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Unveiling quantum Hall transport by Efros-Shklovskii to Mott variable range hopping transition with Graphene

Keyan Bennaceur, Patrice Jacques, Fabien Portier, P. Roche, and D.C. Glattli*
Service de Physique de l'Etat Condens/IRAMIS/DSM (CNRS URA 2464), CEA Saclay, F-91191 Gif-sur-Yvette, France

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The quantum localization in the quantum Hall regime is revisited using Graphene monolayers with accurate measurements of the longitudinal resistivity as a function of temperature and current. We experimentally show for the first time a cross-over from Efros-Shklovskii Variable Range Hopping (VRH) conduction regime with Coulomb interaction to a Mott VRH regime without interactions. This occurs at Hall plateau transitions for localization lengths larger than the interaction screening length set by the nearby gate. Measurements of the scaling exponents of the conductance peak widths with both temperature and current give the first validation of the Polyakov-Shklovskii scenario that VRH alone is sufficient to describe conductance in the Quantum Hall regime and that the usual assumption of a metallic conduction regime on conductance peaks is unnecessary.

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Since its recent discovery [1, 2], the anomalous Quantum Hall Effect (QHE) displayed by relativistic like electrons in a Graphene monolayer has been mostly investigated to search for quantum Hall ferromagnetism [3] or Fractional Quantum Hall effect [4, 5] in very low disorder samples to favor interactions effects. Here we address the opposite regime where disorder is strong enough to hide electron interactions, as it is the case for standard exfoliated Graphene monolayers deposited on the oxide layer of a silicon substrate. Graphene offers a new set of parameters to revisit the quantum phase transition of localization in the Quantum Hall regime. In particular, we show that the screening of interactions by the 300 nm close back-gate provided by highly doped silicon allows to observe for the first time the transition from Efros-Shklovskii (E-S) to Mott Variable Range Hopping (VRH) in QHE for large localization length. The universal scaling exponents of quantum localization at Hall plateau transitions deduced from our whole set of temperature and bias current data, together with the E-S to Mott VRH transition, definitely validate the Polyakov-Shklovskii (P-S) suggestion that VRH transport is sufficient to describe the quantum Hall electrical transport [7].

The integer Quantum Hall effect occurs whenever Landau Levels (LLs) which form due to quantization of carrier cyclotron orbits are fully filled. In a conventional 2D-electron gas (2DEG) positive energy LLs with $E_n = \hbar eB/m^*(n+1/2)$, n integer, lead to LLs filling factors $\nu = k$, and quantized Hall resistance $R_H = h/ke^2$ with k a positive integer and spin degeneracy lifted assumed. In a Graphene monolayer, carriers obey a ultra-relativistic Dirac equation. LLs have both positive and negative energies $E_n = \pm hV_F/l_c\sqrt{(2n)}$ where V_F is the Fermi velocity and $l_c = \sqrt{(\hbar/eB)}$ the magnetic length. Because of the existence of a zero kinetic energy LL and the non lifted four fold valley and spin degeneracy, the filling factors series leading to Hall plateaus becomes $\nu = \pm 4(k+1/2)$. The finite width of the quantized Hall plateaus occurs because of localized states in the bulk provided by potential disorder whose energies spread around the unperturbed LL energies. For filling factors yielding the Hall plateau series mentioned above, the Fermi level E_F lies between two unperturbed LLs, i.e. on localized state energies. The longitudinal conductance σ_{xx} vanishes at zero temperature, preventing electron backscattering through the bulk and ensuring perfect quantization of the Hall current circulating on the sample edges. The transition between two Hall plateaus occurs when the Fermi level lies in the middle of a disorder energy broadened LL. For an infinite size sample, the localized state size ξ diverges at a single energy $E_c \approx E_n$ resulting in backscattering and a longitudinal conductance peak. According to the quantum localization theory [8] $\xi \sim |E - E_c|^{-\gamma} \sim |\nu - \nu_c|^{-\gamma}$ with $\gamma \simeq 7/3$, a value confirmed in many experiments on conventional 2DEGs [10, 13].

