HYR²PICS: Hybrid Regularized Reconstruction for combined Parallel Imaging and Compressive Sensing in MRI.

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Abstract

Both parallel Magnetic Resonance Imaging (pMRI) and Compressed Sensing (CS) are emerging techniques to accelerate conventional MRI by reducing the number of acquired data in the *k*-space. So far, first attempts to combine sensitivity encoding (SENSE) [1] imaging in pMRI with CS have been proposed in the context of Cartesian trajectories. Here, we extend these approaches to non-Cartesian trajectories by jointly formulating the CS and SENSE recovery in a hybrid Fourier/wavelet framework and optimizing a convex but nonsmooth criterion. On anatomical MRI data, we show that HYR²PICS outperforms wavelet-based regularized SENSE reconstruction. Our results are also in agreement with the Transform Point Spread Function (TPSF) or *mutual coherence* criterion that measures the degree of incoherence of *k*-space undersampling schemes.

Overview: Parallel Imaging

In parallel MRI, an array of L coils is used to measure the spin density ρ into the object under investigation.

$$d_\ell = \boldsymbol{\Sigma}_s \boldsymbol{F} \boldsymbol{S}_\ell \rho + n_\ell$$

- $\rho \in \mathbb{C}^n$ is the full FOV image,
- $d_\ell \in \mathbb{C}^m$ the signal received by each coil ℓ,
- $\boldsymbol{S}_\ell : \mathbb{C}^n \to \mathbb{C}^n$ denotes a sensitivity operator,
- $\boldsymbol{\Sigma}_s : \mathbb{C}^n \to \mathbb{C}^m$ represents the sampling operator, $\boldsymbol{\Sigma}_s = [e_{i_1}, \cdots, e_{i_m}]^*$,
- $\boldsymbol{F}$ denotes the discrete Fourier Transform, and $\boldsymbol{F}^* = \boldsymbol{F}^{-1}$,
- n_ℓ an additive Gaussian white noise of variance σ_ℓ, the between-channel covariance is assumed diagonal $\boldsymbol{\Lambda} = \mathrm{diag}\left[\sigma_1^2 \mathbf{Id}, \cdots, \sigma_L^2 \mathbf{Id}\right]$.

Combining Compressed Sensing and pMRI reconstruction

As pointed out in [2], MR images are usually sparse in wavelet domain. Let $\boldsymbol{\Psi} = [\Psi_1, \ldots, \Psi_p] \in \mathbb{C}^{n \times p}$, $p \geq n$ design a 2D orthonormal wavelet transform with ζ the wavelet coefficients such that $\rho = \sum_{i=1}^p \zeta(i) \Psi_i = \boldsymbol{\Psi}\zeta$.

Compressed Sensing and Sparsity

The compressive sensing theory ensures that ρ can be recovered precisely with only few observations $d(k)$, with $k \in \{1, \ldots, m\}$ and $m \ll n$, by computing $\widehat{\rho} = \boldsymbol{\Psi}\widehat{\zeta}$ where

$$\widehat{\zeta} \in \underset{\zeta \in \mathbb{C}^p}{\arg\min} \|\zeta\|_1 \quad \text{s.t.} \quad d(k) = \langle \rho, \boldsymbol{\phi}_k \rangle, \; \forall k = 1 : m.$$

Hybrid regularization

The reconstruction problem can be expressed as the optimization of

$$\widehat{\zeta} \in \underset{\zeta \in \mathbb{C}^p}{\arg\min} \left[\mathcal{J}_{\mathrm{WLS}}(\boldsymbol{\Phi}\boldsymbol{\Psi}\zeta) + \mathcal{J}_S(\zeta) + \lambda_A \mathcal{J}_A(\boldsymbol{\nabla}\boldsymbol{\Psi}\zeta)\right]. \qquad (1)$$

- $\mathcal{J}_{\mathrm{WLS}}(\boldsymbol{\Phi}\rho) = \sum_{\ell=1}^L \sigma_\ell^{-1} \|d_\ell - (\boldsymbol{\Phi}\rho)_\ell\|^2$ denotes the fidelity data term, and $\boldsymbol{\Phi} = [\boldsymbol{\Sigma}_s \boldsymbol{F} \boldsymbol{S}_1; \ldots; \boldsymbol{\Sigma}_s \boldsymbol{F} \boldsymbol{S}_L]$ is the observation operator.
- $\mathcal{J}_S(\zeta)$ is an ℓ_1-like prior which promotes sparsity of the recontructed solution,
- $\mathcal{J}_A(\boldsymbol{\nabla}\rho) = \sum_{i=1}^n \varphi_A\left(\sqrt{(\boldsymbol{\partial}_1\rho)(i)^2 + (\boldsymbol{\partial}_2\rho)(i)^2}\right)$, φ_A is a Huber function and $\lambda_A \geq 0$. $\mathcal{J}_A$ is a Total Variation like prior

Primal Dual Optimization

We make use of the Chambolle-Pock primal-dual method [3], which has the following properties:

- "optimal" $O\left(\frac{1}{k}\right)$ convergence rate for the problem under consideration,
- it only requires matrix-vector multiplications,
- precise enough solutions in around 50 low-cost iterations.

