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**Abstract.** – Electrical conductivity measurements on a 2D packing of metallic beads have been performed to study internal rearrangements in weakly perturbed granular materials. Small thermal perturbations lead to large non gaussian conductance fluctuations. These fluctuations are found to be intermittent in time, with peaks gathered in bursts. The distributions of the waiting time $\Delta t$ between two peaks is found to be a power-law $\Delta t^{-(1+\alpha_t)}$ inside bursts. The exponent $\alpha_t$ is independent of the bead network, the intensity of perturbation and external stress. These bursts are interpreted as the signature of individual bead creep rather than collective vault reorganisations. We propose a simple model giving $\alpha_t = 1 - \zeta$, where $\zeta$ is the roughness exponent of the surface of the beads.

*Introduction.* – Granular materials present interesting and unusual properties [1]. For example, photoelastic visualizations of static confined granular packing under external stress have provided evidence of large inhomogeneities in the distribution of contact forces between grains [2, 3], giving rise to strong force chains (vaults or arches) extending on a scale much larger than the size of an individual grain. Recent experiments have revealed the sensitivity of sound transmission in granular media to small perturbations (such as thermal expansion of the grains) [4]. This sensitivity has been interpreted using a simple theoretical model of vault formation, the 'Scalar Arching Model' [5]. This model predicts that large rearrangements of the force chains network can indeed occur, even for small external perturbations. This sensitivity tends to disappear when the media is subjected to a strong external stress [6].

However, both photoelastic visualization and sound transmission techniques require large external stresses and can not give information on a weakly confined granular media where interesting signatures of *fragility* [7] can be expected. We propose here to study this fragility through conductivity measurements. Experiments on the electrical properties of granular media have been performed in the past : Branly reported [8] the influence of an electromagnetic wave on the electrical resistance of a granular packing. Giraud et al. [9], Marion et al. [10] and Gervois et al. [11] have performed measurements of the electrical conductivity in order to reveal the influence of the contact network on the mechanical properties of the media. More recently, Vandewalle et al. [12] reported power-law distributions of conductivity changes, and argued that this was a signature of large rearrangments of stress paths.

The aim of this work is to investigate whether force chain rearrangements can be observed through the variation of the electrical conductance of a 2D bead packings under small thermal perturbations. Large intermittent fluctuations are indeed observed. They appear in bursts that can be characterized via a statistical analysis of the waiting time separating two fluctuation peaks. The microscopic origin of these electrical fluctuations is then investigated. We argue that most of the interesting statistical features are not related to large force chains rearrangements but to individual microcontacts between two beads that rearrange. The observed non trivial statistics might be related to the self-affine roughness of the surface of the beads.

*Experiment.* – The experimental set-up is illustrated on fig. 1. It consists mainly of a 2D packing of stainless steel beads of diameter $a = 6\,\mathrm{mm} \pm 10\,\mu\mathrm{m}$ confined between two Plexiglas plates (thickness 2 cm). The use of metallic beads of millimetric size allows one to have a good control of geometrical and mechanical properties. We can also change the number of grains in the sample to separate local behaviour from collective behaviour. Beads are carefully arranged to form triangular compact packing. This situation of maximal compacity can easily be reproduced before each experiment. Even if the piling can be viewed as regular, the weak polydispersity and the solid friction between grains is sufficient to entail disorder in the network of contact forces [13]. The size of the packing is $W \times H$ where $W$ is its width and $H$ its height in bead size units. Thermal perturbations are induced by a 75 W lamp standing at a distance $d = 10\,\mathrm{cm}$. The lamp increases the temperature of the beads and of the Plexiglas plates by typically $3\,^\circ\mathrm{C}$. This leads to an expansion of both the beads and the plates: the thermal expansivity for beads is $2\ 10^{-5}\,\mathrm{K}^{-1}$ and $7\ 10^{-5}\,\mathrm{K}^{-1}$ for Plexiglas. The bead network can also be vibrated via a buzzer placed against the Plexiglas plate. The packing is connected to a 9 V battery. A resistor $R_1$ in series insures that the current crossing a contact between any couple of beads is much smaller than 40 mA. Preliminary studies performed on 2 beads indeed show that, as long as this current is below 40 mA, micro-welding is prevented and the contacts behave reversibly like an ohmic resistance. Voltages $V_0$, $V_1$ and $V_2$ (see fig. 1) are recorded via a 12 bytes AD converter: $V_0$ is the emf of the battery, $V_1$ is proportional to the current crossing the packing and $V_2$ represents the fast fluctuating part (faster than $\tau = RC < 0.02\,\mathrm{s}$) of $V_1$. The following quantities can then be deduced:

