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# Universal fluctuation-driven eccentricities in proton-proton, proton-nucleus and nucleus-nucleus collisions

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We show that the statistics of fluctuation-driven initial-state anisotropies in proton-proton, proton-nucleus and nucleus-nucleus collisions is to a large extent universal. We propose a simple parameterization for the probability distribution of the Fourier coefficient  $\varepsilon_n$  in harmonic  $n$ , which is in good agreement with Monte-Carlo simulations. Our results provide a simple explanation for the 4-particle cumulant of triangular flow measured in Pb-Pb collisions, and for the 4-particle cumulant of elliptic flow recently measured in p-Pb collisions. Both arise as natural consequences of the condition that initial anisotropies are bounded by unity. We argue that the initial rms anisotropy in harmonic  $n$  can be directly extracted from the measured ratio  $v_n\{4\}/v_n\{2\}$ : this gives direct access to a property of the initial density profile from experimental data. We also make quantitative predictions for the small lifting of degeneracy between  $v_n\{4\}$ ,  $v_n\{6\}$  and  $v_n\{8\}$ . If confirmed by future experiments, they will support the picture that long-range correlations observed in p-Pb collisions at the LHC originate from collective flow proportional to the initial anisotropy.

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## INTRODUCTION

A breakthrough in our understanding of high-energy nuclear collisions is the recognition [1, 2] that quantum fluctuations in the wavefunctions of projectile and target, followed by hydrodynamic expansion, result in unique long-range azimuthal correlations between outgoing particles. The importance of these fluctuations was pointed out in the context of detailed analyses of elliptic flow in nucleus-nucleus collisions [1, 3]. It was later realized that fluctuations produce triangular flow [2], which has subsequently been measured in nucleus-nucleus collisions at RHIC [4, 5] and LHC [6–8]. Recently, fluctuations were predicted to generate significant anisotropic flow in proton-nucleus collisions [9], which quantitatively explains [10] the long-range correlations observed by LHC experiments [11–13].

Recently, the ATLAS and CMS experiments reported the observation of a nonzero 4-particle cumulant of azimuthal correlations, dubbed  $v_2\{4\}$ , in proton-nucleus collisions [14, 15]. The occurrence of a large  $v_2\{4\}$  in proton-nucleus collisions is not fully understood, even though it is borne out by hydrodynamic calculations with fluctuating initial conditions [16]. Such higher-order cumulants were originally introduced [17, 18] to measure elliptic flow in the reaction plane of non-central nucleus-nucleus collisions, and isolate it from other, “nonflow” correlations. It turns out that the simplest fluctuations one can think of, namely, Gaussian fluctuations, do not contribute to  $v_2\{4\}$  [19]. Since flow in proton-nucleus collisions is thought to originate from fluctuations in the initial geometry, one naively expects  $v_2\{4\} \sim 0$ , *even if* there is collective flow in the system.

In this paper, we argue that the values observed for  $v_2\{4\}$  in p-Pb collisions are naturally explained by non-

Gaussian fluctuations, which are expected for small systems. Our explanation differs from that recently put forward by Bzdak et al. [20] that it is due to symmetry breaking (see Eq. (3) and discussion below). As Bzdak et al., we assume that anisotropic flow  $v_n$  scales like the corresponding initial-state anisotropy  $\varepsilon_n$  on an event-by-event basis. This is known to be a very good approximation in ideal [21] and viscous [22] hydrodynamics. Thus flow fluctuations directly reflect  $\varepsilon_n$  fluctuations. Now,  $\varepsilon_n$  is bounded by unity by definition. On the other hand, Gaussian fluctuations are not bounded, which is the reason why they fail to model small systems. We propose a simple alternative to the Gaussian parameterization which naturally satisfies the constraint  $\varepsilon_n < 1$ . We show that it provides an excellent fit to all Monte-Carlo calculations.

## DISTRIBUTION OF THE INITIAL ANISOTROPY

In each event, the anisotropy in harmonic  $n$  is defined (for  $n = 2, 3$ ) by [23]

$$\begin{aligned}\varepsilon_{n,x} &\equiv -\frac{\int r^n \cos(n\phi) \rho(r, \phi) r dr d\phi}{\int r^n \rho(r, \phi) r dr d\phi} \\ \varepsilon_{n,y} &\equiv -\frac{\int r^n \sin(n\phi) \rho(r, \phi) r dr d\phi}{\int r^n \rho(r, \phi) r dr d\phi},\end{aligned}\quad (1)$$

where  $\rho(r, \phi)$  is the initial transverse density profile near midrapidity in a centered polar coordinate system.