Deducing the scaling exponent from transport measurement requires a model linking transport quantities to ξ. In general one can write $\sigma_{xx} = f(\xi/L(T))$ where f is a universal scaling function and $L(T)$ a characteristic length depending on the conduction mechanisms at finite temperature. Let us first discuss bulk conduction on the Hall plateaus. It is generally accepted that transport occurs via phonon assisted inelastic transitions between localized states, the so-called variable range hopping mechanism. For non-interacting electrons, the VRH Mott's law gives

$$\sigma_{xx} \propto \frac{T_M}{T} \exp(-(T_M/T)^{1/3}) \qquad (1)$$

or equivalently $\sigma_{xx} \propto (\frac{\xi}{L_M(T)})^2 \exp(-(L_M(T)/\xi)^{2/3})$, which defines the characteristic length $L(T)$ labeled as $L_M(T) = \sqrt{1/\pi g(\varepsilon_F) k_B T}$, where $g(\varepsilon)$ is the energy independent density of states at the Fermi energy. However, in the QHE regime screening is poor and Coulomb repulsion must be included. One thus enters the Efros-Shklovskii (E-S) VRH regime, where the density of states $g(E) \propto |E - E_F|$ yields

$$\sigma_{xx} \propto \frac{T_0}{T} \exp(-(T_0/T)^{1/2}) \tag{2}$$

or $\sigma_{xx} \propto (\frac{\xi}{L_{E-S}}) \exp(-(L_{E-S}/\xi)^{1/2})$[14] , with both the length $L_{E-S}(T) = 4\pi\varepsilon_0\varepsilon k_B T/Ce^2$ and the energy $k_B T_0 = Ce^2/4\pi\varepsilon\varepsilon_0\xi$ given by the Coulomb energy. $C \simeq 6.2$ is a numerical constant [15]. Measuring T_0 thus allows to determine ξ and to probe the scaling law far from the conductance peaks. Still in the same regime, passing a current I trough the sample while keeping a fixed low temperature gives an E-S VRH like law for σ_{xx} where the current plays the role of the temperature. This is the P-S model [16] which uses the effective electronic temperature $k_B T \rightarrow eE_H\xi/2$ where the local Hall electric field E_H is proportional to the current I. This leads to

$$\sigma_{xx} \propto \exp(-((E_0/E_H)^{1/2}) \tag{3}$$

and

$$\sigma_{xx} \propto \exp(-((E_1/E_H)^{1/3}) \tag{4}$$

for E-S and Mott's VRH respectively.

Probing the localization scaling law for large localization length requires understanding the conduction mechanism close to the conductance peaks, in the plateau transition region where the Fermi level approaches a unperturbed LL. Historically, the first conduction mechanism proposed for conductance peaks was a metallic regime. The Pruisken model [17] sets the characteristic length $L(T)$ as the phase coherence length $L_\phi(T) = (D\tau_\phi)^{1/2}$. Here D is a diffusion constant and the phase coherence time $\tau_\phi \propto T^{-p}$ follows a non-universal power law with T ($p = 2$ accounts for most observations). The localization scaling exponent γ can be indirectly accessed by the temperature dependence of the Full Width at Half Maximum (FWHM) $\Delta\nu$ of the conductance peaks. The latter is obtained when $\xi(\Delta\nu/2) \simeq L_\phi(T)$ giving $\Delta\nu = (T/T_1)^\kappa$ with the non universal exponent $\kappa = p/2\gamma$. However Polyakov and Shklovskii proposed that the VRH regime should last in the plateau transition region and the FWHM obtained from $\xi(\Delta\nu/2) \simeq L_{E-S}(T)$ (or $T \simeq T_0$)) giving $\Delta\nu = (T/T_1)^\kappa$ and the now universal $\kappa = 1/\gamma$. Here $k_B T_1 = Ae^2/4\pi\varepsilon\varepsilon_0\xi$ with A a numerical constant. Similarly, the dependence of the FWHM with bias current using P-S model is $\Delta\nu = (I/I_1)^\mu$ with $\mu = 1/2\gamma = \kappa/2$ while using the phase coherence length approach $\mu = p/4\gamma$. From the latter discussion we see that, as $p = 2$ is a reasonable exponent for τ_ϕ, the scaling law of the FWHM with temperature can not discriminate between the two scenariios nor that with bias current.

