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Supersymmetric vacua in $N = 2$ supergravity

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Abstract: We use the embedding tensor formalism to analyse maximally symmetric backgrounds of $N = 2$ gauged supergravities which have the full $N = 2$ supersymmetry. We state the condition for $N = 2$ vacua and discuss some of their general properties. We show that if the gauged isometries leave the SU(2) R-symmetry invariant, then the $N = 2$ vacuum must be Minkowski. This implies that there are no AdS backgrounds with eight unbroken supercharges in the effective $N = 2$ supergravity of six-dimensional $\mathrm{SU}(3)\times\mathrm{SU}(3)$ structure compactifications of type II string theory and M-theory. Combined with previous results on $N = 1$ vacua, we show that there exist $N = 2$ supergravities with a given set of gauged Abelian isometries that have both $N = 2$ and $N = 1$ vacua. We also argue that an analogue of our analysis holds in five and six spacetime dimensions.

Keywords: Extended Supersymmetry, Supergravity Models, Flux compactifications

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Contents

1 Introduction

The analysis of Minkowski and Anti-de Sitter (AdS) supersymmetric vacua in gauged extended supergravity has received much attention in recent years. In this paper we consider such maximally-symmetric backgrounds of $N = 2$ supergravities in four spacetime dimensions ($d = 4$) and their "cousins" in $d = 5, 6$ which also have eight supercharges.

The general conditions for $N = 2$ vacua in electrically gauged $N = 2$ supergravities, together with a few illustrative examples, were given recently in [1]. By using the embedding tensor formalism, introduced in [2] and applied to $N = 2$ gauged supergravity in [3],[1] we extend the analysis of [1] by allowing for the possibility of electrically and magnetically charged fields in the spectrum. We derive the conditions for $N = 2$ vacua with Abelian and non-Abelian factors in the gauge group and show that solutions generically exist. However, it is not guaranteed that these solutions lie inside the physical domain of the Kähler cone and thus are physically acceptable. For the special case of hypermultiplets that are gauged with respect to isometries which do not induce an SU(2) R-symmetry rotation, we show that AdS vacua with eight unbroken supercharges are not possible. It is straightforward to extend our analysis to spacetimes with $d = 5, 6$.

We shall specifically study the class of isometries that are present in quaternionic-Kähler manifolds which are in the image of the c-map and appear at the tree level of type II compactifications in string theory [5, 6]. These manifolds can be viewed as a graded Heisenberg algebra fibred over a special-Kähler base. We show that no $N = 2$ AdS vacua can occur for gauged isometries in the fibre, which in turn implies that there are no AdS vacua in the low-energy effective $N = 2$ action of six-dimensional $\mathrm{SU}(3) \times \mathrm{SU}(3)$-structure compactifications of type II string theory and M-theory that preserve eight supercharges.

[1] Related work on tensor fields in $N = 2$ supergravity has been performed in [4].

This means in particular that SU(3) × SU(3)-structure backgrounds with four- and five-dimensional $N = 2$ AdS vacua as found in [7–9] do not have any description in terms of $N = 2$ gauged supergravity. The conditions for $N = 2$ Minkowski vacua are linear in the fibre coordinates and holomorphic in the coordinates on the special-Kähler manifold suggesting that generically a solution exists. However, the Kähler cone condition is not automatically satisfied for these solutions.

$N = 2$ supergravities with $N = 1$ vacua were first discovered in refs. [10–12] and later systematically analysed in [13–15]. It is of interest to determine under what conditions these supergravities can also admit $N = 2$ vacua in their field space. We again find that the conditions are linear in the fibre coordinates and holomorphic in the special-Kähler coordinates, leaving the Kähler cone condition as the non-trivial requirement to find a physically acceptable solution. We give two examples of special-Kähler manifolds with cubic prepotential, one of which contains either an $N = 1$ or an $N = 2$ vacuum inside the Kähler cone but never both at the same time. The second example can accommodate both $N = 1$ and $N = 2$ vacua inside the Kähler cone, as long as the charges are chosen appropriately.

This paper is organised as follows. In section 2 we briefly introduce $N = 2$ gauged supergravity in order to set the stage for the analysis. In section 3 we record the conditions for vacua with the full $N = 2$ supersymmetry and determine some of their properties. In section 4 we extend the analysis to supergravities with eight supercharges in $d = 5, 6$. In section 5 we consider the special case of gauged isometries in the fibre of quaternionic-Kähler manifold which are in the image of the c-map. Finally, in section 6 we address the question of simultaneously having $N = 2$ and $N = 1$ vacua in the same gauged supergravity.

2 Gauged supergravity with eight supercharges

Let us start with a brief summary of gauged $N = 2$ supergravity in $d = 4$.[2] Its spectrum consists of a gravitational multiplet, $n_{\rm v}$ vector multiplets and $n_{\rm h}$ hypermultiplets.[3] The gravitational multiplet contains the spacetime metric $g_{\mu\nu}$, two gravitini $\Psi_{\mu\mathcal{A}}, \mathcal{A} = 1, 2$ and the graviphoton A^0_μ. Each vector multiplet contains a vector A^i_μ, two gaugini $\lambda^{i\mathcal{A}}$ and a complex scalar t^i, where $i = 1, \ldots, n_{\rm v}$ labels the vector multiplets.[4] Finally, a hypermultiplet consists of two hyperini ζ_α and four scalars q^u, where $\alpha = 1, \ldots, 2n_{\rm h}$ and $u = 1, \ldots, 4n_{\rm h}$. The scalar field space is parametrised by (t^i, q^u) and splits into the product

$$M = M_{\rm v} \times M_{\rm h} \ . \tag{2.1}$$

The first component $M_{\rm v}$ is a special-Kähler manifold of complex dimension $n_{\rm v}$ spanned by the scalars t^i in the vector multiplets. This implies that the metric obeys

$$g_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} K^{\rm v} \ , \qquad \text{with} \qquad K^{\rm v} = -\ln {\rm i} \left(\bar{X}^\Lambda \Omega_{\Lambda\Sigma} X^\Sigma \right) \ , \tag{2.2}$$

[2]For a more comprehensive review see, for example, ref. [16].

[3]We neglect the possibility of tensor multiplets, as they can be dualised into hypermultiplets (or vector multiplets, if they are massive).