Problem (1) can be rewritten as: $\min_{x \in \mathbb{C}^p} \mathcal{F}(\boldsymbol{A}x) + \mathcal{G}(x).$

- $\mathcal{F}(y) = \mathcal{J}_{\mathrm{WLS}}(y_1) + \lambda_A \mathcal{J}_A(y_2)$ with $y = [y_1; y_2] \in \mathbb{C}^{mL} \times \mathbb{C}^{2n}$,
- $\mathcal{G}(x) = \mathcal{J}_S(x)$ and $\boldsymbol{A} = [\boldsymbol{\Phi}\boldsymbol{\Psi}; \boldsymbol{\nabla}\boldsymbol{\Psi}]$.

Chambolle-Pock implementation.

$$\begin{cases} y_{k+1} = \overbrace{(\mathbf{Id} + \sigma\boldsymbol{\partial}\mathcal{F}^\dagger)^{-1}}^{=\mathrm{prox}_{\sigma\mathcal{F}^\dagger}} (y_k + \sigma \boldsymbol{A}\bar{x}_{k+1}) & \text{Dual descent} \\ x_{k+1} = \underbrace{(\mathbf{Id} + \tau\boldsymbol{\partial}\mathcal{G})^{-1}}_{=\mathrm{prox}_{\tau\mathcal{G}}} (x_k - \tau \boldsymbol{A}^* y_{k+1}) & \text{Primal descent} \\ \bar{x}_{k+1} = 2x^{k+1} - x^k & \text{Correction step (due to the scheme asymmetry)} \end{cases}$$

- $\sigma\tau = L^2$ where L is the highest singular value of $\boldsymbol{A}$,
- $(\mathbf{Id} + \boldsymbol{\partial}\mathcal{F})^{-1}(u) = \underset{v \in \mathbb{R}^n}{\arg\min}\, \mathcal{F}(v) + \|v - u\|^2/2$ is the resolvent (or proximal operator) of $\mathcal{F}$ at point u.

Optimizing the undersampling scheme

Mutual coherence.

$$\mu = \mu(\boldsymbol{\Phi}\boldsymbol{\Psi}) = \max_{1 \leq i, j \leq p, i \neq j} \frac{|\langle \boldsymbol{\Phi}\boldsymbol{\Psi}e_i, \boldsymbol{\Phi}\boldsymbol{\Psi}e_j \rangle|}{\|\boldsymbol{\Phi}\boldsymbol{\Psi}e_i\| \cdot \|\boldsymbol{\Phi}\boldsymbol{\Psi}e_j\|}$$

One of the typical results relating coherence to sparsity states that if $\rho = \boldsymbol{\Psi}\zeta$ is s-sparse and $s \leq \frac{1+1/\mu}{2}$ then the exact recovery of ρ can be achieved by solving $\widehat{\zeta} \in \underset{\zeta \in \mathbb{C}^p}{\arg\min} \|\zeta\|_1$ s.t. $d(k) = \langle \rho, \boldsymbol{\phi}_k \rangle$, $\forall k = 1 : m$. [4, 5].

The smaller the coherence, the fewer samples are needed for perfect reconstruction (noise-free case).