A further criterion is thus needed to discriminate the two scenarios, and that is addressed experimentally in this Letter. The idea originates from the Aleiner Shklovskii (A-S) [18] prediction that a cross-over from E-S to Mott VRH occurs when interactions are screened, for example by a gate parallel to the 2DEG at a distance d. This requires $\xi > 2d$ which is likely to occur on conductance peaks for sample size $\gg 2d$. In the Mott VRH regime the FWHM conductance peak now becomes $\Delta\nu \propto (T/T_2)^\kappa$ with $\kappa = 1/2\gamma$ and $\Delta\nu \propto (I/I_1)^\mu$ with $\mu = 1/3\gamma$. On the contrary in the Pruisken scenario screening is not expected to impact the temperature dependence of the FWHM. This yet never observed E-S to Mott cross-over in the QH regime would definitely establish the P-S scenario, that VRH describes transport almost everywhere in the QHE regime even close to the maximum of the conductance peaks and that the phase coherence length approach is not appropriate.

Previous measurements performed in conventional 2DEGs, including Si-MOSFETs, and InAs/InGAAs or GaAs/AlGAAs heterojunctions have been able to probe the scaling exponent of ξ. Experiments using direct determination of ξ from the E-S VRH [13] and even more directly by geometrical comparison with sample width [11] have given $\gamma \simeq 2.3$. Probing the scaling law using the conductance peak width is less direct and showed a dispersion in the extracted values of κ. Works combining temperature and bias current have shown excellent agreement with the P-S model [11, 13, 19, 21]. Recently the scaling law has been studied in Graphene using temperature, but no current bias study was done [20]. Except on the $n = 0$ LL level the results were found compatible with $\gamma = 2.3$. So far no experiments in the QHE have shown the E-S to Mott cross-over. It has been only observed in highly disordered 2D electron systems in zero field [6]. Here, the cross-over occurs not because of screening but for energies well above the Coulomb gap, restoring a constant density of states.

In this paper we present a complete set of data in temperature and bias current on the QHE regime performed on Graphene monolayers. The silicon back-gate with $d = 300$nm gives for the first time access to the cross-over from Mott to ES VRH regime on conductivity peaks for the highest filling factors where ξ is large. All the scaling exponents γ, κ and μ agree with the P-S and A-S prediction in the screened and unscreened regime. This provides a definitive confirmation of these models. The scaling exponent found to be $\gamma \simeq 2.3$ is the same for first two $n = \pm1, \pm2$ LLs including the $n = 0$ LL where no anomalous behavior is observed as found in Ref.[20].

Four samples (S1 to S4) have been fabricated using exfoliation of natural graphite flakes [22] . All samples have been deposited on the 300nm thick oxide layer of highly doped silicon wafer which serves as a back gate. Contacts where made using e-beam lithography and evaporating 5/70 nm Ti/Au in high vacuum. After processing, S1 and S2 were covered with PMMA while S3 and S4 were not. Samples S3 and S4 were heated up to 450K in cryogenic vacuum during several hours until the four points resistance reached a steady high value signaling low dopant concentration. Mobilities at 1.10^{12}cm^{-2} are around 3000 $\text{cm}^2V^{-1}s^{-1}$ for S1 and S2, 6000 $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ for S3 and 10000 $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ for S4. Sample S4 showed the quantum Hall plateaus series and a Raman spectrum of a monolayer although the gate efficiency was two times smaller than expected. It is likely a twisted bilayer.