[4]Strictly speaking, the definition of the graviphoton is $X^I \, {\rm Im}\, \mathcal{F}_{IJ} A^J_\mu$, which can be read off from the gravitino variation and depends on the scalar fields in the vector multiplets.

where $X^\Lambda = (X^I, \mathcal{F}_I), I = 0, \ldots, n_\mathrm{v}$ is a $2(n_\mathrm{v}+1)$-dimensional symplectic vector that depends holomorphically on the t^i. $\mathcal{F}_I = \partial\mathcal{F}/\partial X^I$ is the derivative of a holomorphic prepotential $\mathcal{F}$ which is homogeneous of degree 2 and $\Omega_{\Lambda\Sigma}$ is the standard symplectic metric. The physical range of the coordinates t^i is restricted to the Kähler cone defined by

$$\mathrm{i}\left(\bar{X}^\Lambda \Omega_{\Lambda\Sigma} X^\Sigma\right) > 0 \; . \tag{2.3}$$

The second component of the field space M_h, spanned by the scalars q^u in the hypermultiplets, is quaternionic-Kähler and of real dimension $4n_\mathrm{h}$. These manifolds admit a triplet of almost complex structures $I^x, x = 1, 2, 3$ satisfying $I^x I^y = -\delta^{xy}\mathbf{1} + \epsilon^{xyz} I^z$, with the metric being Hermitian with respect to all three I^x. The associated two-forms K^x are the field strength of the SU(2) connection ω^x, i.e.

$$K^x = \mathrm{d}\omega^x + \frac{1}{2}\epsilon^{xyz}\omega^y \wedge \omega^z \; . \tag{2.4}$$

In gauged supergravities the multiplets can be charged under a set of electric and magnetic gauge fields. The corresponding covariant derivatives of the scalars read

$$D_\mu q^u = \partial_\mu q^u - A_\mu^\Lambda \Theta_\Lambda^\lambda k_\lambda^u \, , \qquad D_\mu t^i = \partial_\mu t^i - A_\mu^\Lambda \hat{\Theta}_\Lambda^{\hat{\lambda}} k_{\hat{\lambda}}^i \, , \tag{2.5}$$

where $A_\mu^\Lambda = (A_\mu^I, B_{\mu\, I})$ is a symplectic vector of electric and magnetic gauge fields and k_λ^u ($k_{\hat{\lambda}}^i$), $\lambda = 1, \ldots, n_\mathrm{Kh}$, ($\hat{\lambda} = 1, \ldots, n_\mathrm{Kv}$,) are Killing vectors on M_h (M_v) respectively. Finally, the charges or group theoretical representations of the scalars are specified by the embedding tensors $\Theta_\Lambda^\lambda, \hat{\Theta}_\Lambda^{\hat{\lambda}}$. Note that the t^i transform in the adjoint representation of the gauge group and thus for any non-Abelian factor the gauged $k_{\hat{\lambda}}^i$ have to be non-trivial. Moreover, if the gauged isometries are non-Abelian, the embedding tensor has to transform covariantly, which is ensured by the quadratic constraint

$$f_{\hat{\sigma}\hat{\rho}}^{\hat{\lambda}} \hat{\Theta}_\Lambda^{\hat{\sigma}} \hat{\Theta}_\Sigma^{\hat{\rho}} + \hat{\Theta}_\Lambda^{\hat{\sigma}} (k_{\hat{\lambda}})_\Sigma^\Gamma \hat{\Theta}_\Gamma^{\hat{\lambda}} = 0 \; . \tag{2.6}$$

Here $(k_{\hat{\lambda}})_\Sigma^\Gamma$ is the symplectic transformation induced by the Killing vector $k_{\hat{\lambda}}^i$ via

$$k_{\hat{\lambda}}^i \partial_i X^\Lambda = (k_{\hat{\lambda}})_\Sigma^\Lambda X^\Sigma \, , \tag{2.7}$$

and $f_{\hat{\sigma}\hat{\rho}}^{\hat{\lambda}}$ are the structure constants

$$[k_{\hat{\sigma}}, k_{\hat{\rho}}] = f_{\hat{\sigma}\hat{\rho}}^{\hat{\lambda}} k_{\hat{\lambda}} \; . \tag{2.8}$$

Note that both $(k_{\hat{\lambda}})_\Sigma^\Gamma$ and $f_{\hat{\sigma}\hat{\rho}}^{\hat{\lambda}}$ are independent of the coordinates t^i.

The gauging of isometries requires additional terms in the supersymmetry variations. Since we are looking for maximally symmetric backgrounds it is sufficient to focus on the scalar parts of the fermionic supersymmetry variations given by

$$\begin{aligned} \delta_\epsilon \Psi_{\mu\mathcal{A}} &= D_\mu \epsilon_\mathcal{A}^* - S_{\mathcal{AB}} \gamma_\mu \epsilon^\mathcal{B} + \ldots \, , \\ \delta_\epsilon \lambda^{i\mathcal{A}} &= W^{i\mathcal{AB}} \epsilon_\mathcal{B} + \ldots \, , \\ \delta_\epsilon \zeta_\alpha &= N_\alpha^\mathcal{A} \epsilon_\mathcal{A} + \ldots \, , \end{aligned} \tag{2.9}$$

where $\epsilon^{\mathcal{A}}$ are the supersymmetry parameters and

$$\begin{aligned} S_{\mathcal{AB}} &= \frac{1}{2}\mathrm{e}^{K^{\mathrm{v}}/2}X^{\Lambda}\Theta_{\Lambda}^{\lambda}P_{\lambda}^{x}(\sigma^{x})_{\mathcal{AB}}\,, \\ W^{i\mathcal{AB}} &= \mathrm{i}\mathrm{e}^{K^{\mathrm{v}}/2}g^{i\bar{\jmath}}(\nabla_{\bar{\jmath}}\bar{X}^{\Lambda})\Theta_{\Lambda}^{\lambda}P_{\lambda}^{x}(\sigma^{x})^{\mathcal{AB}} + \mathrm{e}^{K^{\mathrm{v}}/2}\epsilon^{\mathcal{AB}}\bar{X}^{\Lambda}\hat{\Theta}_{\Lambda}^{\hat{\lambda}}k_{\hat{\lambda}}^{i}\,, \\ N_{\alpha}^{\mathcal{A}} &= 2\mathrm{e}^{K^{\mathrm{v}}/2}\bar{X}^{\Lambda}\Theta_{\Lambda}^{\lambda}\mathcal{U}_{\alpha u}^{\mathcal{A}}k_{\lambda}^{u}\,. \end{aligned} \tag{2.10}$$