Fig. 1 displays the histogram of the distribution of  $\varepsilon_2$  in a p-Pb collision at 5.02 TeV obtained in a Monte-Carlo Glauber calculation [24]. We use the PHOBOS implementation [25] with a Gaussian wounding profile [26, 27]. We assume that the initial density  $\rho(r, \phi)$  is a sum of Gaussians of width  $\sigma_0 = 0.4$  fm, centered

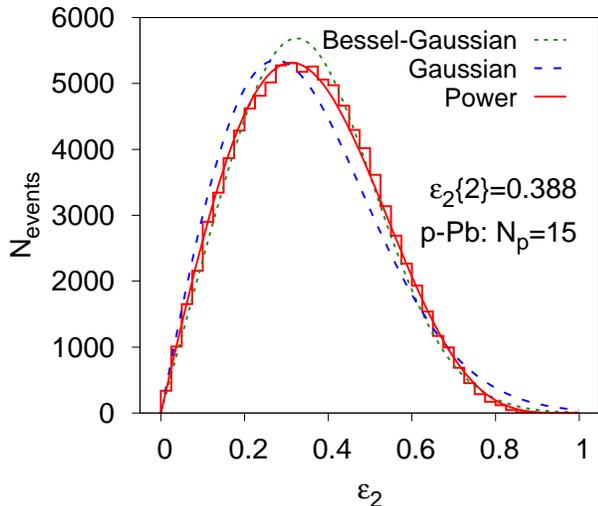


FIG. 1. (Color online) Histogram of the distribution of  $\varepsilon_2$  obtained in a Monte-Carlo Glauber simulation of a p-Pb collision at LHC, and fits using Eqs. (2)-(4).

around each participant nucleon with a normalization that fluctuates [28]. These fluctuations, which increase anisotropies [29], are modeled as in Ref. [20]. We have selected events with number of participants  $14 \leq N \leq 16$ , corresponding to typical values in a central p-Pb collision.

We now compare different parameterizations of this distribution, which we use to fit our numerical results. The first is an isotropic two-dimensional Gaussian (we drop the subscript  $n$  for simplicity):

$$P(\varepsilon) = \frac{2\varepsilon}{\sigma^2} \exp\left(-\frac{\varepsilon^2}{\sigma^2}\right), \quad (2)$$

where  $\varepsilon \equiv \sqrt{\varepsilon_x^2 + \varepsilon_y^2}$  and the distribution is normalized:  $\int_0^\infty P(\varepsilon)d\varepsilon = 1$ . This form is motivated by the central limit theorem, assuming that the eccentricity solely originates from event-by-event fluctuations, and neglecting fluctuations in the denominator. Note that this distribution does not strictly satisfy the constraint  $\varepsilon < 1$ , which follows from the definition (1). When fitting our Monte-Carlo results, we have therefore multiplied Eq. (2) by a constant to ensure normalization between 0 and 1. The rms  $\varepsilon$  has been fitted to that of the Monte-Carlo simulation. Fig. 1 shows that Eq. (2) gives a reasonable approximation to our Monte-Carlo results, but not a good fit.

Bzdak et al. [20] have proposed to replace Eq. (2) by a ‘‘Bessel-Gaussian’’:

$$P(\varepsilon) = \frac{2\varepsilon}{\sigma^2} I_0\left(\frac{2\varepsilon\bar{\varepsilon}}{\sigma^2}\right) \exp\left(-\frac{\varepsilon^2 + \bar{\varepsilon}^2}{\sigma^2}\right). \quad (3)$$

This parameterization introduces an additional free pa-

rameter  $\bar{\varepsilon}$ , corresponding to the mean eccentricity in the reaction plane in nucleus-nucleus collisions [19]. It reduces to (2) if  $\bar{\varepsilon} = 0$ . A nonzero value of  $\bar{\varepsilon}$  is however difficult to justify for a symmetric system in which anisotropies are solely created by fluctuations. In Fig. 1,  $\bar{\varepsilon}$  and  $\sigma$  have been chosen so that the first even moments  $\langle \varepsilon^2 \rangle$  and  $\langle \varepsilon^4 \rangle$  match exactly the Monte-Carlo results, as suggested in [20]. The quality of the fit is not much improved compared to the Gaussian distribution, even though there is an additional free parameter. Note that the Bessel-Gaussian, like the Gaussian, does not take into account the constraint  $\varepsilon < 1$ .