The longitudinal resistivity ρ_{xx} was recorded while varying density for about 50 temperature values ranging from 1.6K (S4) or 4K (S3) to 300K and similar runs are repeated for different fixed high magnetic fields from 6 to 17 Tesla. These extensive measurements have been done on two samples (S3 and S4). Measurements as a function of bias current were also performed ranging from $10nA$ to $100\mu A$ at fixed temperature (4.2K or 1.6K). These measurements have been done on the four samples. Four point measurements were done on S3 (R_{xx}) and S4 (R_{xx} and R_{hall}) whereas 3 point measurements were performed on the other two samples. In order to extract the resistivity from the resistance the aspect ratio was estimated using a numerical electrostatic calculation. For all measurements the density is slowly swept (less than 2 V/mn) to avoid hysteretic effects due to trapped charges in silicon oxide. To get rid of the weak remaining hysteresis the data are taken always in the same sweeping direction. Most of the data presented in this Letter are from sample S3 and some of the data from sample S4. Other samples were used to confirm the reproducibility of the physical properties.

We now present the experimental results. An example of ρ_{xx} measurements is shown in Figure 1 at 16.5 Tesla for S3. The left part shows gate voltage sweeps at low bias for about 50 different fixed temperatures and the right part for a series of fixed current bias at base temperature. The whole data for both positive and negative gate voltages can be found in the supporting material [23]. Let us first focus on the Hall plateau regions. The variation of σ_{xx} with T was shown to follow accurately the E-S VRH law [7], see supporting material [23]. This is well obeyed from 1.6K to $\simeq$ 80K throughout the $\nu = \pm 2$ plateaus for all magnetic fields. We emphasize that both the simple activated law and the 2D Mott's VRH law yields poor fits. Above 100K departure from the E-S VRH law signals thermal activation to the next LLs. From the fit combining activated and E-S VRH law we can extract both the VRH temperature T_0 as shown in Fig.2 and the activation energy Δ. The observed Δ are smaller than those given by the Dirac equation in magnetic field $\Delta = E_{n+1} - E_n = \frac{1}{2}(\sqrt{n+1} - \sqrt{n})\sqrt{2e\hbar v_F^2 B}$ (solid lines) because of LL disorder broadening. At 17T, 200K and 70K LL broadening are found respectively for sample S3 and S4. The values for S3 are comparable with those obtained in [24], see [23] for a complete set of data.

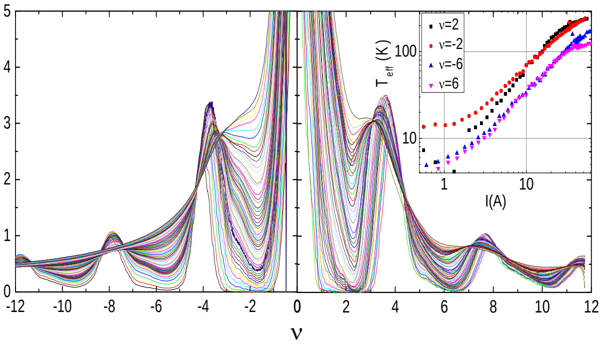

FIG. 1: Longitudinal resistance ρ_{xx} versus filling factor at 16.5T. On the left the temperature is varied from 4.2K to 300K and on the right the bias current from 10nA to 100μA at 4.2K. Inset: effective temperature $T_{eff}(I)$ (see definition in text) versus bias current I for different filling factors on plateaus.