- the conductance $g_p$ of the packing. For a $33 \times 52$ vibrated packing, $g_p \simeq 0.02\,\Omega^{-1}$.
- the fluctuation of conductance $\Delta g_p$ defined as the variation of $g_p$ between 2 acquisition points. Typically, $\Delta g_p$ reaches values up to $5.10^{-3}\,\Omega^{-1}$ to be compared to the detection threshold discussed below, which is typically around $5.10^{-6}\,\Omega^{-1}$.

A series of experiments was performed with the bead packing replaced by a $50\,\Omega$ resistor in order to determine the level of noise of the acquisition system. This is mainly dominated by digital errors on $V_2$ equal to $5\,\mu\mathrm{V}$. A threshold is then defined: $\Delta g_p$ is considered as relevant when the corresponding $\Delta V_2$ is superior to $50\,\mu\mathrm{V}$. Sampling rates $f$ from 0.05 Hz up to 10 kHz have been tried: $f = 500\,\mathrm{Hz}$ is sufficient to separate most successive events (cf fig. 2). For each experiment, data is recorded during a period of 20 minutes. The procedure is the following : (i) the packing is vibrated, (ii) the acquisition is started, (iii) the light is turned on 1 minute after (turned off in experiments with packing prepared with light switched on).

*Results.* – A packing, initially vibrated, keeps a constant conductance as long as its temperature does not change. As soon as it is thermally perturbed (light turned on or off), $g_p$

starts to decrease and to fluctuate. Let us focus firstly on the fluctuations $\Delta g_p$ for a typical experiment (see fig. 2) on a $33 \times 52$ packing. Fluctuation peaks appear in an intermittent way. The distribution of $\Delta g_p$ is to a good approximation symmetrical and can be described (at least for small enough $|\Delta g_p|$) by the following power-law:

$$P(|\Delta g_p|) \sim |\Delta g_p|^{-(1+\alpha_g)} \tag{1}$$

For this experiment, $\alpha_g \simeq 1$, in agreement with the results reported in [12]. For large $|\Delta g_p|$, $P(|\Delta g_p|)$ departs from this power-law behaviour. One can also study, for the same experiment, the temporal structure of the apparition of these peaks. The expanded view fig. 2b of $\Delta g_p(t)$ presents a burst structure, which can be characterised by the distribution of the waiting time $\Delta t$ between two successive events (independently of their sign). Two distinct regimes can be observed in fig. 2d:

- The short $\Delta t$ part of the probability density $P(\Delta t)$ (corresponding to events within the same burst) decays as a power law with $\Delta t$:

  $$P(\Delta t) \sim \Delta t^{-(1+\alpha_t)} \tag{2}$$

  The exponent is found to be $\alpha_t \simeq 0.6$.

- for larger $\Delta t$, $P(\Delta t)$ again departs from this power-law behaviour. The cumulative distribution function $F(\Delta t)$ is less noisy for large $\Delta t$ and shown in the inset of fig. 2d. One clearly sees that $F$ decays exponentially for large $\Delta t$, which is the signature of a Poisson flux. The bursts themselves therefore appear as completely independent events, whereas the events inside bursts are strongly non-Poissonian.