We now introduce the one-parameter power law distribution:

$$P(\varepsilon) = 2\alpha\varepsilon(1 - \varepsilon^2)^{\alpha-1}, \quad (4)$$

where  $\alpha > 0$ . Eq. (4) reduces to Eq. (2) for  $\alpha \gg 1$ , with  $\sigma^2 \equiv 1/\alpha$ . The main advantage of Eq. (4) over previous parameterizations is that the support of  $P(\varepsilon)$  is the unit disc: it satisfies for all  $\alpha > 0$  the normalization  $\int_0^1 P(\varepsilon)d\varepsilon = 1$ . In the limit  $\alpha \rightarrow 0^+$ ,  $P(\varepsilon) \simeq \delta(\varepsilon - 1)$ .

Eq. (4) is the *exact* [30]<sup>1</sup> distribution of  $\varepsilon_2$  for  $N$  identical pointlike sources with a 2-dimensional isotropic Gaussian distribution, with  $\alpha = (N - 1)/2$ , if one ignores the recentering correction. In a more realistic situation, Eq. (4) is no longer exact. We adjust  $\alpha$  to match the rms  $\varepsilon$  from the Monte-Carlo calculation. Fig. 1 shows that Eq. (4) (with  $\alpha \simeq 5.64$ ) agrees much better with Monte-Carlo results than Gaussian and Bessel-Gaussian distributions.

## CUMULANTS

Cumulants of the distribution of  $\varepsilon$  are derived from a generating function, which is the logarithm of the two-dimensional Fourier transform of the distribution of  $(\varepsilon_x, \varepsilon_y)$ :

$$G(k_x, k_y) \equiv \ln\langle \exp(ik_x\varepsilon_x + ik_y\varepsilon_y) \rangle, \quad (5)$$

where angular brackets denote an expectation value over the ensemble of events. If the system has azimuthal symmetry, by integrating over the relative azimuthal angle of  $\mathbf{k}$  and  $\varepsilon$ , one obtains

$$G(k) = \ln\langle J_0(k\varepsilon) \rangle, \quad (6)$$

where  $k \equiv \sqrt{k_x^2 + k_y^2}$  and  $\varepsilon \equiv \sqrt{\varepsilon_x^2 + \varepsilon_y^2}$ . The cumulant to a given order  $n$ ,  $\varepsilon\{n\}$ , is obtained by expanding

<sup>1</sup> See Eq. (3.10) of [30]. What is derived there is the distribution of anisotropy in momentum space, but the algebra is identical for the distribution of eccentricity.

Eq. (6) to order  $k^n$ , and identifying with the expansion of  $\ln J_0(k\varepsilon\{n\})$  to the same order. This uniquely defines  $\varepsilon\{n\}$  for all *even*  $n$ . One thus obtains [3]  $\varepsilon\{2\}^2 = \langle \varepsilon^2 \rangle$ ,  $\varepsilon\{4\}^4 = 2\langle \varepsilon^2 \rangle^2 - \langle \varepsilon^4 \rangle$ . Expressions of  $\varepsilon\{6\}$  and  $\varepsilon\{8\}$  are given in [20].

TABLE I. Values of the first eccentricity cumulants for the Gaussian (2), Bessel-Gaussian (3) and power law (4) distributions.

	Gauss	BG	Power
$\varepsilon\{2\}$	$\sigma$	$\sqrt{\sigma^2 + \bar{\varepsilon}^2}$	$\frac{1}{\sqrt{1 + \alpha}}$
$\varepsilon\{4\}$	0	$\bar{\varepsilon}$	$\left[ \frac{2}{(1 + \alpha)^2(2 + \alpha)} \right]^{1/4}$
$\varepsilon\{6\}$	0	$\bar{\varepsilon}$	$\left[ \frac{6}{(1 + \alpha)^3(2 + \alpha)(3 + \alpha)} \right]^{1/6}$
$\varepsilon\{8\}$	0	$\bar{\varepsilon}$	$\left[ \frac{48 \left( 1 + \frac{5\alpha}{11} \right)}{(1 + \alpha)^4(2 + \alpha)^2(3 + \alpha)(4 + \alpha)} \right]^{1/8}$

