Mechanism for the $\alpha \to \epsilon$ phase transition in iron

Bertrand Dupé,† Bernard Amadon,* Yves-Patrick Pellegrini, and Christophe Denoual
CEA, DAM, DIF, F-91297 Arpajon, France
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The mechanism of the $\alpha$-$\epsilon$ transition in iron is reconsidered. A path in the Burgers description of the bcc/hcp transition different from those previously considered is proposed. It relies on the assumption that shear and shuffle are decoupled and requires some peculiar magnetic order, different from that of $\alpha$ and $\epsilon$ phases as found in Density-Functional Theory. Finally, we put forward an original mechanism for this transition, based on successive shuffle motion of layers, which is akin to a nucleation-propagation process rather than to some uniform motion.

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I. INTRODUCTION

The bcc-hcp transition in iron has recently been the subject of intense experimental and theoretical works. At room temperature and pressure, $\alpha$-iron is a bcc metal with ferromagnetic (FM) order. Upon pressure, iron exhibits a phase transition at $\approx 13$ GPa to hcp structure, with no magnetic order. The hysteresis at the transition is large, and as pressure is changed, the transformation occurs rapidly, suggesting that the transition is non-diffusive and martensitic. Recent work underlines the importance of antiferromagnetic (AFM) fluctuations in hcp $\epsilon$ iron.

In order to describe $\alpha$ iron from an ab-initio point of view, Dynamical Mean Field Theory (DMFT) in combination with Density Functional Theory (DFT) or other advanced schemes have been used. In particular, the localization of electrons in the non-bonding $e_g$ state but not in the bonding $t_{2g}$ state is a key point for understanding paramagnetic bcc iron. However, DFT ab initio techniques alone can handle paramagnetic bcc iron and the thermodynamical fundamentals of the $\alpha \to \epsilon$ transition. The Generalized Gradient Approximation (GGA) correctly captures the basic Stoner mechanism for the reduction of magnetism, under the increase of atom coordination during the $\alpha \to \epsilon$ transition, as well as the larger spin polarization of $e_g$ orbitals with respect to $t_{2g}$ in the $\alpha$ phase.

The $\alpha \to \epsilon$ transition can be viewed as combining an anisotropic compression (shear) in the (100) direction of bcc with a shuffle in the (011) plane, that corresponds to a zone-boundary transverse phonon mode (see, e.g., Ref. [5]). Due to its martensitic nature, the transformation is triggered above the thermodynamical threshold, i.e., when the $\alpha$ and $\epsilon$ enthalpies are equal, which appears above $\approx 10$-13 GPa as computed in the GGA.

However, its ab-initio description is debated. Although the broad outline of the transition has been settled, a detailed description faces the problem of accurately computing the relevant energy barrier. Ekman et al. demonstrated that the transition is first-order, with no dynamical precursors, and that it is caused by the effect of pressure on the magnetism of iron. Johnson and Carter have shown that a minimization with respect to the $\alpha$ and $\epsilon$ phases as found in Density-Functional Theory. Finally, we put forward an original mechanism for this transition, based on successive shuffle motion of layers, which is akin to a nucleation-propagation process rather than to some uniform motion.

II. COMPUTATIONAL SETUP AND METHODOLOGY

The two-atom unit cell is replicated four times in the $\langle 001 \rangle_{\text{hcp}} \parallel \langle 110 \rangle_{\text{bcc}}$ direction into an eight-atom supercell, with periodic boundary conditions (PBCs). The unit cell is represented in Fig. While with lattice vectors $\vec{R}_1$, $\vec{R}_2$, and $\vec{R}_3$. Three parameters are necessary to determine the unit cell: the angle $\alpha$, the value of $c = R_2/4$ and the value of $R_1$. Three macroscopic quantities can be used to compute them: the volume, the $c/a = R_2/(4R_3)$ ratio and the shear $\epsilon$. In our paper, the volume, namely, 71.5 bohr$^3$/atom, is such that hcp and bcc...
energies coincide. The \( c/a \) ratio is kept constant and equal to \( \sqrt{8/3} \approx 1.633 \), which is the value in the bcc structure and in the ideal hcp compact structure. The shear is defined by \( \epsilon = (2 \sin \alpha - \frac{1}{\sqrt{3}})/(1 - \frac{1}{\sqrt{3}}) \) and is equal respectively to 0 and 1, respectively, in the bcc and the hcp structures. For each value of the shear, we computed the energy for three values of the \( c/a \) ratio, namely, 1.605, \( \sqrt{8/3} \approx 1.633 \), and 1.668 to check that the sheared bcc structure is stable with respect to a change of \( c/a \) and that \( c/a = \sqrt{8/3} \) is close to the minimum. The results are insensitive to the small difference between this value and the true minimum. Finally, the shuffle \( \eta \) is proportional to the distance of one blue atom between its position during the transition (see Fig. 2) and the reference position in bcc.

