of run S7-1 did not quantitatively affect our results. We are therefore confident that our simulations are numerically converged.

2.4. Units, diagnostics, and notations

Length units are given in au, surface densities in g cm$^{-2}$, time units in orbital periods at 100 au $\approx T_{100}$ (or equivalently in units of $10^5$ yr), and accretion rates in $M_\odot$ yr$^{-1}$.

In the following, several diagnostics are used to measure the behaviour of the disc coupled to the inflow. We first introduce two averages, an azimuthal average $\langle \cdot \rangle$ and a temporal average $\langle\langle \cdot \rangle\rangle$. We start the temporal average from $t = 10 T_{100}$ to allow the system to relax from the initial condition (this corresponds to $10^4$ orbits at the inner boundary).

These averaging procedures allow us to define fluctuating quantities $\delta v = v - \overline{v}/\overline{\Sigma}$. The first diagnostic is the Shakura & Sunyaev (1973) $\alpha$ parameter $\alpha \equiv \overline{\Sigma} \delta v \delta \theta / \overline{P}$. We also use the disc radius $R_d$ defined as the location where the disc rotation velocity drops below 90% of the Keplerian velocity $c_K$. This definition might look rather arbitrary, but it nevertheless leads to a well-defined disc radius since the rotation velocity drops very rapidly with radius at the transition region between the disc and the inflow. Each simulation is integrated for $3 \times 10^4$ orbits at the inner boundary, which corresponds to $30 T_{100}$.

Our runs are labelled “XY-Z” where X = S or A denotes symmetric ($\theta_{inj} = 2 \pi$) or asymmetric ($\theta_{inj} = \pi/2$) inflows, Y is the accretion rate at $R_{out}$ and Z is the specific angular momentum $L_{out}$. Our simulations all have $\varepsilon = 0.1$, except for runs labelled “thin”, which assume $\varepsilon = 0.05$.

3. Results

We first concentrate on symmetric inflows. We then explore the impact of asymmetric inflows and of the disc thickness in a second part.

3.1. Symmetric accretion

3.1.1. Accretion shock

The initial evolution of the symmetric accretion run S7-1 is presented in Fig. 1. The falling material initially forms an accretion shock that propagates inward. At $t = 2 T_{100}$, the shock stalls at $R = R_d$. The stationary shock then develops an instability, which appears as a short-wavelength non-axisymmetric ondulation of the shock front ($t = 2.8 T_{100}$). This instability then quickly saturates and generates strong spiral waves that propagate inward on the long term ($t > 3.5 T_{100}$).

The origin of this instability is tightly linked to the structure of the shock. Spirals are continuously produced from $R \leq R_d$, and the instability survives for the entire duration of the simulation. The instability mechanism becomes clear when the average specific angular momentum profile $\langle L \rangle$ and the average potential vorticity $\langle \zeta \rangle \equiv \langle (2\Omega + \nabla^2) / \overline{\Sigma} \rangle$ are plotted as a function of radius (Fig. 2). We find that at the location of the stationary shock a strong velocity gradient develops, which leads to an extremely peaked $\zeta$. This sharp structure appears because the specific angular momentum of the falling material is lower than that of the disc material at $R = 200$. A potential vorticity layer is therefore unavoidable if matter keeps falling onto the disc.

As is well known, such a hydrodynamical structure naturally leads to a Kelvin-Helmholtz instability (KHI, also known as Rossby wave instability (RWI) in the astrophysical context)2. We note, however, that the shear rate is locally so strong that it also locally violates the Rayleigh criterion: in the shock region, the specific angular momentum is decreasing outward (Fig. 2). As stated above, this configuration is unavoidable given that the inflow has a lower angular momentum than the disc. The instability driving the spiral is therefore a mixture of KHI (RWI) and Rayleigh centrifugal instability. It has a growth rate close to the specific angular momentum of the inflow that is lower than that of the disc material at $R = 200$. A potential vorticity layer is therefore unavoidable if matter keeps falling onto the disc.