Here $\mathcal{U}^{\mathcal{A}\alpha}$ are the vielbein one-forms on M_{h}, the $(\sigma^{x})_{\mathcal{B}}^{\mathcal{A}}$ are the Pauli matrices, and $\nabla_i X^{\Lambda} := \partial_i X^{\Lambda} + (\partial_i K^{\mathrm{v}})X^{\Lambda}$. Finally, P_{λ}^{x} are the Killing prepotentials defined by

$$-2k_{\lambda}^{u}K_{uv}^{x} = \nabla_v P_{\lambda}^{x}\,, \tag{2.11}$$

where ∇_v is the SU(2)-covariant derivative and the two-forms K^x are defined in (2.4). The matrices given in (2.10) also determine the scalar potential V in the Lagrangian

$$V = -6S_{\mathcal{AB}}\bar{S}^{\mathcal{AB}} + \frac{1}{2}g_{i\bar{\jmath}}W^{i\mathcal{AB}}W_{\mathcal{AB}}^{\bar{\jmath}} + N_{\alpha}^{\mathcal{A}}N_{\mathcal{A}}^{\alpha}\,. \tag{2.12}$$

To conclude, a gauged supergravity is specified by the spectrum of vector- and hyper-multiplets, their respective field spaces and the embedding tensor which determines the charged directions in field space.

3 Vacua with $N = 2$ supersymmetry

We shall now give the conditions for vacua which have the full $N = 2$ supersymmetry. This requires that all fermionic supersymmetry variations (2.9) vanish, which, for a maximally symmetric spacetime, translates into the conditions

$$S_{\mathcal{AB}}\epsilon^{\mathcal{B}} = \frac{1}{2}\mu\epsilon_{\mathcal{A}}^{*}\,, \qquad W^{iAB} = 0\,, \qquad N^{\alpha A} = 0\,, \tag{3.1}$$

where $\Lambda = -3|\mu|^2$ is the cosmological constant of the $N = 2$ vacuum. These conditions have been discussed before for electric gaugings in [1].

Let us start by analysing the second condition in (3.1). Since $(\sigma^x)^{\mathcal{AB}}$ and $\epsilon^{\mathcal{AB}}$ are linearly independent, this condition together with the definition (2.10) implies [1]

$$(\nabla_i X^{\Lambda})\,\Theta_{\Lambda}^{\lambda}P_{\lambda}^{x} = 0\,, \tag{3.2}$$

$$\bar{X}^{\Lambda}\hat{\Theta}_{\Lambda}^{\hat{\lambda}}k_{\hat{\lambda}}^{i} = 0\,. \tag{3.3}$$

Equation (3.3) only depends on the t^i and has a trivial solution $k_{\hat{\lambda}}^{i} = 0$ with the property that any non-Abelian factor of the gauge group is unbroken in the vacuum. If, on the other hand, the background has $k_{\hat{\lambda}}^{i} \neq 0$, the gauge group is spontaneously broken and (3.3) can only be fulfilled by tuning some of the t^i's appropriately. Contracting (3.3) with $\partial_i X^{\Sigma}$ and using (2.7) yields

$$\bar{X}^{\Lambda}\hat{\Theta}_{\Lambda}^{\hat{\lambda}}(k_{\hat{\lambda}})_{\Gamma}^{\Sigma}X^{\Gamma} = 0\,, \tag{3.4}$$

which, upon further multiplication with $\hat{\Theta}_{\Sigma}^{\hat{\rho}}$ and use of (2.6), results in

$$\mathrm{i}\,\bar{X}^{\Lambda}(\hat{\Theta}_{\Lambda}^{\hat{\lambda}} f_{\hat{\lambda}\hat{\sigma}}^{\hat{\rho}}\,\hat{\Theta}_{\Gamma}^{\hat{\sigma}})X^{\Gamma} = 0\,. \tag{3.5}$$

This gives a number of real quadratic equations for X^{Λ}, which fix $n_{\mathrm{r}} = \mathrm{rk}(T(t,\bar{t}))$ real degrees of freedom at some point t^i, where we defined the $n_{\mathrm{Kv}} \times (4n_{\mathrm{v}}+4)$-matrix

$$T_{\hat{\Lambda}}^{\hat{\rho}}(t,\bar{t}) = \Big(-\hat{\Theta}_{\Lambda}^{\hat{\lambda}} f_{\hat{\lambda}\hat{\sigma}}^{\hat{\rho}}\,\hat{\Theta}_{\Gamma}^{\hat{\sigma}}\,\mathrm{Im}(X^{\Gamma}(t)),\hat{\Theta}_{\Lambda}^{\hat{\lambda}} f_{\hat{\lambda}\hat{\sigma}}^{\hat{\rho}}\,\hat{\Theta}_{\Gamma}^{\hat{\sigma}}\,\mathrm{Re}(X^{\Gamma}(t))\Big)\,, \qquad \hat{\Lambda} = 1,\ldots,4n_{\mathrm{v}}+4\,. \tag{3.6}$$

As a consequence n_{r} gauge bosons become massive by "eating" n_{r} real scalar degrees of freedom leaving n_{r} massive short BPS vector multiplets.[5]

Before we analyse (3.2) let us turn to the third condition in (3.1). Since the vielbein on the quaternionic-Kähler manifold is invertible we infer from (2.10) that $N^{\alpha A} = 0$ implies

$$X^{\Lambda}\Theta_{\Lambda}^{\lambda} k_{\lambda}^{u} = 0\,, \tag{3.7}$$

which is similar to (3.3) but now couples the vector- and hypermultiplet sector. Furthermore, in contrast to (3.3), equations (3.7) are holomorphic conditions on the t^i. As before there is the trivial solution $k_{\lambda}^{u} = 0$ but (3.7) can also be satisfied by tuning further vector scalars t^i appropriately. More precisely, from the Killing vectors k_{λ} that are non-zero at the vacuum locus $n_{\mathrm{c}} = \mathrm{rk}(\Theta_{\Lambda}^{\lambda} k_{\lambda}^{u})$ holomorphic conditions arise for the vector multiplet scalars t^i which in turn imply that there are n_{c} further massive gauge boson.[6] As we shall see shortly, these massive gauge bosons reside in long non-BPS vector multiplets. Note that the combined conditions following from (3.3) and (3.7) have to be compatible and solvable by tuning at most n_{v} complex scalars.