The dependence with current bias shows accurate agreement with the E-S VRH like law, Eq.3, following the P-S prediction. Comparing Eqs. 2 and 3 we can write $(\frac{\varepsilon_{H0}}{\varepsilon_H})^{\alpha_1} = (\frac{T_0}{T})^{\alpha_2} + k$, where k is a constant. If the E-S law is obeyed for bias currents one should have $\alpha_1 = \alpha_2 = 1/2$, where $\alpha_2 = 1/2$ was already established. In order to check this, we define the effective temperature $T_{eff}(I)$ such that $\rho_{xx}(I) = \rho_{xx}(T_{eff})$. On Fig.1, right inset, T_{eff} is plotted as a function of the bias current in the logarithmic scale for $\nu = \pm 2, \pm 6$. It is clear that below 100K (no thermal activation) $T_{eff} \propto I$ showing that $\alpha_1 = \alpha_2$ and a VRH like law for current is well obeyed by σ_{xx}. Fig.2 shows $T_0^{1/2}$ extracted from fits of $\sigma_{xx}(T)$ using Eq.3 for a continuous series of filling factors except on the very maximum of the conductance peaks. From these measurements we can extract the localization length via T_0 using: $\xi(\nu) = Ce^2/4\pi\epsilon_r\epsilon_0 k_B T_0(\nu)$. In the inset of Fig.2 the smallest localization length ξ_{min} found in the middle of the plateaus is plotted as a function of

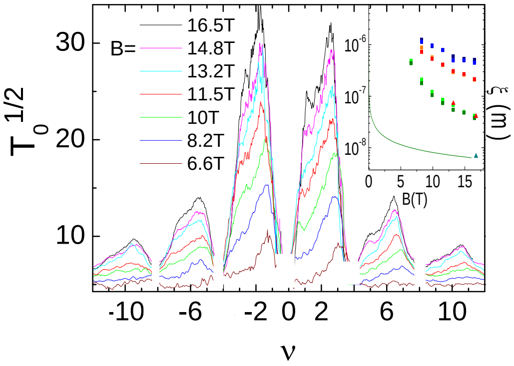

FIG. 2: Square root of E-S VRH temperature T_0 versus filling factor for sample S3 at different magnetic fields ranging from 6.6T to 16.5T. The inset shows ξ_{min} as a function of the magnetic field. Green, red, and blue data are for $\nu = \pm 2, \pm 6$, and $\nu = \pm 10$ respectively. Squares are for sample S3 and triangles for S4. The solid curve corresponds to l_c.

the magnetic field. The solid lines show the magnetic length l_c for comparison. In sample S3, $\xi_{min} \sim 40$ nm for $\nu = \pm 2$ at $B = 16.5$ Tesla is around seven times larger than l_c whereas in sample S4 ξ_{min} is of the order of l_c. Smaller localized states can be expected due to higher mobility of S4 or, if it is a twisted bilayer, due to screening of the silicon oxide charge impurities by the lower layer. ξ values in S3 are consistent with measurements of Ref.[25] but quite below those found in Refs.[20, 26].

In Fig.3, lower graph, we show $\xi(\nu)$ extracted from a E-S VRH analysis of the data for continuous values of the filling factor and for different magnetic fields. The line at $\xi = 2d = 600$nm signals the limit of validity for ξ extracted from the E-S VRH law. Indeed for larger ξ we expect screening of the interactions and a cross over from E-S to Mott VRH law. This is what is observed as shown by a log plot of $T\sigma_{xx}$ as a function of $1/\sqrt{T}$ on the upper part of 3. For $\xi < 600$nm a linear variation is found while for $\xi > 600$nm the variation is no longer linear and is well fitted by the Mott's law. This yet never observed cross-over from E-S to Mott's law is one of the two main results of our experiment.

Before going further in the study of the Mott's regime, it is important to determine the universal scaling exponent γ in the E-S VRH regime for which ξ can be reliably known. Fig. 4, upper graph, shows $T_0^{2.3}$ as a function of $\nu - \nu_C$ for $\nu_C \simeq \pm 2$ for samples S3 and S4. The linear variation indicates that $\gamma = 2.3$ is a reasonable exponent. This result is in agreement with Ref. [20].

