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A common way to obtain Standard-Model-like Lagrangians in String Theory is to place D3 branes inside flux compactifications. The bosonic and fermionic masses and couplings of the resulting gauge theory are determined by the ten-dimensional metric and the fluxes respectively, and the breaking of supersymmetry is soft. However, not any soft-supersymmetry-breaking Lagrangian can be obtained this way, since the String Theory equations of motion impose certain relations between the soft couplings. We show that these relations imply that the sums of the squares of the boson and of the fermion masses are equal and that, furthermore, one- and two-loop quantum corrections do not spoil this equality. This makes problematic the use of D3 branes for constructing models for physics beyond the Standard Model.

D-branes provide a very nice mechanism to embed supersymmetric gauge theories in type II string theory. There is an extensive literature on using branes extended along a 3+1-dimensional space and wrapping some cycles (or a point) in a six-dimensional (internal) manifold to construct four-dimensional effective theories that have a field content similar to that of the Standard Model (for reviews see [1–3]). In these constructions, the low-energy excitations on the branes give the gauge theory sector, with masses and couplings related to the low-energy closed string modes of the internal manifold.

Phenomenologically-relevant models arise when non-trivial background fluxes on the internal space are turned on. Whenever these fluxes break supersymmetry in the bulk, this is communicated to the gauge sector through the bulk fields (via, for example, gravity-mediation supersymmetry-breaking scenarios), generating soft terms for the matter fields.

The main advantage of soft supersymmetry breaking compared to spontaneous breaking, is that the former can avoid the supertrace sum rule

$$\sum_{\text{bosons}} m_{\text{b}}^2 = \sum_{\text{fermions}} m_{\text{f}}^2 \,, \qquad (1)$$

and hence avoid the existence of supersymmetric particles much lighter than the top quark, which is essentially ruled out by recent LHC results.

The simplest low-energy theories can be obtained using D3-branes transverse to the six dimensional manifold - these are $U(N)$ gauge theories whose field content and symmetries are determined by the geometry of the internal space. To obtain theories that are more relevant phenomenologically, one usually places the D3 branes at singularities in the internal space, which breaks the $U(N)$ gauge symmetry into standard-model- or GUT-like gauge groups. As already mentioned, fluxes on the internal manifold induce soft-supersymmetry breaking terms in the gauge theory [4, 5] and one may therefore hope to use these branes to construct realistic models of physics beyond-the-standard-model (BSM).

The purpose of this letter is to show that, even if the breaking of supersymmetry on the D3 branes is soft, the soft terms still obey (1), not only at tree level, but also (at

least) at one and two loops. Hence, the tree-level zero-supertrace condition appears to be a universal feature of any D3 brane in equilibrium in a flux compactification whose metric, dilaton and fluxes obey the equations of motions of supergravity. By explicit calculations we checked that the supertrace also vanishes at one and two loops when the D3 branes are at a generic minimum, and at one loop when the D3-brane is on top of $\mathbb{Z}_2$ and $\mathbb{Z}_3$ orbifold singularities. This appears to be a feature of other $\mathbb{Z}_N$ singularities as well. Hence, our result indicates that any field theory built using such D3 branes will have this feature and hence will not be a feasible candidate for describing BSM physics.

1. SOFTLY BROKEN $\mathcal{N} = 1$ THEORIES

We are interested in $\mathcal{N} = 1$ theories that descend from $\mathcal{N} = 4$ Super Yang Mills (SYM), that can be found on the world-volume of D3-branes extended along the space-time directions and sitting at a point in some six-dimensional compactification space. These theories have three chiral multiplets Φ^i, $i = 1, 2, 3$, transforming in the adjoint representation of the gauge group $U(N)$ (for N branes) and a superpotential

$$W = \frac{1}{2} m_{ij} \Phi^i \Phi^j + \frac{1}{6} g \epsilon_{ijk} \Phi^i \Phi^j \Phi^k \,, \qquad (2)$$

where, to simplify notation, we have omitted the trace over the color indices. The last term is the superpotential of the original $\mathcal{N} = 4$ SYM, and the first one corresponds to a generic mass term that, as we will see, is generated by the fluxes on the six-dimensional space. Supergravity fluxes can also induce soft supersymmetry breaking terms. Generically, the Lagrangian containing both supersymmetric and soft SUSY-breaking terms has the form (up to cubic terms)