In order to determine the origin of these electrical fluctuations, several experiments have been performed with different packing: (1) regular $33 \times 52$ packing, (2) disordered packing of 1690 beads, (3) regular $33 \times 32$ packing, (4) regular $33 \times 16$ packing, (5) regular $33 \times 2$ packing. Finally, (6) is a regular $33 \times 2$ packing where all metallic beads have been replaced by insulating glass beads except 3 beads forming a triangle in the middle. For all these experiments, $P(|\Delta g_p|)$ decays as a power-law but the exponent $\alpha_g$ depends on the bead network geometry, and lies between 0.8 and 1.6 (see fig. 3a). On the contrary, the power-law decay of $P(\Delta t)$ does not depend on the packing, and leads to $\alpha_t \sim 0.6$ (see fig. 3b). For the packing of 3 beads, the power-law behaviour actually extends over the three decades of the time distribution: only few bursts are present. The exponent is found to be $\alpha_t = 0.6 \pm 0.05$. When the number of beads increases, the value of $\Delta t$ above which $P(\Delta t)$ starts to deviate from a power-law decreases. It can be interpreted by the increase of independent burst number making the inter-burst waiting time mix more and more with the intra-burst statistics. The intensity of the thermal perturbation has been changed by modifying the distance of the lamp ($d = 5\,\mathrm{cm}$, $10\,\mathrm{cm}$ and $20\,\mathrm{cm}$) without affecting the value of $\alpha_t$. Moreover, for packing (5), the weight of the upper electrode has been varied ($M = 108\,\mathrm{g}$, $M = 206\,\mathrm{g}$, $M = 304\,\mathrm{g}$ and $M = 404\,\mathrm{g}$). The number of events decreases, but $P(\Delta t)$ remains a power-law with the same exponent $\alpha_t = 0.6$.

*Discussion.* – The origin of these non trivial electric fluctuations should be found in the geometry of electrical paths. A natural idea would be to relate these to force chains rearrangements. Indeed the Scalar Arching Model (SAM) introduced by two of us [5] suggests the existence of large force chains rearrangements in a packing subjected to small thermal perturbations. The SAM predicts a broad distribution of the apparent weight fluctuation $\Delta W_a$

measured at the bottom of the packing. This broad distribution of $\Delta W_a$ could in principle be related to the observed broad distribution of $\Delta g_p$. Moreover, the model also predicts avalanching contact reorganisations whose electrical signature could indeed be the bursts of fluctuations peaks seen in the thermally perturbed packing, each $\Delta g_p$ peak corresponding to the creation or the breakdown of a contact somewhere in the packing. However, in this scenario, the intra-burst structure should become less and less pronounced when the number of beads decreases, and should disappear completely for a packing of 3 beads. This is clearly not the case (see fig. 3b).

The fact that the power-law nature of the distributions is independent of the geometry of the packing, and still holds for three beads, suggests that the origin of these electrical fluctuations is local. The direct imaging of the surface of the beads with an Atomic Force Microscope allowed us to measure the average roughness $R$ of the beads. We have found $R = 90\,\mathrm{nm}$, to be compared to Hertzian deformation $\delta$ in the packing. For contact forces on the order of $10^{-2}$ Newton, $\delta \simeq 10\,\mathrm{nm}$ which is smaller than $R$. The electrical contact between 2 beads is thus completely dominated by the surface roughness. We propose then the following scenario to explain the observed behaviour of $g_p$ and $\Delta g_p$ in our experiment: when the packing is vibrated, the surface roughness of the two surfaces in contact 'adapt' to one another and the effective contact surface is high. Therefore, $g_p$ is maximal. As soon as the packing is heated, beads expand on the order of $0.5\,\mu\mathrm{m}$. This thermal expansion is hindered by the roughness induced friction between beads. Therefore, most of the time the contact between two beads does not change, until the accumulated stress due to the thermal expansion is sufficient to 'unpin' the rough surfaces and make them slip over each other. This leads to the burst structure of the signal. Inside each burst, individual peaks of $\Delta g_p$ correspond to the creation or the loss of a micro-contact between the two slipping beads. These micro-contact avalanches globally lead to a smaller effective contact surface, thereby making $g_p$ smaller.