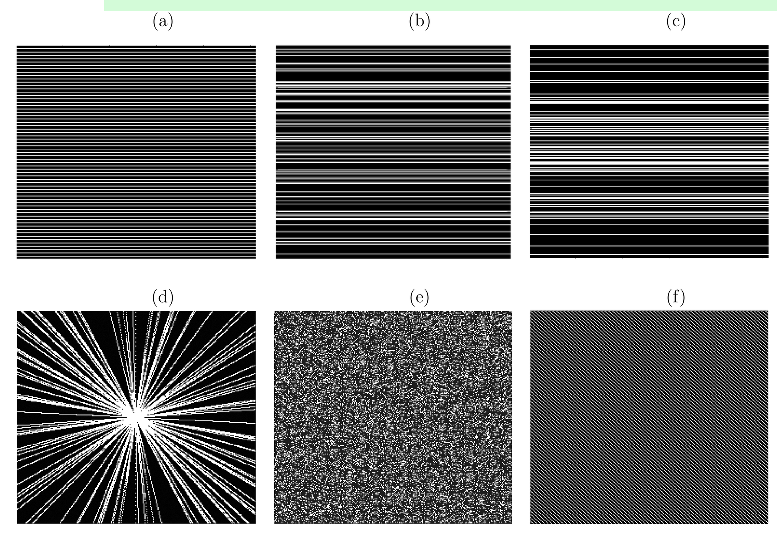


Figure 1: **Various k-space sampling schemes for $R = 4$:** selected points appear in white color. (a) pMRI line undersampling. (b)-(c): Pseudo-random line undersampling with uniform (b) and Gaussian (c) distributions. (d): Radial scheme with uniformly random angles. (e): 2D random points. (f): checkerboard scheme.

Figure 2: Mutual coherence μ in pMRI for $R = 2$ and $R = 4$.

Undersampling scheme	μ ($R=2$)	μ ($R=4$).
Regular lines	0.2770	0.6774
ND lines	0.4850	0.7471
UD lines	0.3336	0.4602
Random points	0.1040	0.1739
Checkerboard	02373	0.4020
UD angled radial	0.2526	0.6233
Equi-angled radial	0.2637	0.4234
Spiral scheme	0.3214	0.3499