Now let us turn to (3.2) which can be nicely combined with the first equation in (3.1). Noting that the matrix $(X^I, \nabla_i X^I)$ is invertible in special geometry we can rewrite the two conditions together as [13]

$$(\Theta_I^{\lambda} - \mathcal{F}_{IJ}\Theta^{J\lambda})P_{\lambda}^{x} = -\mathrm{e}^{-K^{\mathrm{v}}/2}(\partial_I K^{\mathrm{v}})\,\hat{\mu}\,a^x\,, \tag{3.8}$$

where a^x is an arbitrary real vector on S^2 and $\hat{\mu}$ is related to μ by a phase. From the definition of the Kähler potential (2.2) we have $X^I\partial_I K^{\mathrm{v}} = 1$ and $(\partial_i X^I)\partial_I K^{\mathrm{v}} = 0$, which means that the right-hand side in (3.8) gives only a contribution to the gauging of the graviphoton $X^I\,\mathrm{Im}\,\mathcal{F}_{IJ}A_{\mu}^{J}$.[7] The non-vanishing prepotential of this gauging therefore determines the cosmological constant, while the prepotentials of all other gaugings should vanish in an $N=2$ vacuum. We can easily solve (3.8) for the prepotentials. Since $\mathrm{Im}\,\mathcal{F}_{IJ}$ is required by special geometry to be invertible, (3.8) is equivalent to

$$\Theta_{\Lambda}^{\lambda}P_{\lambda}^{x} = -\frac{1}{2}\mathrm{e}^{K^{\mathrm{v}}/2}\Omega_{\Lambda\Sigma}\,\mathrm{Im}(\hat{\mu}\bar{X}^{\Sigma})\,a^x\,, \tag{3.9}$$

where we used (2.2) and $\Theta_{\Lambda}^{\lambda} = (\Theta_I^{\lambda}, -\Theta^{J\lambda})$.

[5] Note that for $k_{\hat{\lambda}}^{i} = 0$ we have $T_{\hat{\Lambda}}^{\hat{\rho}} = 0$ and therefore $n_{\mathrm{r}} = 0$ so that the gauge group remains unbroken.

[6] Note that electric gaugings give rise to linear equations, while magnetic gaugings are non-linear in the standard coordinates on M_{v}.

[7] This explicit expression for the graviphoton is found from its appearance in the gravitino variation.

JHEP08(2012)039

In general (3.9) corresponds to $3n_c$ real conditions for the hypermultiplet scalars which in turn become massive. As we observed above n_c gauge bosons also become massive by each eating the forth real scalar field of a hypermultiplet. We thus see that the Higgs mechanism leads to a long massive vector multiplet which contains altogether five massive scalars — three from hypermultiplets and two from vector multiplets. Those scalar fields which do not participate in the Higgs mechanism are flat directions of the vacuum and thus define its $N = 2$ moduli space. Note that we need $n_h \geq n_c$ in order to have an $N = 2$ vacuum.

Let us now consider the special case of isometries k_λ which do not induce an SU(2) R-symmetry rotation on the fermions, i.e. isometries of M_h whose Lie derivative on the SU(2) connection vanishes

$$\mathcal{L}_k \omega^x = 0 \ . \tag{3.10}$$

For such isometries the Killing prepotentials are given in terms of the SU(2) connection by [17]

$$P^x = \omega^x(k) \ . \tag{3.11}$$

Inserted into $S_{\mathcal{AB}}$ the hyperino condition (3.7) implies

$$S_{\mathcal{AB}} \sim X^\Lambda \Theta_\Lambda^\lambda P_\lambda^x (\sigma^x)^{AB} = \omega^x (X^\Lambda \Theta_\Lambda^\lambda k_\lambda)(\sigma^x)_{\mathcal{AB}} = 0 \ . \tag{3.12}$$

From eq. (3.1) we then infer that the cosmological constant must vanish and all $N = 2$ vacua in such theories are necessarily Minkowski. It can be easily checked that the isometries in the fibre of quaternionic-Kähler manifolds which are in the image of the c-map, and which we discuss in more detail in section 5, have this property [13, 17]. Note, however, that there are also examples where (3.10) is not fulfilled [1, 19].

Before we proceed, let us address the issue of the SU(2)-covariance of our result. Both (3.10) and (3.11) do not transform covariantly under local SU(2) rotations and therefore one might worry that they only hold for a particular choice of coordinates.[8] Indeed, the Killing prepotentials can be written more generally as [18]

$$P_\lambda^x = \omega^x(k_\lambda) + W_\lambda^x \ , \tag{3.13}$$

where W_λ^x is the so-called compensator field that makes the right-hand side transform non-trivially as an SU(2) vector and that is defined via

$$\mathcal{L}_{k_\lambda} K^x = \epsilon^{xyz} K^y W_\lambda^z \ . \tag{3.14}$$

As a consequence of (3.10) the left-hand side of this equation vanishes and the compensator field vanishes in this particular SU(2) frame. However, in the $N = 2$ locus (3.7) implies that

$$X^\Lambda \Theta_\Lambda^\lambda P_\lambda^x \Big|_{N=2} = X^\Lambda \Theta_\Lambda^\lambda W_\lambda^x \Big|_{N=2} \ , \tag{3.15}$$

where each side transforms as an SU(2) vector. This means that

$$X^\Lambda \Theta_\Lambda^\lambda \mathcal{L}_{k_\lambda} K^x \Big|_{N=2} = 0 \ , \tag{3.16}$$

[8] We thank the referee and S. Vandoren for drawing our attention to this subtlety.

is an SU(2)-covariant condition (see also appendix A.3 of [13] for similar manipulations), which follows from the non-covariant equation (3.10). Furthermore, the condition (3.16) implies that $S_{\mathcal{AB}}$ is vanishing and that the $N = 2$ vacuum must be Minkowski.