Expressions of the first four cumulants are listed in Table I. For the power law distribution (4), these results are obtained by expanding the generating function (6):

$$G(k) = \ln \left[ \int_0^1 J_0(k\varepsilon) P(\varepsilon) d\varepsilon \right] = \ln \left[ \frac{2^\alpha \alpha!}{k^\alpha} J_\alpha(k) \right]. \quad (7)$$

General results have been obtained previously in the case of  $N$  pointlike sources and in the large  $N$  limit for  $\varepsilon_2\{2\}$  [31] and  $\varepsilon_2\{4\}$  [32]. Our results derived from Eq. (4) are exact for a Gaussian distribution of sources and therefore agree with these general results for  $N \gg 1$ . Similar results have also been derived for  $\varepsilon_3\{2\}$  and  $\varepsilon_3\{4\}$  [33], but not for cumulants of order 6 or higher.

Fig. 2 displays the cumulants  $\varepsilon\{2\}$  to  $\varepsilon\{8\}$  as a function of  $N$ , as predicted by Eq. (4) for pointlike sources.<sup>2</sup> These results are similar to those obtained in full Monte-Carlo Glauber calculations [20]. In the limit  $N \gg 1$ , the power law distribution yields  $\varepsilon\{k\} \propto N^{(1-k)/k}$ . It thus predicts a strong ordering  $\varepsilon\{8\} \ll \varepsilon\{6\} \ll \varepsilon\{4\} \ll \varepsilon\{2\} \ll 1$ , unlike the Bessel-Gaussian which predicts  $\varepsilon\{4\} = \varepsilon\{6\} = \varepsilon\{8\}$ . For fixed  $N$ , however, the cumulant expansion quickly converges, as illustrated in Fig. 2. In practice, for typical values of  $N$  in p-Pb collisions, one observes  $\varepsilon\{4\} \simeq \varepsilon\{6\} \simeq \varepsilon\{8\}$ , in agreement with numerical findings of Bzdak et al. [20]. This rapid convergence can be

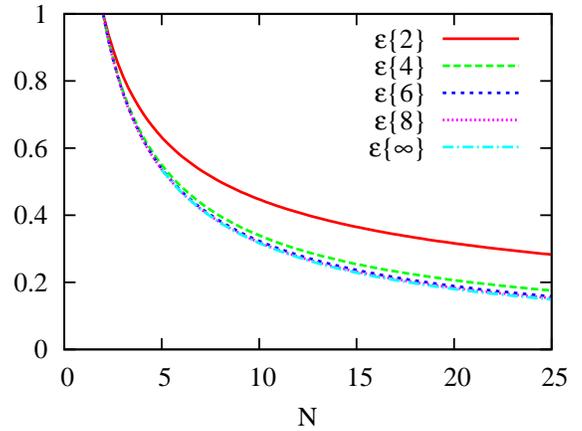


FIG. 2. (Color online) Cumulants of the eccentricity distribution as a function of the number of participants  $N$  for the power law distribution (4), where we have set  $\alpha = (N - 2)/2$ .

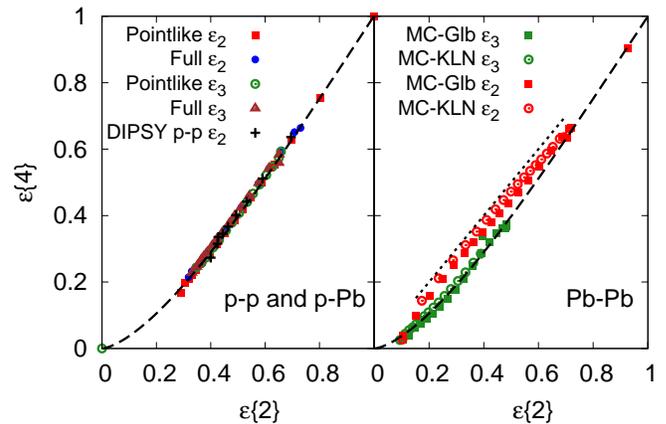


FIG. 3. (Color online)  $\varepsilon\{4\}$  versus  $\varepsilon\{2\}$ . The dashed line in both panels is Eq. (9). Left: p-Pb collisions. “Full” refers to Gaussian sources associated with each participant, and fluctuations in the weights of each source. “Pointlike” refers to pointlike identical sources. DIPSY results for p-p collisions are replotted from [35]. Right: Pb-Pb collisions. The dotted line is  $\varepsilon\{4\} = \varepsilon\{2\}$ , corresponding to a nonzero mean eccentricity, and negligible fluctuations.