Reconstruction results

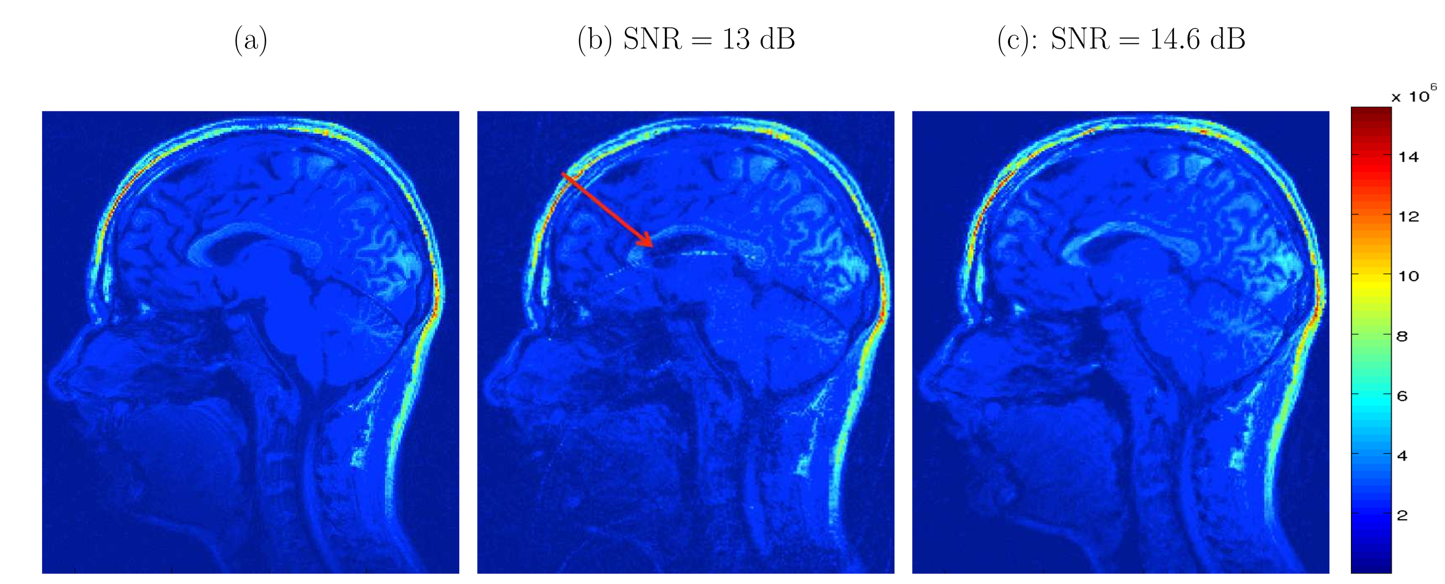
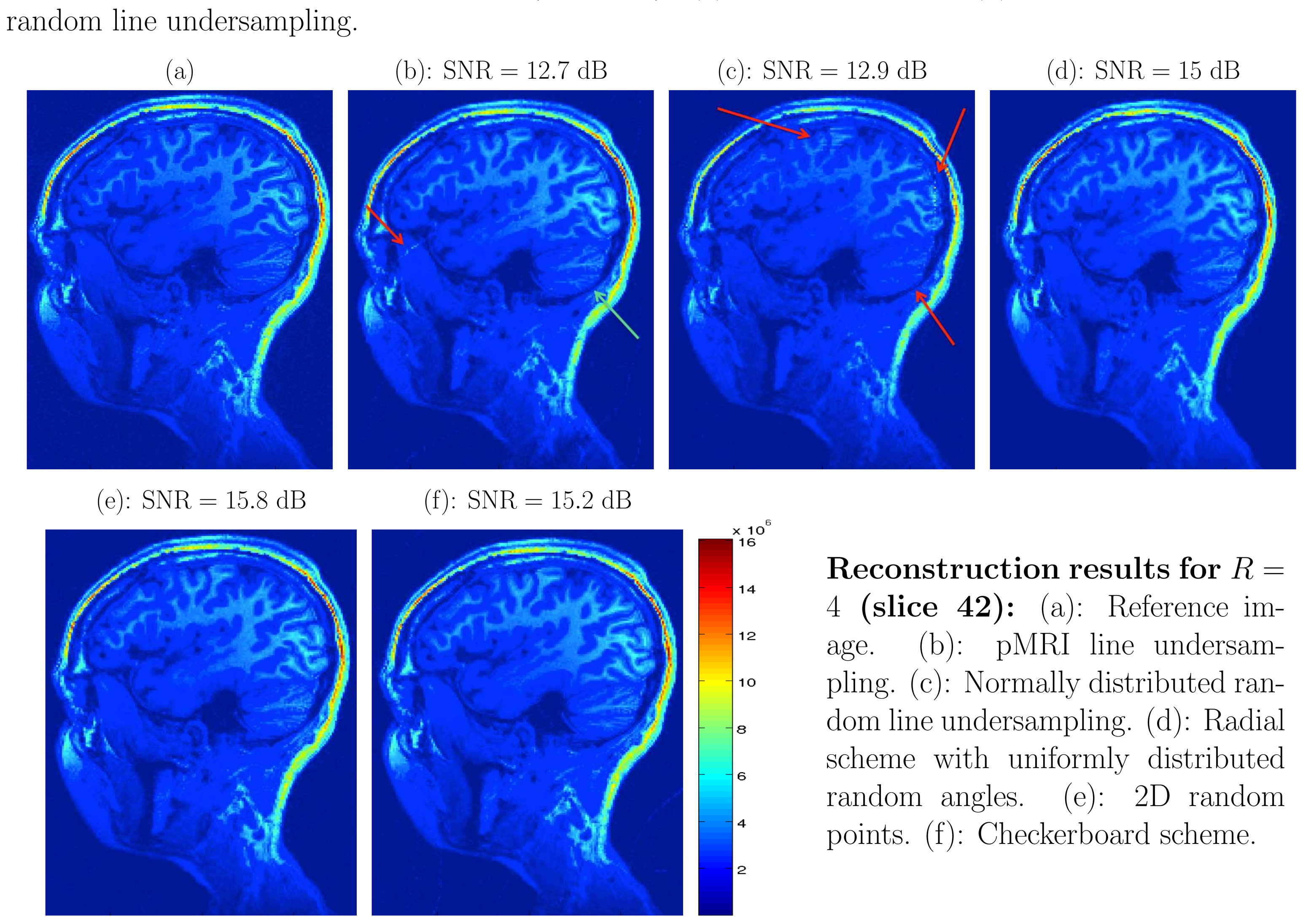


Figure 3: **Reconstruction results for $R = 4$ (slice 82):** (a): Reference image. (b):SENSE imaging. (c): ND random line undersampling.



Reconstruction results for $R = 4$ (slice 42): (a): Reference image. (b): pMRI line undersampling. (c): Normally distributed random line undersampling. (d): Radial scheme with uniformly distributed random angles. (e): 2D random points. (f): Checkerboard scheme.

Conclusion

- Joint formulation of compressive sensing with SENSE imaging;
- Hybrid regularization combing "analysis" and "synthesis" priors;
- Optimal primal-dual optimization algorithm;
- Reconstruction results in agreement with mutual coherent assessment performed prior to the reconstruction.

Perspectives

- 3D wavelets for 3D T1-weigthed MPRAGE MRI sequences;
- Non cartesian trajectories (eg spiral) for sampling low frequencies more densely;
- New solutions to the synthesis problem in compressive sensing [6]: Optimal algorithms for designing sparse k-space trajectories while allowing exact image reconstruction by ℓ_1 minimization in the noise-free case.

References

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