4 $N = 2$ supergravities in $d = 5, 6$

The analysis of the previous section can be repeated in five and six dimensions for supergravities with the same number (eight) of supercharges. The hypermultiplet sector is unchanged while the vector multiplets have only one real scalar in $d = 5$ or none at all in $d = 6$. As a result the matrices appearing in fermionic supersymmetry variations (2.9) change.

Five-dimensional $N = 2$ gauged supergravity has been discussed for example in [20–22] and references therein. Here we will restrict to the case with no tensor multiplets and comment on the more general case later. The $N = 2$ vacua again arise as solutions of (3.1), with the major difference relative to $d = 4$ being that there are no magnetically charged fields, as there are no magnetic gauge vectors. In addition, the scalar matrices previously defined in (2.10) now read

$$\begin{aligned} S_{\mathcal{AB}} =& h^I \Theta_I^\lambda P_\lambda^x (\sigma^x)_{\mathcal{AB}} \,, \\ W^{i\mathcal{AB}} =& -\frac{\sqrt{3}}{\sqrt{2}} g^{ij} \partial_j h^I \Theta_I^\lambda P_\lambda^x (\sigma^x)^{\mathcal{AB}} \,, \\ N_\alpha^{\mathcal{A}} =& \frac{\sqrt{6}}{4} h^I \Theta_I^\lambda \mathcal{U}_{\alpha u}^{\mathcal{A}} k_\lambda^u \,, \end{aligned} \tag{4.1}$$

and depend in the vector multiplets only on a set of real coordinates h^I (instead of the complex coordinates X^I) that obey the cubic condition

$$d_{IJK} h^I h^J h^K = 1 \ . \tag{4.2}$$

Analogously to the derivation of (3.7), the hyperino condition $N_\alpha^{\mathcal{A}} = 0$ leads to

$$h^I \Theta_I^\lambda k_\lambda^u = 0 \ . \tag{4.3}$$

These are $n_{\rm r} = {\rm rk}(\Theta_I^\lambda k_\lambda^u)$ real equations on the h^I which fix the scalars of $n_{\rm r}$ vector multiplets. Furthermore, $(h^I, \partial_j h^I)$ is again an invertible matrix so that, similarly to (3.9), we can combine the gaugino and gravitino equation into

$$\Theta_I^\lambda P_\lambda^x = d_{IJK} h^J h^K \mu a^x \,, \tag{4.4}$$

where μ is real and $d_{IJK} h^J h^K$ replaces $\partial_I K$ in (3.9) by virtue of the cubic condition (4.2). This fixes $3n_{\rm r}$ hypermultiplet scalars, consistent with the Higgs mechanism and we end up with $n_{\rm r}$ long massive vector multiplets. Note that, analogously to four dimensions, a supersymmetric AdS vacuum exists only if the Lie derivative on the SU(2) connection is non-zero for at least one of the gauged Killing vectors. The story gets more involved in the presence of tensor multiplets [20]. However, let us stress that the cosmological constant is only affected by gaugings in the hypermultiplets, and therefore our discussion concerning the existence of supersymmetric AdS vacua still applies.

JHEP08(2012)039

We now turn to gauged supergravities with eight supercharges in $d = 6$ which are discussed, for example, in [23, 24]. In this case there are no scalars in the vector multiplet sector. Moreover, due to chirality of the supergravity no scalar contributions arise in the hyperino or gravitino variation, in contrast to (2.9). From the gaugino variation one finds similarly to (3.7) the condition

$$\Theta_i^\lambda P_\lambda^x = 0\,, \tag{4.5}$$

which again are $3\,\mathrm{rk}(\Theta)$ real conditions on the hypermultiplet scalars, as required by the Higgs mechanism. Furthermore, supersymmetric AdS is not a solution, as gaugings do not give a contribution to the cosmological constant.

5 Gauging the isometries of the c-map

A large class of known quaternionic-Kähler manifolds are those that lie in the image of the c-map [5, 6]. These manifolds are fibrations of a graded Heisenberg algebra over a special-Kähler manifold and they are of interest as the fibre admits a large number of isometries. Furthermore, they appear in the low-energy effective action of type II and M-theory compactifications on six-dimensional $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure manifolds where fluxes, torsion and non-geometric fluxes precisely gauge these isometries (see e.g. [17, 25–30]). Therefore the vacua in these gauged supergravities coincide with the vacua for $\mathrm{SU}(3)\times\mathrm{SU}(3)$-structure compactifications of type II and M-theory to four and five dimensions, respectively.

Let us denote the $(n_\mathrm{h} - 1)$ complex coordinates of the special-Kähler base space by z^a, the analogue of the holomorphic symplectic vector X^Λ by $Z^{\tilde{\Lambda}} = (Z^A, \mathcal{G}_A)$ and the corresponding Kähler potential by K^h. The c-map adds an additional $(2n_\mathrm{h} + 2)$ real fibre coordinates $(\phi, \tilde{\phi}, \xi^{\tilde{\Lambda}})$ where $\xi^{\tilde{\Lambda}} = (\xi^A, \tilde{\xi}_A)$ is a $2n_\mathrm{h}$-dimensional symplectic vector.[9] The isometries of the fibre are generated by the Killing vectors[10]

$$k_{\tilde{\phi}} \;=\; -2\,\frac{\partial}{\partial\tilde{\phi}}\,, \qquad k_{\tilde{\Lambda}} \;=\; \frac{\partial}{\partial\xi^{\tilde{\Lambda}}} + \Omega_{\tilde{\Lambda}\tilde{\Sigma}}\xi^{\tilde{\Sigma}}\,\frac{\partial}{\partial\tilde{\phi}}\,, \tag{5.1}$$

which form a graded Heisenberg algebra with the only non-trivial commutator being

$$[k_{\tilde{\Lambda}}, k_{\tilde{\Sigma}}] = \Omega_{\tilde{\Lambda}\tilde{\Sigma}} k_{\tilde{\phi}}\ . \tag{5.2}$$

Since these Killing vectors are everywhere linearly independent, eq. (3.7) simplifies to

$$X^\Lambda \Theta_\Lambda^{\tilde{\Lambda}} = 0\,, \qquad X^\Lambda \Theta_\Lambda^{\tilde{\phi}} = 0\ . \tag{5.3}$$

This gives $n_\mathrm{c} = \mathrm{rk}(\Theta)$ holomorphic conditions on M_v, giving a mass to n_c vector multiplets in the Higgs mechanism. Furthermore, eq. (5.3) defines the $N = 2$ vector moduli space of the vacuum.