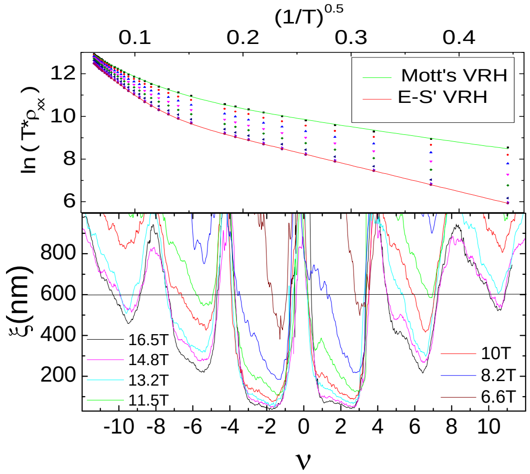

FIG. 3: Lower graph: localization length ξ versus filling factor for S3 for different magnetic fields. The horizontal solid line $\xi = 600$nm shows the threshold above which screening of interactions by the gate plays a role. Upper figure: plots of $T\rho_{xx}$ versus $1/\sqrt{T}$ at 14.8T for various filling factors near $\nu = -6$ and ξ above and below 600nm. The red solid curve is a E-S law's fit of the data while the green solid curve uses Mott's law.

The second important result of this work comes from the study of the scaling exponent of the FWHM $\Delta\nu$ of resistivity peaks between Hall plateaus with both temperature and bias current. Here, as $\xi > 2d$ in this regime, Mott's VRH law is obeyed and we expect the exponent values will allow to discriminate between the E-S VRH and the phase coherence length scenario. The FWHM of resistance peaks are plotted for both S3 and S4 on Fig.4 on the lower left and lower right part respectively for $\nu = [\pm 2, \pm 6], [\pm 10, \pm 6], [-2, +2]$. The figures clearly show a universal behavior of $\Delta\nu$ at temperature below 100K and bias current below 10μA. κ is found to be equal to 0.23 ± 0.02 and μ to 0.13 ± 0.01 which are both in good agreement with Mott's VRH confirming Mott's law at the edge of the quantum Hall plateaus. If the phase coherence length approach was relevant, we would have found $\kappa = 0.42$ and $\mu = 0.21$.

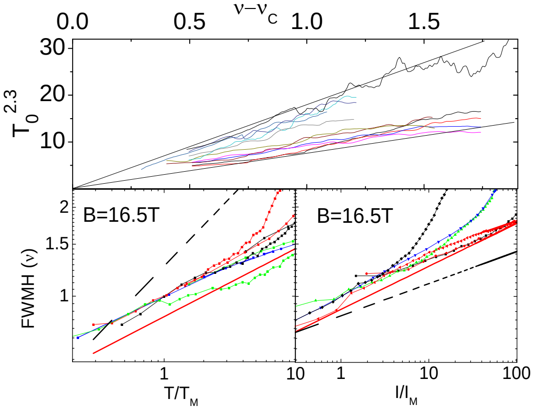

FIG. 4: Upper figure: $T_0^{-2.3}$ as a function of $\nu - \nu_C$ for $\nu = [\pm 2, \pm 6]$ and $\nu = [\pm 2, 0]$ at different magnetic field, the black curve is for S4 while the others are for S3. Lower part: FWHM of the ρ_{xx} peaks for temperature measurement (left) and for bias current measurement (right). Blue, green and red curves are data for the peaks between $\nu = \pm 10$ and ± 6, $\nu = \pm 6$ and ± 2, and $\nu = -2$ and 2 respectively. Squares are for S3 and triangle for S4. The solid red and dashed black lines correspond to the expected exponent for E-S scenario and for the A-S scenario in the Mott regime respectively. The Pruisken scenario is also represented by the dashed black lines.

To conclude we have studied the quantum localization in the quantum Hall regime using Graphene monolayers. The standard localization length scaling exponents was found for all studied Landau Levels. More important, our first observation of an Efros-Shklovskii to Mott VRH cross-over in the quantum Hall regime found on conductance peaks allows to discriminate between the Polyakov-Shklovskii and the Pruisken phase coherence length scenarios describing the conduction on the plateau transition.

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[*] Electronic address: christian.glattli@cea.fr; Also at Laboratoire Pierre Aigrain, 24 rue Lhomond, 75231 Paris Cedex 05, France

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