$$\mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}} = -(mm^\dagger)_i{}^j \phi^i \bar{\phi}_j - \left(\frac{1}{2} m_{ij} \psi^i \psi^j + \text{h.c.}\right)$$

$$- (m_{\text{soft}}^2)_i{}^j \phi^i \bar{\phi}_j - \left(\frac{1}{2} b_{ij} \phi^i \phi^j + \hat{m}_i \psi^i \lambda + \frac{1}{2} \tilde{m} \lambda \lambda + \text{h.c.}\right)$$

$$- \left(\frac{1}{2} m_{il} \epsilon^{ljk} \phi^i \bar{\phi}_j \bar{\phi}_k \right.$$

$$\left. + \frac{1}{2} c_{ij}^k \phi^i \phi^j \bar{\phi}_k + \frac{1}{6} a_{ijk} \phi^i \phi^j \phi^k + \text{h.c.}\right). \qquad (3)$$

Here ϕ^i and ψ^i are the bosonic and fermionic components of the chiral field Φ^i and λ is the gaugino. The first and third lines contain the supersymmetric terms coming from the superpotential (2), while the second and fourth are soft supersymmetry breaking terms: bosonic masses, scalar bilinear terms, quadratic couplings between the chiral fermions and the gaugino[1], gaugino mass, trilinear c and A-terms[2].

More interesting models for phenomenology are obtained using singular six-dimensional manifolds and putting branes at the singularities. A simple class of such models comes from $\mathbb{Z}_p$ orbifolds of six-dimensional flat space for which the gauge symmetry is enhanced to $U(pN)$, where p is the number of images of a single brane under the $\mathbb{Z}_p$ symmetry that gives the singularity, and then broken to subgroups of $U(pN)$ by splitting the branes and image branes in different stacks [6]. For the simplest example of a $\mathbb{Z}_2$ singularity with N branes one obtains in the end a theory with $U(N) \times U(N)$ gauge group, while the adjoint matter of the original theory splits into two bi-fundamentals of the two different gauge groups plus two adjoint chiral fields, each charged under one of the gauge groups.

Knowing the action of the symmetry group at the singular point allows one to obtain the Lagrangian of the orbifolded theory from that of the "original" SUSY $\mathcal{N} = 1$ theory (2). The structure of the softly broken theory is the same as in (3), where now the matter fields are in the bifundamentals of the different gauge groups. Generically, the orbifold symmetry constrains some of the couplings in (2) or (3) to be equal for the different gauge groups, or to be zero if they do not respect the symmetry.

2. SUSY AND SOFT TERMS FOR D3-BRANES IN FLUXES

In the gauge theories that live on the world-volume of D3-branes in flux backgrounds, both the supersymmetric masses m_{ij} and the soft supersymmetry breaking terms arise from the supergravity fields. These are the ten-dimensional metric g_{10}, the dilaton ϕ, as well as the gauge fields: a pair of two-form gauge fields B_2 and C_2, and a four-form C_4. It is convenient to combine the field strengths of B_2 and C_2 into a complex 3-form

$$G_3 = F_3 - i\, e^{-\phi} H_3 = \mathrm{d}C_2 - i\, e^{-\phi} \mathrm{d}B_2 \ , \qquad (4)$$

which will play a crucial role in the gauge-theory Lagrangian. A general string theory compactification with fluxes has a warped metric of the form

$$g_{10}(x,y) = \begin{pmatrix} e^{2\alpha(y)} g_4(x) & 0 \\ 0 & g_6(y) \end{pmatrix} \qquad (5)$$

where x and y are the four ("external") and six ("internal") coordinates, α is the warp factor and g_4 is the $4d$ Minkowski metric.

The supersymmetric masses m_{ij} and the soft fermionic masses $\hat{m}$ and $\tilde{m}$ are generated by the bulk supergravity fields, and the precise relation between them is most easily obtained by computing the fermionic D-brane action [7]. It is important to note that the $\mathcal{N} = 1$ gauge theory (2) descends from $\mathcal{N} = 4$ SYM and hence it has a memory of the original $SU(4)$ R-symmetry of the $\mathcal{N} = 4$ theory. In particular the three fermions ψ^i in the chiral multiplets can be combined with the gaugino λ to reconstruct the $\mathcal{N} = 4$ fermions in the **4** of $SU(4)$. Then the fermionic masses can similarly be combined into a 4×4 mass matrix