Simple models of pinning [14, 15] can lead to a power-law distribution of the 'trapping' time between successive events; however, in these models the exponent $\alpha_t$ is found to depend on the external driving force. The fact that $\alpha_t$ is independent of both the strength of the perturbation and of the external stress suggests a purely geometrical interpretation: if the bead surface can be considered to be self-affine with a Hurst exponent $\zeta$, the distance $\Delta\ell$ separating two successive 'spikes' in a given direction is power-law distributed with an exponent $\alpha_\ell = 1-\zeta$ on the interval $[a_1, a_2]$ where $a_1$ is a microscopic length below which the surface is flat, and $a_2$ is at most the diameter of an Hertzian contact: $a_2 \simeq 1\,\mu\mathrm{m}$. The typical speed $V$ with which a bead thermally expands is $1\,\mu\mathrm{m}$ per second. If this speed is constant in time, the time between successive micro-contact closing/opening is therefore a power law distribution with an exponent $\alpha_t = 1 - \zeta$ in the interval $[a_1/V, a_2/V = 1\,\mathrm{s}]$, which is precisely the experimental interval of time scales. We have determined $\zeta$ from AFM frames via the moving average method [16]: the moving average of a series $z(x)$ on a length interval $L$ is defined as :

$$\overline{z}(x) = \frac{1}{L}\sum_{i=0}^{L-1} z(x+i) \tag{3}$$

For a self affine signal with an Hurst exponent of $\zeta$, the density $\rho$ of crossing point between two moving averages on length scales $L_1$ and $L_2 > L_1$ is found to scale as:

$$\rho \sim \frac{1}{L_2}[(\Delta L)(1-\Delta L)]^{\zeta-1} \tag{4}$$

where $\Delta L = (L_2 - L_1)/L_2$.

AFM frames are stored as $256 \times 256$ matrices. $L_2$ is fixed to 128 and $\rho(\Delta L)$ has been determined for rows and columns averaging (see fig. 4). In both case, we find $\zeta = 0.35 \pm 0.05$, and therefore $1 - \zeta \simeq 0.65$. This value is consistent with $\alpha_t = 0.6$.

*Conclusion.* – In summary, a series of electrical measurements has been performed to test the SAM predictions of large force chains rearrangements in a 2D metallic beads packing under small perturbations. As soon as the packing is heated or cooled, its conductance starts to decrease and to fluctuate. These fluctuations are intermittent and gather in bursts as predicted by the SAM. However, the waiting time distribution inside bursts are independent of beads network, perturbation intensity and external applied stress. Consequently, the origin of these fluctuations should be found in local microcontact rearrangements at each beads rather than collective vaults rearrangements, as suggested in [12]. However, some information on these collective rearrangments might lie in the deviation from a pure power-law distribution which is clearly observed on figure 4 when one increases the number of grains. These collective effects might also be needed to account for the variation of the exponent $\alpha_g$ with the geometry. Work in this direction is underway.

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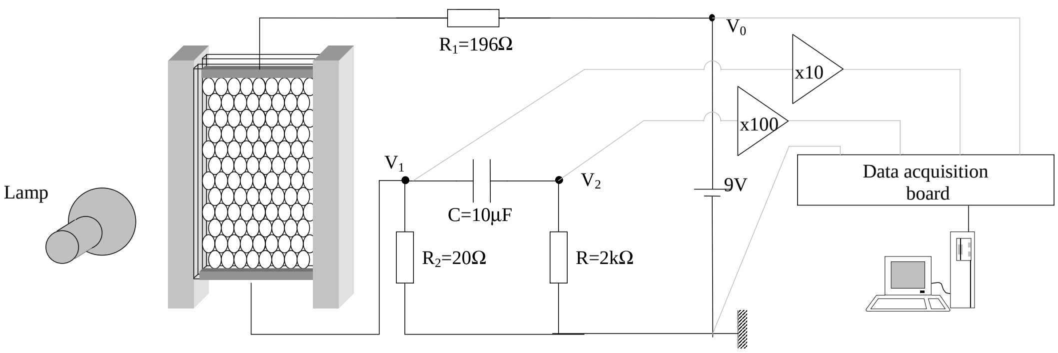