traced back to the fact that the generating function  $G(k)$  in Eq. (7) has a singularity at the first zero of  $J_\alpha(k)$ , denoted by  $j_{\alpha 1}$ . This causes the cumulant expansion to quickly converge to the value [34]

$$\varepsilon\{\infty\} = \frac{j_{01}}{j_{\alpha 1}}. \quad (8)$$

This asymptotic limit is also plotted in Fig. 2. It is hardly distinguishable from  $\varepsilon\{6\}$  and  $\varepsilon\{8\}$  for these values of  $N$ .

<sup>2</sup> Here, we assume that the recentering correction effectively reduces by one unit the number of independent sources. We thus replace  $N$  by  $N - 1$  in the exact result of Ref. [30].

## TESTING UNIVERSALITY

The power law distribution (4) predicts the following parameter-free relation between the first two cumulants:

$$\varepsilon\{4\} = \varepsilon\{2\}^{3/2} \left( \frac{2}{1 + \varepsilon\{2\}^2} \right)^{1/4}. \quad (9)$$

This relation can be used to test the universality of the distribution (4). For p-Pb collisions at 5.02 TeV, we run two different types of Monte-Carlo Glauber calculations: a full Monte-Carlo identical to that of Fig. 1, and a second one where fluctuations and smearing are switched off (identical pointlike sources). We calculate  $\varepsilon_2$  and  $\varepsilon_3$  for each event. Events are then binned according to the number of participants  $N$ , mimicking a centrality selection. For p-p collisions at 7 TeV, we use published results [35] obtained with the event generator DIPSY [36], which are binned according to multiplicity. Results are shown in Fig. 3 (left). Each symbol of a given type corresponds to a different bin. All Monte-Carlo results are in very good agreement with Eq. (9). A closer look at the results show that the “full” Monte-Carlo Glauber calculations are above the line by  $\sim 0.015$  (for both  $\varepsilon_2$  and  $\varepsilon_3$ ), the “pointlike” results for  $\varepsilon_3$  by  $\sim 0.005$ , and the “pointlike” results for  $\varepsilon_2$  (where our result is exact, up to the recentering correction) by  $\sim 0.002$ . DIPSY results are above the line by  $\sim 0.01$ .

For Pb-Pb collisions at 2.76 TeV (Fig. 3 right), we use the results obtained in Ref. [37] using the Monte-Carlo Glauber [25] and Monte-Carlo KLN [38] models. These results are in 5% centrality bins. For  $\varepsilon_3$ , both models are in very good agreement with Eq. (9) (within 0.01 or so). Note that Pb-Pb collisions probe this relation closer to the origin, in the large  $N$  limit where more general results are available [33]. These general results predict  $\varepsilon\{4\} \propto \varepsilon\{2\}^{3/2}$  for  $N \rightarrow \infty$ , but with a proportionality constant that depends on the density profile. Our results show that it is in practice very close to the value predicted by Eq. (9), namely,  $2^{1/4}$ .

Monte-Carlo results for  $\varepsilon_2$  in Pb-Pb differ from Eq. (9). This is expected, since  $\varepsilon_2$  in mid-central Pb-Pb collisions is mostly driven by the almond shape of the overlap area between colliding nuclei [30], not by fluctuations. In the limiting case where fluctuations are negligible,  $\varepsilon_2\{4\} = \varepsilon_2\{2\}$ . Our results show that fluctuations dominate only for the most central and most peripheral bins.

We conclude that the power law distribution (4) is a very good approximation to the distribution of fluctuation-driven eccentricities, irrespective of the details of the model. This could be checked explicitly with other initial-state models [29, 39].