We focus our paper on the shuffle part of the Burgers mechanism. Because of the different physical timescales involved, we assume that the lattice shear deformation mode \( \epsilon \) and the shuffle mode \( \eta \) are uncoupled in such manner that, with respect to shear, shuffle is instantaneous. Volume and \( c/a \) ratio are, thus, kept constant during the shuffle. The path proposed in Ref. 7 is defined by the equality of the energies of the sheared bcc and hcp phases. In Ref. 7, however, the energetics of the shuffle is not explored. On the other hand, Liu and Johnson couple shear and shuffle modes. Their path is schematically represented in black in Fig. 2. In this figure, the shear \( \epsilon \) is fixed to half the value necessary to go from the bcc \( \alpha \) phase to the hcp \( \epsilon \) phase. It is the shear at the transition state computed by Ekman et al. at constant volume. This choice guarantees that the shuffle mode goes through the transition state (TS) of Ref. 8 and corresponds to the energetically most favorable path, represented in solid red in Fig. 2.

In this paper, we are concerned by the shuffle mechanism along the BC line (see letters in Fig. 2). Since the shuffle of a large number of atoms can be described in several different ways, our aim is to understand whether the shuffle of all the layers is simultaneous or not.

### A. Simultaneous shuffle

Let us first consider simultaneous shuffle (S). By AAAAAAÅ, we denote the stacking of the eight atoms in the bcc sheared phase, and by BB BBBB BB, we denote their stacking in the hcp sheared phase. In this notation, A and Å, and B and B, correspond to atoms in the bcc- or hcp-like configurations, respectively. The atoms in the A and Å layers have 8 neighbors, whereas atoms in the B and B layer have 12 neighbors. During the simultaneous shuffling of all four atoms of the Å-type layer into a B-type layer, Å-type layers are transformed into B-type layers. The transformation is, thus, AAAAAÅ Å → BB BBBB BB [Fig. 3(b)].

### B. Consecutive shuffle

Consider next the consecutive individual shuffle of layers (C), schematically represented in Fig. 4(b). First, we compute the individual shuffling of layer 2 from Å to B. It corresponds to the transformation AAAAAÅ Å → IBIAÅÅÅ. Due to the PBCs, this creates intermediate (I) configurations for the first and third layers. It corresponds to an atom layer sandwiched between one B and one Å layer. Their local environment is, thus, neither bcc nor hcp.

In a second step, starting from the latter configuration, we study the shuffling of the second layer up to the new configuration IBIBIÅÅ. During this transformation, the fourth layer shuffles from configuration A to configuration B. Thus, the third layer now has a full hcp-like configuration around it.
and now is, thus, labeled B. The process can be pursued until all layers have shuffled.

C. Computational details

We use the Projector Wave Augmented\(^{26}\) implementation\(^{27}\) of ABINIT (Ref. \(^{28}\)) in the GGA PBE approximation of DFT. Atomic data include 3s and 3p semi-core states,\(^{28}\) the cut-off radius is 2.0 a.u., the energy cutoff for the plane-wave expansion is 20 Ha, and the convergence criterion for the charge-density residual is \(10^{-9}\). With these atomic data, results of Ref. \(^{27}\) are reproduced. A 18x4x26 special \(k\)-point mesh\(^ {28}\) and a Gaussian smearing of \(5.10^{-4}\) Ha for electronic occupation\(^{30}\) are used in order to obtain a good estimate of the energetic and magnetic properties.