[9] For more details see, for example, [5, 6, 13].

[10] We neglect the Killing vector in the ϕ direction, as this isometry is broken in string compactifications by one-loop corrections [31].

Let us continue with the constraints in the hypermultiplet sector. The isometries generated by (5.1) fulfil (3.10) and therefore the $N = 2$ vacuum is necessarily Minkowski. Inserting (3.11) into (3.9) we arrive at

$$\Theta_{\Lambda}^{\tilde{\Lambda}}\omega^{x}(k_{\tilde{\Lambda}}) = 0\,, \qquad \Theta_{\Lambda}^{\tilde{\phi}}\omega^{x}(k_{\tilde{\phi}}) = 0\;. \tag{5.4}$$

The explicit form of the SU(2) connection is given by [6, 13]

$$\begin{aligned} \omega^1 - \mathrm{i}\,\omega^2 &= 2\mathrm{e}^{K^{\mathrm{h}}/2+\phi} Z^A(\mathrm{d}\tilde{\xi}_A - \mathcal{F}_{AB}\mathrm{d}\xi^B)\,, \\ \omega^3 &= \frac{1}{2}\mathrm{e}^{2\phi}(\mathrm{d}\tilde{\phi} + \tilde{\xi}_A\mathrm{d}\xi^A - \xi^A\mathrm{d}\tilde{\xi}_A) - \mathrm{i}\,\mathrm{e}^{K^{\mathrm{h}}}\left(Z^A(\mathrm{Im}\,\mathcal{G}_{AB})\mathrm{d}\bar{Z}^B - \bar{Z}^A(\mathrm{Im}\,\mathcal{G}_{AB})\mathrm{d}Z^B\right)\;. \end{aligned} \tag{5.5}$$

Inserted into (5.4) yields

$$\Theta_{\Lambda}^{\tilde{\Lambda}}\Omega_{\tilde{\Lambda}\tilde{\Sigma}}Z^{\tilde{\Sigma}} = 0\,, \tag{5.6}$$

$$\Theta_{\Lambda}^{\tilde{\Lambda}}\Omega_{\tilde{\Lambda}\tilde{\Sigma}}\xi^{\tilde{\Sigma}} = \Theta_{\Lambda\tilde{\phi}}\;. \tag{5.7}$$

The first equation is completely analogous to (5.3) and gives n_{c} holomorphic conditions on the special-Kähler base of M_{h}. The second equation leads to n_{c} real conditions on the fibre of M_{h}. The other n_{c} fibre scalars are eaten by the gauge vectors so that altogether there are n_{c} long massive vector multiplets leaving $n_{\mathrm{v}} - n_{\mathrm{c}}$ vector and $n_{\mathrm{h}} - n_{\mathrm{c}}$ hypermultiplets unfixed and massless.

Note that (5.3) and (5.6) are holomorphic equations of the special-Kähler coordinates and (5.7) gives real, linear equations for the fibre. Therefore they are generically solvable but it is not automatic that the solution lies inside the Kähler cones for both the X^I and Z^A (cf. (2.3)). We will see this feature more explicitly in the next section when we discuss some examples.

6 $N = 2$ and $N = 1$ vacua in the same gauged supergravity

In ref. [13] the issue of spontaneous $N = 2 \to N = 1$ supersymmetry breaking was considered and the possible $N = 1$ vacua of $N = 2$ supergravities were classified. It is of interest to determine under which conditions a given gauged supergravity can have simultaneously $N = 2$ and $N = 1$ vacua in its field space.[11] Supersymmetry then implies that both vacua are completely stable [32, 33]. In the following we derive these conditions and give two explicit examples. As we will see, they are separated in scalar field space and can lie in the same or in different chambers of the Kähler cone.

We will concentrate in the following on supergravities that are in the image of the c-map. For this class the $N = 1$ Minkowski solutions of [13] can be stated in terms of the embedding tensor as

$$\Theta_{\Lambda}^{\tilde{\Lambda}} = \mathrm{Re}\left(\bar{C}_{\Lambda}D^{\tilde{\Lambda}}\right)\,, \qquad \Theta_{\Lambda}^{\tilde{\phi}} = \mathrm{Re}\left(\bar{C}_{\Lambda}\hat{D}\right)\,, \tag{6.1}$$

[11] We thank Z. Komargodski for a remark which inspired the following analysis.

where the solution is parametrised by two complex lightlike vectors C_Λ and $D^{\tilde\Lambda}$ satisfying

$$\bar{C}_\Lambda \Omega^{\Lambda\Sigma} C_\Sigma = 0\,, \qquad \bar{D}^{\tilde\Lambda}\Omega_{\tilde\Lambda\tilde\Sigma} D^{\tilde\Sigma} = 0\,, \tag{6.2}$$

and

$$C^J \mathcal{F}_{JI}(t_{N=1}) = C_I\,, \qquad D^B \mathcal{G}_{BA}(z_{N=1}) = D_A\,, \qquad D^{\tilde\Lambda}\,\Omega_{\tilde\Lambda\tilde\Sigma}\,\xi^{\tilde\Sigma}_{N=1} = \hat{D}\,. \tag{6.3}$$

$\hat{D}$ is a constant and the last equation fixes two of the scalars $\xi^{\tilde\Sigma}$. The first two equations in (6.3) generically fix all scalars t^i and z^a, but for special theories there can be a moduli space spanned by $t_{N=1}$ and $z_{N=1}$, respectively [15]. The structure of the embedding tensor given in (6.1) defines the gauged supergravity and the conditions (6.2) and (6.3) ensure that it has $N = 1$ vacua. Let us now consider under what conditions these supergravities can also have $N = 2$ vacua.