1 We also ran simulations with the HLLC Riemann solver. In this case, however, the noise level of the background spirals in the absence of inflow is $\alpha \sim 10^{-4}$ because there is almost no numerical dissipation. We therefore only present results obtained with HLL for which the background noise level is much lower.

2 In our case, the RWI is not due to a density bump but to a strong and localised shear. Nevertheless, the general RWI criterion, $\zeta^{-1}$ having a maximum (Lovelace et al. 1999), is satisfied.
G. Lesur et al.: Spiral-driven accretion in protoplanetary discs

![Fig. 1. Density maps as a function of time for run S7-1. From left to right: t = 2T_{100}, t = 2.8T_{100}, t = 6T_{100}.](image)

![Fig. 2. Averaged potential vorticity ⟨ζ⟩ (plain blue) and specific angular momentum L (red dashed) of run S7-1. Note the sharp negative angular momentum gradient associated with a strong potential vorticity peak at R ∼ 200.](image)

![Fig. 3. Profiles of α as a function of radius, measured from R_d. The spiral stress increases with increasing M_{inf} and with decreasing initial angular momentum L_{out}.](image)

![Fig. 4. α measured at R = R_d/10 as a function of M_{inf}. Colours represent different L_{out} or H/R, diamonds correspond to symmetric inflow, and stars correspond to asymmetric accretion. The dashed line are fits given by Eq. (4).](image)

3.1.2. Angular momentum transport

Spiral waves generate a positive Reynolds stress through the entire disc, which we quantify as an α parameter (Fig. 3). We find that these spirals produce a very strong transport close to R ∼ R_d with α ∼ 1. They propagate inward from R_d and dissipate through shocks, reaching α ∼ 10^{-3} at R_d/2 and α ∼ 10^{-4} at R_d/10. A generation region (0.6R_d < R < R_d) can be defined where the inflow mixes with the disc, spiral modes are excited, and α values are similar for all of our simulations. Likewise, there is a propagation region (R < 0.6R_d) where the disc density structure is unaffected by the inflow except for propagating spiral waves.

We see that α in the generation region is not affected by M_{inf}, but α in the propagation region (R < 0.6R_d) clearly is. Moreover, for our strongest M_{inf} (run S6-1 for instance), we observe a bump of transport for R ∼ 0.6R_d, directly at the transition between these two regions. This is because the mass of the generation region has increased significantly due to the inflow while that of the propagation region has not, forming a jump in surface density at the interface. As a result, the generation region has a higher inertia and excites stronger waves at the interface with the propagation region. We note that smaller angular momentum in the inflow also leads to somewhat stronger spirals. These findings can be summarised by representing α at R = R_d/10 as a function of M_{inf} (Fig. 4).

From these results, we can estimate that in the symmetric inflow case

\[ \alpha(R_d/10) \approx \alpha_0 \left( \frac{M_{inf}}{10^{-7} M_\odot \text{yr}^{-1}} \right)^\gamma. \]  

(4)

For inflows with significant rotation (L_{out} = 1) we have \( \alpha_0 = 1.3 \times 10^{-4} \) and \( \gamma = 0.4 \), whereas for inflows without initial
ical attenuation of a factor 3 between \( \varepsilon \) and the momentum of a Keplerian disc at several locations (Table 1).

### Notes.
- \( M_{\text{out}} \) is measured in \( M_\odot \text{ yr}^{-1} \), and \( L_{\text{out}} \) in units of specific angular momentum of a Keplerian disc at \( R = 100 \): \( L_{\text{out}} \).

As is well known, standard accretion disc theory allows determining the angular momentum of the falling material is always smaller than that of the outer disc so that a radial shear layer is always present, a possibility only explored in one simulation by Bae et al. (2015). Whether an external inflow might be at the origin of these spiral pattern is an open question that we defer to a future publication.