III. RESULTS

A. Simultaneous shuffle

Energies for the simultaneous mechanism are displayed in Fig. 3(a). The reaction coordinate \(\rho\) is taken proportional to the shuffle \(n\) of the supercell atoms and varies from 0 (bcc-like) to 4 (all atoms in hcp-like environment). Near equilibrium \((\rho = 0)\), the energy increases quadratically as expected. The curve exhibits a cusp at 60\% shuffle amplitude due to a change of magnetism from FM to AFMI.\(^{31}\) During this transition, the magnetic moment—reproduced in the inset of Fig. 3(c)—changes abruptly from \(2.2\mu_B/\text{atom}\) to \(1.1\mu_B/\text{atom}\) in agreement with Ref. \(^8\) where, however, AFMI order was not considered. This confirms the importance of magnetism for this transition.\(^{27,29}\) Without any change in magnetism, the transition would not occur. In the rightmost part of the curve, the energy decreases to a local minimum corresponding to a deformed hcp phase. We computed the same curve with a cell of four atoms allowing for the creation of AFMII order.\(^{31}\) We find, in agreement with Refs. \(^{32}\) and \(^8\) that AFMII is more stable for the hcp structure \((\rho = 4)\) compared to AFMI. However, and interestingly, we also find that past the cusp, the AFMI structure is more stable than the AFMII. This highlights the important role of complex magnetism for the DFT description of the transition path and not only for the stable structure.\(^{27}\) A complete study of magnetism, especially noncollinear, lies beyond the goal of our paper. We nevertheless show that an AFMI description of the transition path is particularly adapted to the present case. We call \(E^{\text{S1}}(\rho)\) the combination of the FM and AFMI curves. The energy profile qualitatively agrees with previous results, involving a smaller unit cell with two atomic layers (Figs. 5 and 6 of Ref. \(^5\) even if Ekman \emph{et al.} have considered a NM ground state for the hcp phase and the PW91 GGA functional.

B. Consecutive shuffle

Energy plots associated with the consecutive mechanism are drawn in Fig. 3(a). From \(E^{\text{S1}}(\rho)\), one can trivially compute the energy of shuffle per atom and, thus, can simulate the consecutive motion of four layers under the assumption that the energy for the shuffling of only one layer is equal to that for shuffling all layers, divided by the number of layers. This approximation would be correct if there were no interactions between layers. For this mechanism, because all atoms are not shuffling at the same time, \(\rho\) describes the consecutive shuffling of all the layers. Formally, from value \(n - 1\) to value \(n\), \((\rho - (n - 1))\) is, thus, proportional to the displacement of the atoms of the \(n\)th layer. As a consequence, for \(\rho = 0\) and \(\rho = 4\), whatever the mechanism, all atoms are in the same positions. The expression for \(E^{\text{CIS}}(\rho)\) between \(n - 1\) and \(n\) is as follows:

\[
E^{\text{CIS}}(\rho) = \frac{E^{\text{S1}}(4\rho - (n - 1))}{4} + \frac{(n - 1)E^{\text{S1}}(\rho = 4)}{4}.
\]

As a consequence, \(E^{\text{CIS}}(0)\), \(E^{\text{CIS}}(1)\), \(E^{\text{CIS}}(2)\), \(E^{\text{CIS}}(3)\), and \(E^{\text{CIS}}(4)\) lie on the same line (dashed red in Fig. 4). Trivially, this mechanism reduces the total energy barrier: In Fig. 4 the energy barrier \(E^{\text{S1}}(\rho \simeq 3.53)\) is lower than the energy barrier \(E^{\text{S1}}(\rho \simeq 2.30)\) in the inset of Fig. 3(c). Finally, the third curve \((E^{\text{CIS}})\) corresponds to the explicit calculation of the consecutive motion of four layers.

C. Discussion

We focus next on the comparison between the model of independent layers (CIS\(_1\)) and the true calculation (C\(_1\)). The two curves show the same main tendencies with four energy
barriers corresponding to the shuffling of the four atoms and three metastable structures corresponding to the shuffling of only one, two or three layers. However, the energy of the three metastable configurations differs by only \( E_i = E^{C_1}(1) - E^{CBS_1}(1) = 60 \text{ meV} \) and is, thus, a constant independent of the number of shuffled layers. This suggests that \( E_i \) comes from the two interfaces (in blue in Fig. 4) between shuffled and unshuffled layers. The number of such interfaces is constant and independent of the number of layers.

To assess the electronic origin of these interface energies, Fig. 5 represents the evolution of the d-magnetic moment for the eight atoms of the supercell as a function of the reaction coordinate \( \rho \). The local moments strongly depend on local coordination, in agreement with the generalized Stoner mechanism for the appearance of magnetism as a function of bandwidth. For example, when the fourth atom goes from a bcc-like local configuration to a more compact hcp-like local configuration, its local moment changes from 2.2 \( \mu_B \) to -1.1 \( \mu_B \) with no substantial difference with the simultaneous mechanism. At the same time, the moment of the neighboring atom 5 changes from 2.2 \( \mu_B \) to 1.7 \( \mu_B \) only, because atom 5 has now a local environment intermediate between the hcp and bcc. Once atom 6 has moved, atom 5 ends up (for \( \rho = 3 \)) with a full hcp-like local environment and a moment of 1.1 \( \mu_B \). This demonstrates: (i) the absolute correlation between magnetism and total energy in the transition and (ii) the link between interfacial energy and layers with intermediate local moment and coordination. The interfacial energy only depends on the number of interfaces, which is constant in the present paper. Remarkably, energy barriers are not shifted by this interfacial energy.