Fig. 1 – experimental set-up

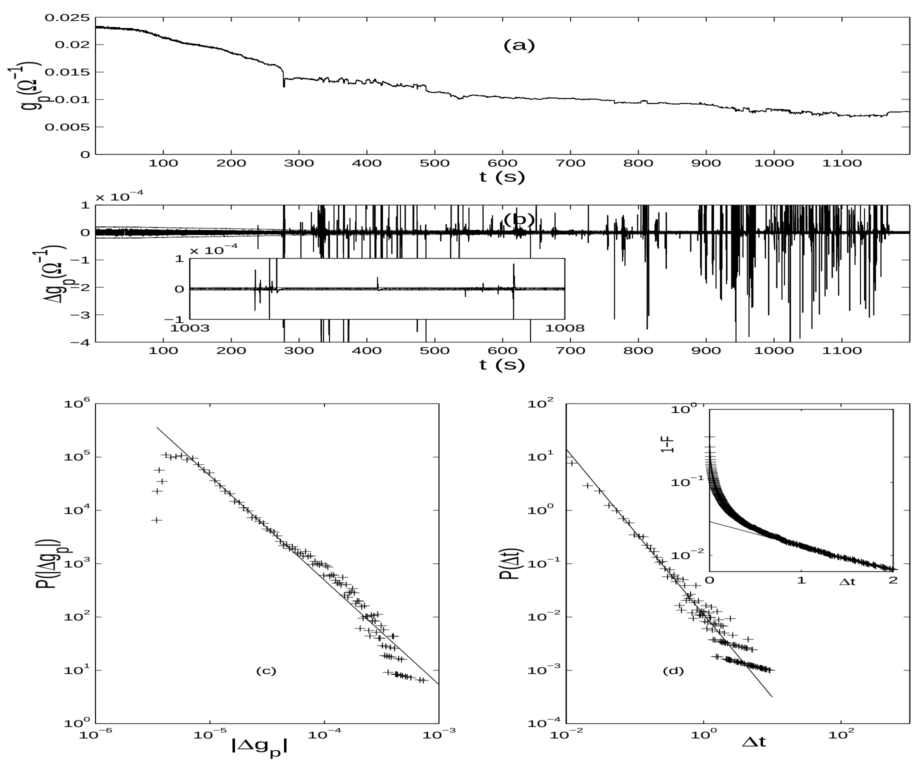


Fig. 2 – Analysis of $\Delta g_p$ for a $33 \times 52$ regular packing heated with the lamp. (a) Temporal evolution of the $g_p$. (b) Temporal evolution of $\Delta g_p$. Inset, expanded view on a few seconds scale (c) Log-log plot of the distribution $P(|\Delta g_p|)$. The straight line is a power law fit using Eq. 1 with $\alpha_g = 1$.(d) Log-log plot of the probability density function $P(\Delta t)$. The straight line is a power law fit using Eq. 2, with $\alpha_t = 0.61$. The inset shows the cumulative distribution which decays asymptotically as an exponential.