### Acknowledgements.
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### References


### 4. Discussion

We explored the impact of an external inflow on a protoplanetary disc using 2D hydrodynamical simulations. We found that this interaction leads to the generation of strong spiral waves that propagate on long distances (typically down to radii smaller than \( R_{\text{out}} / 10 \)). The resulting \( \alpha \) at \( R_{\text{out}} / 2 \) is larger than \( 10^{-3} \) and reach a few \( 10^{-4} \) at \( R_{\text{out}} / 10 \) for inflow mass rates higher than \( 10^{-8} M_\odot \text{ yr}^{-1} \). This stress varies strongly with \( M_{\text{infl}} \) and \( H/R \), but not significantly affected by the geometry of the inflow.

As shown in the introduction, this study is similar in nature to the work of Bae et al. (2015). However, several important differences should be emphasised. First, our simulations only assumed a radial inflow and no vertical inflow. Second, the angular momentum of the falling material is always smaller than that of the outer disc so that a radial shear layer is always present, a possibility only explored in one simulation by Bae et al. (2015). Because of these differences, we do not observe the formation of vortices, most probably because the instability at \( R_0 \) involves a mixture of the RWI and of Rayleigh centrifugal instability thanks to the strong shear layer. We also obtained higher \( \alpha \) values than did Bae et al. (2015) in the generation region, probably due to the same shear layer.

Finally, spiral waves such as we discussed here are now detectable with infrared continuum observations. Several authors have already reported direct observations of spiral patterns at the outer edge of evolved protostellar discs (e.g. Muto et al. 2012; Benisty et al. 2015). Whether an external inflow might be at the origin of these spiral pattern is an open question that we defer to a future publication.

### Table 1. List of simulations.

<table>
<thead>
<tr>
<th>Run</th>
<th>( M_{\text{infl}} )</th>
<th>( L_{\text{out}} )</th>
<th>( \alpha(R_0/2) )</th>
<th>( \alpha(R_0/10) )</th>
<th>( R_0 )</th>
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<tbody>
<tr>
<td>S8-1</td>
<td>10^{-6}</td>
<td>1</td>
<td>1.6 × 10^{-3}</td>
<td>5.3 × 10^{-3}</td>
<td>255</td>
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<tr>
<td>S7-1</td>
<td>10^{-7}</td>
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<td>4.5 × 10^{-3}</td>
<td>1.4 × 10^{-4}</td>
<td>199</td>
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<tr>
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<td>1</td>
<td>1.0 × 10^{-2}</td>
<td>3.5 × 10^{-4}</td>
<td>167</td>
</tr>
<tr>
<td>A7-1</td>
<td>10^{-7}</td>
<td>1</td>
<td>3.5 × 10^{-3}</td>
<td>3.8 × 10^{-4}</td>
<td>184</td>
</tr>
<tr>
<td>S8-1-thin</td>
<td>10^{-6}</td>
<td>1</td>
<td>3.8 × 10^{-4}</td>
<td>1.3 × 10^{-5}</td>
<td>225</td>
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<tr>
<td>S7-1-thin</td>
<td>10^{-7}</td>
<td>1</td>
<td>8.3 × 10^{-4}</td>
<td>3.4 × 10^{-5}</td>
<td>164</td>
</tr>
<tr>
<td>S6-1-thin</td>
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<td>1</td>
<td>1.5 × 10^{-3}</td>
<td>1.1 × 10^{-4}</td>
<td>135</td>
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<tr>
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<td>2.8 × 10^{-5}</td>
<td>264</td>
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<td>177</td>
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<tr>
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<tr>
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<td>5.0 × 10^{-4}</td>
<td>90</td>
</tr>
</tbody>
</table>

By comparing \( \alpha(r) \) obtained in the asymmetric case to the symmetric case, we find that asymmetric accretion in general drives a slightly more efficient angular momentum transport, probably thanks to the excitation of low \( m \) modes of the instability by the stream that propagates more easily. At \( R = R_{\text{out}} / 10 \), we obtain an \( \alpha \) about two to four times larger than the one obtained from symmetric accretion simulations (Fig. 4). This suggests that the symmetric accretion scenario given by Eq. (4) constitutes a lower bound to spiral-driven angular momentum transport.