Clearly, the embedding tensor in (6.1) has just rank two, so that generically there should also exist an $N = 2$ vacuum. Inserting the $\mathcal{N} = 1$ solutions (6.1) into (5.3), (5.6) and (5.7) we find the $N = 2$ condition to be

$$\begin{aligned} X^\Lambda_{N=2}\, \mathrm{Re}\,\big(\bar{C}_\Lambda D^{\tilde\Lambda}\big) &= 0\,, & X^\Lambda_{N=2}\, \mathrm{Re}\,\big(\bar{C}_\Lambda \hat{D}\big) &= 0\,,\\ \mathrm{Re}\,\big(\bar{C}_\Lambda D^{\tilde\Lambda}\big)\,\Omega_{\tilde\Lambda\tilde\Sigma}\, Z^{\tilde\Sigma}_{N=2} &= 0\,, & \mathrm{Re}\,\big(\bar{C}_\Lambda D^{\tilde\Lambda}\big)\,\Omega_{\tilde\Lambda\tilde\Sigma}\, \xi^{\tilde\Sigma}_{N=2} &= \mathrm{Re}\,\big(\bar{C}_\Lambda \hat{D}\big)\,, \end{aligned} \tag{6.4}$$

where the subscript $N = 2$ indicates that we evaluate the quantity in the $N = 2$ vacuum. Using the fact that (6.3) holds at some point in scalar field space and that $\mathrm{Im}\,F_{IJ}$ and $\mathrm{Im}\,G_{AB}$ are invertible, it follows that neither of the complex vectors C_Λ and $D^{\tilde\Lambda}$ are proportional to a real vector. Therefore, the most general solution of (6.4) is

$$X^\Lambda_{N=2}\bar{C}_\Lambda = X^\Lambda_{N=2} C_\Lambda = 0\,, \quad D^{\tilde\Lambda}\,\Omega_{\tilde\Lambda\tilde\Sigma}\, Z^{\tilde\Sigma}_{N=2} = \bar{D}^{\tilde\Lambda}\,\Omega_{\tilde\Lambda\tilde\Sigma}\, Z^{\tilde\Sigma}_{N=2} = 0\,, \quad D^{\tilde\Lambda}\,\Omega_{\tilde\Lambda\tilde\Sigma}\,\xi^{\tilde\Sigma}_{N=2} = \hat{D}\,. \tag{6.5}$$

We see that the condition on the fibre coordinates $\xi^{\tilde\Sigma}$ is the same for $N = 1$ and $N = 2$ vacua while the conditions on the scalars t^i and z^a are less restrictive for $N = 2$ vacua. Generically, two complex t^i and two complex z^a are fixed by (6.5). Therefore, gauged supergravities which admit an $N = 1$ vacuum could easily also have an $N = 2$ vacuum. However, it is not obvious that both vacua lie within the same Kähler cone where (2.3) holds.

Before we discuss examples where both types of vacua are realised, let us discuss their positions in field space. On the one hand one expects that different vacua should not intersect in field space. On the other hand one easily imagines a point in field space which could fulfil both the $N = 2$ and $N = 1$ conditions (6.5) and (6.3) simultaneously. However, the Kähler cone condition (2.3) ensures that $N = 1$ and $N = 2$ vacua are always separated in field space. To see this we combine (6.3) and (6.5) to arrive at

$$\bar{X}^I (\mathrm{Im}\,\mathcal{F})_{IJ} C^J = 0\,, \tag{6.6}$$

while (6.3) implies

$$\bar{C}^I (\mathrm{Im}\,\mathcal{F})_{IJ} C^J = 0\,. \tag{6.7}$$

Eq. (6.7) states that C^I is lightlike while (6.6) means that C^I and X^I are orthogonal to each other. In the Kähler cone defined by (2.3), X^I is timelike, contradicting one of these two statements. Therefore, both conditions cannot be fulfilled simultaneously as long as (2.3) holds. Hence, $N = 1$ and $N = 2$ vacua can only coincide outside the physical region of the Kähler cone. Of course, the same reasoning also holds for the special-Kähler base space in the hypermultiplet sector.

We shall now consider the STU model as a first example, where the scalar manifolds are given by

$$M_{\rm v} = \left(\frac{Sl(2,\mathbb{R})}{\mathrm{SO}(2)}\right)^3 , \qquad M_{\rm h} = \frac{\mathrm{SO}(4,4)}{\mathrm{SO}(4)^2} . \tag{6.8}$$

This means that both the special-Kähler manifold $M_{\rm v}$ for the vector multiplets as well as the special-Kähler base underlying the quaternionic-Kähler manifold $M_{\rm h}$ are described by the holomorphic prepotential

$$\mathcal{F} = \frac{X^S X^T X^U}{X^0} = STU , \tag{6.9}$$

where we have defined the complex coordinates $S = \frac{X^S}{X^0}$, $T = \frac{X^T}{X^0}$, $U = \frac{X^U}{X^0}$ and chosen $X^0 = 1$. Since the equations (6.3) and (6.5) are identical for both special-Kähler manifolds, we will only focus on $M_{\rm v}$ in the following. The discussion for $M_{\rm h}$ is completely analogous. The Kähler potential can be computed from (6.9) and is given by

$$K = -\ln(-\mathrm{i}(\bar{S} - S)(\bar{T} - T)(\bar{U} - U)) , \tag{6.10}$$

so that the Kähler cone condition (2.3) reads

$$\mathrm{Im}\, S \,\mathrm{Im}\, T \,\mathrm{Im}\, U > 0 . \tag{6.11}$$

This gives various domains where either all imaginary parts are positive or two imaginary parts are negative and the third one is positive. In [15] we already discussed the $N = 1$ vacuum of this model. In order to find a vacuum inside the Kähler cone, we choose

$$C_S = \frac{C^T C^U}{C^0} , \qquad C_T = \frac{C^S C^U}{C^0} , \qquad C_U = \frac{C^S C^T}{C^0} , \qquad C_0 = -\frac{C^S C^T C^U}{(C^0)^2} , \tag{6.12}$$

with $C^0 \neq 0$. Furthermore, condition (6.2) gives

$$\mathrm{Im}\, \frac{C^S}{C^0} \,\mathrm{Im}\, \frac{C^T}{C^0} \,\mathrm{Im}\, \frac{C^U}{C^0} = 0 . \tag{6.13}$$

This means that one of the three imaginary parts, say $\mathrm{Im}\, \frac{C^U}{C^0}$, must vanish. Then the $N = 1$ solution is at [15]

$$S_{N=1} = \frac{C^S}{C^0} , \qquad T_{N=1} = \frac{C^T}{C^0} , \tag{6.14}$$

with U arbitrary. On the other hand, from (6.5) we infer that a possible $N = 2$ vacuum would be located at

$$S_{N=2} = \frac{C^S}{C^0} , \quad T_{N=2} = \frac{\bar{C}^T}{\bar{C}^0} , \qquad \text{or at} \qquad S_{N=2} = \frac{\bar{C}^S}{\bar{C}^0} , \quad T_{N=2} = \frac{C^T}{C^0} . \tag{6.15}$$

Checking the Kähler cone condition (6.11) we see that the $N = 1$ and $N = 2$ solutions can never be both in the same chamber of the Kähler cone. Therefore, we find either an $N = 1$ or an $N = 2$ vacuum inside the Kähler cone, depending on the choice of C^I.