$$M_{IJ} = \begin{pmatrix} m_{ij} & \hat{m}_i \\ \hat{m}_i^{\mathrm{T}} & \tilde{m} \end{pmatrix} \qquad (6)$$

with $I = 1, .., 4$. This matrix transforms in the **10** of $SU(4) \cong SO(6)/\mathbb{Z}_2$, and can equivalently be encoded in an imaginary anti-self dual (IASD) three-form on the six-dimensional space, T_{ABC} $(A, B, C = 1, ..6)$ which also has 10 independent components[3] and transforms in the **10** of $SO(6)$. The map between the mass matrix and this three-form is given by

$$\begin{aligned} T_{ABC} &= -\frac{1}{2\sqrt{2}} \mathrm{Tr} \left(M\eta^A \eta^{B\dagger} \eta^C \right) , \\ M_{IJ} &= \frac{1}{12\sqrt{2}} T_{ABC} (\eta^{A\dagger} \eta^B \eta^{C\dagger})_{IJ} , \end{aligned} \qquad (7)$$

where the six matrices η^A that intertwine between $SU(4)$ and $SO(6)$ are usually called 't Hooft symbols, or generalized Weyl matrices, (their explicit expression is given in Appendix A) and the numerical coefficients are chosen to match the conventions of [8]. The splitting of the matrix M into its $\mathcal{N} = 1$ components (6) corresponds to selecting one supersymmetry among the four of $\mathcal{N} = 4$ SYM and is equivalent to choosing a set of complex coordinates on the six-dimensional internal space. The IASD 3-form T splits into components with different number of holomorphic and antiholomorphic indices. The fundamental $SU(4)$ index I splits under $SU(4)_R \to SU(3) \times U(1)_R$ into $I = (i, 4)$ and the fundamental $SO(6)$ index A splits into $A = (i, \bar{\imath})$. We thus find

$$\begin{aligned} m_{il} &= \tfrac{1}{2} T_{i\bar{\jmath}\bar{k}} \epsilon^{\bar{\jmath}\bar{k}}{}_l \\ \hat{m}_i &= -\tfrac{i}{2} T_{ij\bar{k}} J^{j\bar{k}} \\ \tilde{m} &= \tfrac{1}{6} T_{ijk} \epsilon^{ijk} \end{aligned} \qquad (8)$$

where $J_{i\bar{\jmath}}$ is the symplectic structure associated to the choice of $SU(3)$ subgroup (in our conventions $J_{1\bar{1}} = J_{2\bar{2}} = J_{3\bar{3}} = i$).

For D3-branes in flux backgrounds, the tensor T is the IASD piece of the complex 3-form flux G_3 introduced in equation (4) of [7]

$$T_3 = e^{4\alpha} (\star_6 G_3 - iG_3) . \qquad (9)$$

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[1] These are not usually considered in the literature as there are no chiral fermions transforming in the adjoint representation. As we will discuss later, the fluxes giving rise to these terms are not allowed in the most typical situations.

[2] The c-term is not usually considered because it can lead to quadratic divergencies if there are gauge singlets. For D3-branes in fluxes, it arises from the same fluxes as $\hat{m}_i$.

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[3] In our conventions $(*T)_{ABC} \equiv \frac{1}{3!} \epsilon_{ABC}{}^{DEF} T_{DEF} = -iT^{ABC}$.

Here we have used the notation of [8] and the Hodge star $\star_6$ on the six-dimensional space is defined in Footnote 3. D3 branes in Calabi-Yau compactifications have a moduli space corresponding to the fact that the brane can sit at any point inside the CY. The same is true if one adds to the background an imaginary self-dual flux G_3. However, introducing an IASD component generically uplifts this moduli space and as a result the branes only have a finite number of minima. The equations of motion imply that the tensor T_3 is position-independent, and therefore the masses are the same, regardless of where the branes sit.

From Eqs. (8) and (9), we see that the (1,2) component of the IASD fluxes gives rise to m_{il}, which can be included in a supersymmetric Lagrangian, while the (3,0) component gives a gaugino mass $\tilde{m}$ that breaks supersymmetry softly on the brane. The flux terms that would give rise to $\hat{m}_i$ cannot arise in fluxed Calabi-Yau compactifications [9], but can appear when the branes are in more general backgrounds.