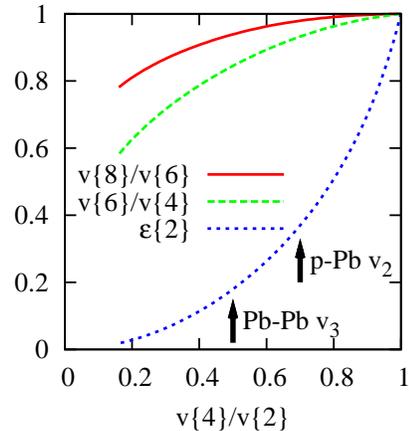


FIG. 4. (Color online) Predictions of the model for ratios of higher order cumulants and  $\varepsilon\{2\}$  as a function of the measured  $v\{4\}/v\{2\}$ . Typical values for  $v_3$  in Pb-Pb [6, 41] and  $v_2$  in p-Pb collisions [15] are indicated by arrows.

## APPLICATIONS

We now discuss applications of our result. The distribution of  $\varepsilon_n$  is completely determined by the parameter  $\alpha$  in Eq. (4). This parameter can be obtained directly from experimental data. Assuming that anisotropic flow is proportional to eccentricity in the corresponding harmonic,  $v_n \propto \varepsilon_n$ , which is proven to be a very good approximation for  $n = 2, 3$  [22], one obtains

$$\frac{v\{4\}}{v\{2\}} = \frac{\varepsilon\{4\}}{\varepsilon\{2\}} = \left( \frac{2}{2 + \alpha} \right)^{1/4}. \quad (10)$$

The first equality has already been checked against Monte-Carlo models and experimental data [40, 41]. The second equality directly relates the parameter  $\alpha$  in Eq. (4) to the measured ratio  $v\{4\}/v\{2\}$ .

This in turn gives a prediction for ratios of higher-order flow cumulants, which scale like the corresponding ratios of eccentricity cumulants. These predictions are displayed in Fig. 4. One can also directly obtain the rms eccentricity  $\varepsilon\{2\}$ , which is a property of the initial state.

The ratio  $v_3\{4\}/v_3\{2\}$  in Pb-Pb is close to 0.5 in mid-central collisions [6, 41]. We thus predict  $v_3\{6\}/v_3\{4\} \simeq 0.84$  and  $v_3\{8\}/v_3\{6\} \simeq 0.94$  in the same centrality. We also obtain  $\varepsilon_3\{2\} \simeq 0.17$ , which is a typical prediction from Monte-Carlo models in the 10%-20% or 20%-30% centrality range [42].

Similarly, the ratio  $v_2\{4\}/v_2\{2\} \sim 0.7$  measured in p-Pb collisions [14, 15] implies  $v_2\{6\}/v_2\{4\} \simeq 0.93$  and  $v_2\{8\}/v_2\{6\} \simeq 0.98$ , that is, almost degenerate higher-order cumulants. We obtain  $\varepsilon_2\{2\} \simeq 0.37$ , in agreement with Monte-Carlo Glauber models [20].

## CONCLUSIONS

We have proposed a new parameterization of the distribution of the initial anisotropy  $\varepsilon_n$  in proton-proton, proton-nucleus and nucleus-nucleus collisions which, unlike previous parameterizations, takes into account the condition  $\varepsilon_n < 1$ . This new parameterization is found in good agreement with results of Monte-Carlo simulations when  $\varepsilon_n$  is created by fluctuations of the initial geometry. Our results explain the observation, in these Monte-Carlo models, that cumulants of the distribution of  $\varepsilon_n$  quickly converge as the order increases. This is because the Fourier transform of the distribution of  $\varepsilon_n$  has a zero at a finite value of the conjugate variable  $k$ . This, in turn, is a consequence of the fact that the probability distribution of  $\varepsilon_n$  has compact support (that is,  $\varepsilon_n < 1$ ).

The consequence of this universality is that while the rms  $\varepsilon_n$  is strongly model-dependent [42], the probability distribution of  $\varepsilon_n$  is fully determined once the rms value is known — in particular, the magnitudes of higher-order cumulants such as  $\varepsilon_n\{4\}$ . Assuming that anisotropic flow  $v_n$  is proportional to  $\varepsilon_n$  in every event, we have predicted the values of  $v_3\{6\}$  and  $v_3\{8\}$  in Pb-Pb collisions, and the values of  $v_2\{6\}$  and  $v_2\{8\}$  in p-Pb collisions.

If future experimental data confirm our prediction, these results will strongly support the picture that the long-range correlations observed in proton-nucleus and nucleus-nucleus collisions are due to anisotropic flow, which is itself proportional to the anisotropy in the initial state. This picture, furthermore, will be confirmed irrespective of the details of the initial-state model.

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