For \( \epsilon_1 = 0.5 \), one notices that the energy progressively increases for all the metastable states up to the final energy gain of -0.5eV. An additional calculation is performed for a shear of \( \epsilon_3 = 0.875 \) (green lines in Fig. 2 and Fig. 4). The resulting energy exhibits the same characteristics as for \( \epsilon_1 = 0.5 \), and notably an interface energy of 100 meV but with an overall gain after shuffle of +0.45 eV. For \( \epsilon_3 = 0.875 \) and only around \( \rho = 3 \), a new magnetic order—not stable for \( \epsilon_1 = 0.5 \)—diminishes the interface energy to 52 meV. It embodies both FM and AFM interactions between the layers and highlights the energetic proximity of several magnetic orders. Further studies, with larger supercells, would be necessary to study this magnetic order, but this is not the goal of the present paper.
Two shear thresholds for the bulk transformation can be estimated. A simple linear interpolation between the shuffle curves $C_1$ and $C_3$ produces the following energy for a shear $\epsilon_2$ such that $\epsilon_1 \leq \epsilon_2 \leq \epsilon_3$ and the shuffle $\rho$:

$$E(\rho, \epsilon_2) = E^{C_1}(\rho) + \frac{E^{C_3}(\rho) - E^{C_1}(\rho)}{\epsilon_3 - \epsilon_1}(\epsilon_2 - \epsilon_1).$$

A lower nucleation-threshold (LT) shear $\epsilon_2^{LT} \approx 0.7$ is defined as the shear above which the gain $E(\rho = 0, \epsilon_2) - E(\rho = 4, \epsilon_2)$ becomes positive. To complete the transformation, the interface energy has to be overcome for the first layer only, leading to shuffling of subsequent layers at no additional cost. For a large number of layers, thermodynamical equilibrium, thus, drives the transition to completion. Also, an upper-nucleation threshold (UT) shear of $\epsilon_2^{UT} \approx 0.84$ is estimated from the linear interpolation. It is defined as the shear above which the gain $E(\rho = 0, \epsilon_2) - E(\rho = 1, \epsilon_2)$ for the first metastable state becomes positive. Above this shear, even the first shuffle occurs at no energy cost. Then, all subsequent shuffles are strongly favored, possibly leading to propagation and to a global transition.

Consequences on the kinetics of the $\alpha$-$\epsilon$ transition are as follows. Shuffle can be viewed as a thermally-activated kinetic process due to vibrations of atoms.\textsuperscript{5,6} As emphasized above, the energy barrier per atom $E_0$, which defines the activation energy, is independent of the interfacial energy so that the shuffle of a single layer has, thus, no supplementary activation cost. In this perspective, the above provides an alternative mechanism for the transition in which the bulk shuffle transformation nucleates from one single shuffle event, possibly initiated by some initial shear $\epsilon_2$, such as, e.g., in a shock wave. This mechanism considerably reduces the amount of energy needed to trigger the transformation, the dimensionality of the nucleation process being reduced from 3 to 2.

In order to describe a transition corresponding to a shuffle for a domain, a complete thermodynamical description of this mechanism would be necessary. This would require relaxing atoms after each shuffle, and is beyond the scope of our paper.

### IV. CONCLUSION

In conclusion, we utilize the fact that shuffle—or optical phonon mechanism—and macroscopic compression shear—have in general different timescales, which allows for their decoupling. This is true for iron but also for other systems such as Zr, Ba, or related transitions, such as Pu.\textsuperscript{7} Second, in iron bcc-hcp transition, we show that the transition-path description requires some peculiar magnetic order different from the magnetic order of equilibrium phases. Third, we highlight a new mechanism for this transition based on successive shuffle motion of layers. It indicates that each layer itself could move by developing “shuffle dislocations” in a manner akin to slip motion in plasticity. Our paper suggests that detailed investigations of nucleation mechanisms would be required to resolve the frontier between the military and the thermally activated characters of solid-solid phase transitions.

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\* bernard.amadon@cea.fr

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AFMI is the AFM coupling of layers in the [0001] direction, whereas AFMII is an AFM coupling inside the [0001] plane: See, e.g., Fig 1 of Ref. 8.