The soft SUSY-breaking trilinear terms c and a in (3) were computed in [10] using the bosonic non-abelian D-brane action, and are again entirely determined by T_3:

$$c_{ij}^k = T_{ij\bar{k}} = \delta_{[i}^k \hat{m}_{j]} \ , \quad a_{ijk} = T_{ijk} = \tilde{m}\epsilon_{ijk} \ . \quad (10)$$

Note that in a general theory with soft supersymmetry breaking there is no relation between the boson trilinear couplings and the fermion masses, but in the theories that live on the worldvolume of D3 branes, these are always linked: the fermion mass matrix completely determines the boson trilinear couplings. This relation is a crucial ingredient of the calculation that establishes our result.

The scalar masses and the b-terms in (3) are trickier to determine. In the D3-brane world-volume action they are obtained from the potential felt by the D3 brane, $V = e^{4\alpha} - \mathcal{C}$, where $e^{4\alpha}$ is the warp factor and $\mathcal{C}$ is the 4-form potential C_4 along the spacetime directions. By Taylor expanding V around one of its minima, y^0,

$$e^{4\alpha} - \mathcal{C} = (e^{4\alpha} - \mathcal{C})|_{y^0}$$
$$+ \frac{1}{2}\partial_{AB}^2(e^{4\alpha} - \mathcal{C})|_{y^0}(y - y^0)^A(y - y^0)^B + ... \quad (11)$$

and identifying the distance to the brane with the scalar fields on the brane $(y - y^0)^A \sim \phi^A$, one can calculate all the $(6 \times 7)/2 = 21$ entries of the matrix $\partial_{AB}^2(e^{4\alpha} - \mathcal{C})|_{y^0}$, which give the 21 boson masses [4]. When choosing complex coordinates the boson masses split under $SU(4)_R \to SU(3) \times U(1)_R$ as

$$\mathbf{20 + 1 = 8 + 1 + 6 + \bar{6}} \quad (12)$$

corresponding respectively to the traceless part of $m_{i\bar{j}}^2$, its trace, as well as b_{ij} and its complex conjugate:

$$(m_{\mathrm{SUSY}}^2 + m_{\mathrm{soft}}^2)_{i\bar{j}} = \partial_{i\bar{j}}^2(e^{4\alpha} - \mathcal{C})|_{y^0} \ , \ b_{ij} = \partial_{ij}^2(e^{4\alpha} - \mathcal{C})|_{y^0} \ . \quad (13)$$

Unlike the trilinear terms, which are completely determined by the background three-form fluxes, only one of the 21 boson mass components can be related to these fluxes, via the equations of motion. When the branes are

at a minimum of the potential V, a particular combination of the bulk equations of motion (see eq. (2.30) of [11]) allows to fix the trace of the boson mass matrix in terms of the thee-form T_3 that determines the fermion masses:

$$\nabla^2(e^{4\alpha} - \mathcal{C}) = \frac{1}{6}e^{4\alpha+\phi}\,|\star_6 G_3 - iG_3|^2 = \frac{1}{6}e^{-4\alpha+\phi}\,|T_3|^2 \quad (14)$$

where $|T_3|^2 = \frac{1}{6}T_{ABC}\bar{T}^{ABC}$. The other 20 components of the boson masses are not fixed by T_3, but are determined by the features of the geometry near the location of the brane. Thus they are both model- and location-dependent.

Using (13), (14) and (8) we finally arrive at the main formula of this letter:

$$\begin{aligned}\mathrm{Tr}(m_{\mathrm{SUSY}}^2 + m_{\mathrm{soft}}^2) &= \mathrm{Tr}(mm^\dagger) + 2\,\hat{m}_I\bar{\hat{m}}^I + \tilde{m}^2 \\ &= \mathrm{Tr}(MM^\dagger) \ , \end{aligned} \quad (15)$$

where M is defined in (6). In other words, we find

$$\mathrm{Tr}[\mathrm{boson\ masses}^2] = \mathrm{Tr}[\mathrm{fermion\ masses}^2] \ . \quad (16)$$

Hence, the actions of all D3-branes extended along the spacetime directions and sitting at equilibrium inside the compact manifold obey at tree level the zero-supertrace sum rule (1). In light of the recent LHC results, this makes problematic the use of these D3 branes in constructing BSM models[4]. We explain in the next section that this feature persists when one and two- loop quantum corrections are taken into account.