Let us now give an example where $N = 1$ and $N = 2$ vacua do exist in the same theory and, moreover, in the same domain of the Kähler cone. We consider a supergravity with the field space

$$M_{\rm v} = \frac{Sl(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{SO}(2,n+2)}{\mathrm{SO}(2)\times\mathrm{SO}(n+2)}\,, \qquad M_{\rm h} = \frac{\mathrm{SO}(4,n+4)}{\mathrm{SO}(4)\times\mathrm{SO}(n+4)}\,. \tag{6.16}$$

$M_{\rm h}$ is in the image of the c-map where the special Kähler base coincides with $M_{\rm v}$ [5]. Thus the holomorphic prepotential for both spaces is given by

$$\mathcal{F} = \frac{X^S(X^T X^U + X^m X^m)}{X^0} = STU + Sy^m y^m\,, \qquad m = 1,\dots,n\,, \tag{6.17}$$

where again the first expression is in terms of X^I and the second one in terms of holomorphic coordinates with $X^0 = 1$. As before, we will focus on $M_{\rm v}$ in the following with the discussion for $M_{\rm h}$ being completely analogous. The Kähler potential is given by

$$K = -\ln \mathrm{i}(\bar S - S) - \ln\big(-(T-\bar T)(U-\bar U) - (y^m - \bar y^m)(y^m - \bar y^m)\big)\,, \tag{6.18}$$

so that the Kähler cone condition (2.3) reads

$$\mathrm{Im}\, S(\mathrm{Im}\, T\, \mathrm{Im}\, U + \mathrm{Im}\, y^m\, \mathrm{Im}\, y^m) > 0\,. \tag{6.19}$$

In the following we will concentrate on the domain where

$$\mathrm{Im}\, S > 0\,, \qquad \mathrm{Im}\, T\, \mathrm{Im}\, U + \mathrm{Im}\, y^m\, \mathrm{Im}\, y^m > 0\,. \tag{6.20}$$

In [15] the condition (6.3) was discussed in detail for the example (6.17). The vector C^Λ parametrising the embedding tensor was defined to be

$$\begin{aligned} C_S &= \frac{C^T C^U}{C^0}\,, & C_T &= \langle S\rangle C^U\,, & C_U &= \langle S\rangle C^T\,,\\ C_m &= 2\langle S\rangle C^m\,, & C_0 &= -\langle S\rangle \frac{C^T C^U}{C^0}\,, & C^S &= \langle S\rangle C^0\,, \end{aligned} \tag{6.21}$$

with $C^0 \neq 0$. The $N = 1$ vacuum is located at

$$S = \langle S\rangle\,, \qquad \left(T - \frac{C^T}{C^0}\right)\left(U - \frac{C^U}{C^0}\right) + \left(y^m - \frac{2C^m}{C^0}\right) y^m = 0\,. \tag{6.22}$$

If $\mathrm{Im}\langle S\rangle > 0$, condition (6.2) gives

$$\mathrm{Im}\,\frac{C^T}{C^0}\,\mathrm{Im}\,\frac{C^U}{C^0} = -\frac{C^m \bar C^m}{2|C^0|^2}\,. \tag{6.23}$$

If we take $\mathrm{Im}\,\frac{C^T}{C^0} > 0$, then one point of the $N = 1$ vacuum is given by

$$T = \frac{C^T}{C^0}\,, \qquad y^m = 0\,, \tag{6.24}$$

JHEP08(2012)039

and therefore an $N = 1$ vacuum exists.

Now let us discuss the $N = 2$ vacuum. From (6.5) we obtain two equations that read

$$\begin{aligned}
&(S - \langle S\rangle)\left(\left(T - \frac{C^T}{C^0}\right)\left(U - \frac{C^U}{C^0}\right) + y^m\left(y^m - \frac{2C^m}{C^0}\right)\right) = 0\,, \\
&(\bar{S} - \langle S\rangle)\left(\left(\bar{T} - \frac{C^T}{C^0}\right)\left(\bar{U} - \frac{C^U}{C^0}\right) + \bar{y}^m\left(\bar{y}^m - \frac{2C^m}{C^0}\right)\right) = 0\,.
\end{aligned} \tag{6.25}$$

The first one is easily satisfied by $S = \langle S\rangle$. The second one is then more difficult to solve since (6.20) demands $\operatorname{Im} S > 0$. Here we only display one point of the $N = 2$ vacuum to prove that it exists inside the Kähler cone. This point is

$$\begin{aligned}
S &= \langle S\rangle\,, & U &= \operatorname{Re}\frac{C^U}{C^0} + 3\,\mathrm{i}\operatorname{Im}\frac{C^U}{C^0}\,, \\
T &= \operatorname{Re}\frac{C^T}{C^0} + 3\,\mathrm{i}\operatorname{Im}\frac{C^T}{C^0}\,, & y^m &= 2\,\mathrm{i}\operatorname{Im}\frac{C^m}{C^0}\,,
\end{aligned} \tag{6.26}$$

where we set $\operatorname{Re}\frac{C^m}{C^0} = 0$. By using (6.13), one can check that the point (6.26) solves (6.25) and therefore gives an $N = 2$ solution. Furthermore, (6.26) lies inside the Kähler cone defined by (6.20). Therefore, we have an $N = 1$ and an $N = 2$ vacuum in the same $N = 2$ gauged supergravity.

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