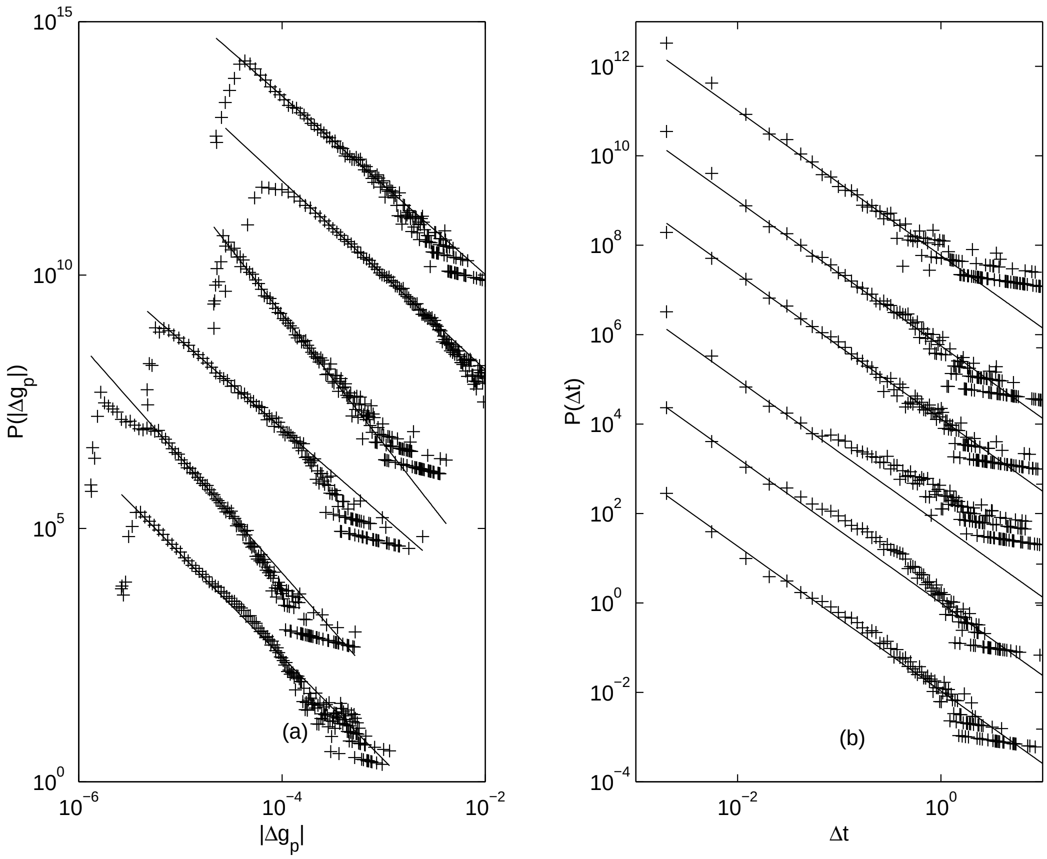


Fig. 3 – Log-log plot of (a) $P(|\Delta g_p|$ and (b) $P(\Delta t)$ for packing (1), (2), (3), (4), (5), and (6) from bottom to top. Data have been shifted for clarity. The straight lines are power-law fits. The exponent $\alpha_t$ has been fixed for all $P(\Delta t)$ to $\alpha_t = 0.6$ minimising errors on the fit for the three bead packing (6). $\alpha_g$ is equal, from bottom to top, to 1.0, 1.1, 0.8, 1.6, 0.9, and 0.8 respectively.

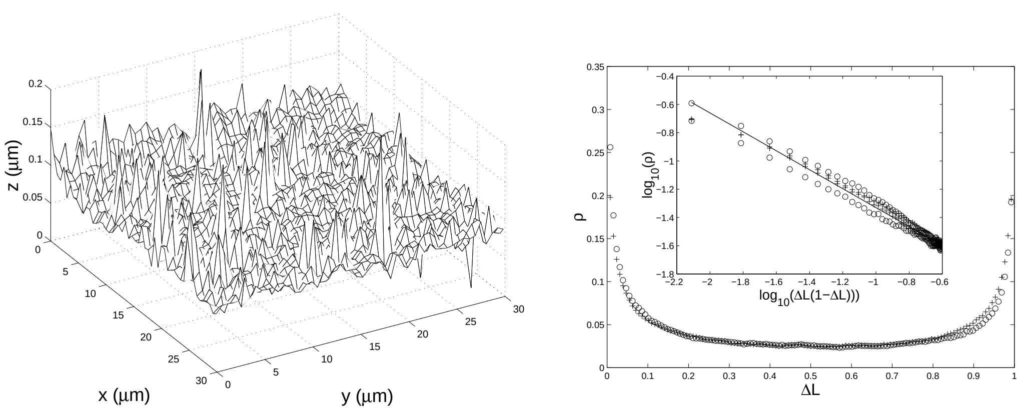


Fig. 4 – (a) AFM picture of the bead surface (b) density $\rho$ as a function of the relative difference $\Delta L$ with $L_2 = 128$. $o$ corresponds to row averaging and $+$ correspond to column averaging. Inset : $\log_{10}(\rho)$ as a function of $\log_{10}(\Delta L(1 - \Delta L))$. The straight line corresponds to a fit using Eq. 4