We close this section by a comment. All the parameters in the Lagrangian (3) can alternatively be computed purely within $\mathcal{N} = 1$ supergravity with chiral fields including a hidden sector (moduli) on top of the observable sector (brane fields). After breaking supersymmetry spontaneously in the hidden sector via F-terms (which can be done by turning on three-form fluxes), integrating out the moduli fields and taking the limit of infinite Planck mass while keeping the gravitino mass finite, one obtains a softly-broken $\mathcal{N} = 1$ gauge theory for the visible sector. The parameters of the latter are given in terms of the F-terms, the superpotential and the Kähler potential of the original $\mathcal{N} = 1$ supergravity theory[5] [14, 15]. Comparing these with those obtained from the D3-brane action, one finds [4] that they all agree, except for the boson masses. Furthermore, it is only for non-scale supersymmetry breaking and zero supersymmetric masses that the supertrace obtained by the supergravity calculation is zero; generically it is not. It would be interesting to understand why the supergravity calculation fails to reproduce this feature of the D3-brane action.

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[4] Note that the trace of the fermion masses includes that of the gaugini. However, having the latter heavy enough to overcome the problem of the supertrace rule brings in the little hierarchy problem and is therefore not a way out.

[5] For D3-branes in CY compactifications the Kähler potential is of sequestered form if the complex structure moduli are integrated out [4], as done in KKLT [12] or in large volume scenarios [13].

3. QUANTUM CORRECTIONS

The one-loop beta-functions for all the coupling constants including the "non-standard soft-supersymmetry breaking" terms $\hat{m}$ and c in (3) were computed in [16]. By using the relation between the soft trilinear terms and the fermion masses (10), we find that all the one-loop beta-functions, except the ones for the boson masses, vanish exactly. The one-loop beta-function for the trace of the boson mass matrix also vanishes if and only if (16) holds, which is precisely what happens for the D3-brane world-volume theories. We have checked this for branes at a regular point of the internal manifold, and also for branes at $\mathbb{Z}_2$ and $\mathbb{Z}_3$ singularities.

The two-loop beta-functions were computed in [17] and [18]. We find that for D3-branes at nonsingular points in the internal manifold all these beta-functions again vanish when the supertrace of the square of the masses vanish (there might be additional regularization scheme-dependent conditions; for example in [18] the mass of the fictitious "ϵ-scalar" should be set to zero).

It is very likely that all beta-functions vanish perturbatively at all loops. Indeed, the fermionic masses (8) are given by a constant (position independent) tensor, and therefore we do not expect them to run with the energy scale (corresponding to the radial distance away from the branes). Furthermore, since the trace of the bosonic masses is equal to the trace of the fermionic ones classically and at one and two loops, and the latter do not run, we expect this equality to hold at all loops. When the branes are placed in an $SO(3) \times SO(3)$ invariant background that only has (1,2) but no (3,0) components, this expectation can also be confirmed by explicit calculations [19]: the theory on their worldvolume is simply $\mathcal{N} = 4$ broken to $\mathcal{N} = 1$ by the introduction of supersymmetric chiral multiplet masses, and broken to $\mathcal{N} = 0$ only by a certain traceless bosonic bilinear. Using some clever superspace tricks, this theory was shown in [20] to have vanishing beta-functions at all loops.

It is worth stressing that our analysis also holds for D3-branes at orbifold singularities. Explicit tree-level and one-loop calculation for the $\mathbb{Z}_2$ and $\mathbb{Z}_3$ model confirm our expectations. It would be interesting to see if this result extends also to other types of singularities and to other types of branes.

Appendix A: 't Hooft symbols

We have used a basis where the 't Hooft matrices η_{ij}^A are

$$\eta^1 = -i \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix} \quad \eta^2 = \begin{pmatrix} 0 & -\sigma_0 \\ \sigma_0 & 0 \end{pmatrix} \quad \eta^3 = i \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}$$

$$\eta^4 = i \begin{pmatrix} 0 & -\sigma_1 \\ \sigma_1 & 0 \end{pmatrix} \quad \eta^5 = i \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix} \quad \eta^6 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix} ,$$

where $\sigma_{1,2,3}$ are the Pauli matrices, σ_0 is the 2×2 unit matrix and we have chosen the complex coordinates

$$z^i = \frac{1}{\sqrt{2}}(x^1 + ix^4, x^2 + ix^5, x^3 + ix^6) .$$

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