Understanding the RBMK behavior at low power

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Technical note : Understanding the RBMK behavior at low power

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Abstract : It is well known that the RBMK (Reaktor Bolshoy Moshchnosti Kanalniy) reactors could be unstable at low power. But what does it mean precisely ? By using a 2x2 system of non-linear ordinary differential equations and a more accurate 8x8 system derived in [1] we show that naturally (i-e without using the control rods), with the same reactivity injection, the lower the initial power then the higher the final power, which is a rather unusual behaviour.

1. Introduction :

About the Chernobyl accident, INSAG-7 [2] has no answer to the following question :

- How does the power level of the RBMK affect the course of the accident ?

Our paper aims at giving a qualitative answer to this basic question.

For a nuclear reactor without reactivity feedback, i.e. the so-called "zero" power reactor for which reactivity feedback can be neglected, the linear theory applies.

Let ρ denote the reactivity injected in the core, the linear theory predicts that if $\rho > 0$ (resp. $\rho < 0$) then the number of neutrons $n(t)$ increases (resp. decreases) exponentially.

In reactivity accidents, the kinetics is fast and the reactivity feedback consists mainly of the Doppler effect. In the Nordheim-Fuchs model (see [3]) the fuel is supposed to be adiabatic (no heat exchange with the coolant, e.g. water for PWR), which is valid for times of the order of 1s only.

In the present paper we consider slower reactivity injections, so that the counter reactions consist both of the Doppler effect, and the coolant effect.

In the RBMK, the moderator is graphite which has a great thermal inertia, so that its temperature changes slowly (~ 45 min)(see [4]). On the contrary heat exchange between fuel and coolant are faster, say at a timescale of 1 to 10s. Since the RBMK is overmoderated, and since it is a boiling water reactor, the coolant density decreases significantly when the core energy increases which reduces the neutron absorption in water and then tends to increase the core reactivity, while the Doppler effect tends to reduce it. So the overall reactivity feedback coefficient is the difference of two terms : the Doppler effect dominates above 50% NP (NP= Nominal Power), whereas the coolant effect dominates below 50%NP. The coolant effect depends on void coefficient and then on burn-up.

From now on, we shall denote by $n(t)$ the ratio of the core power at time t to the nominal power. For normal operations $(t) \in (0,1)$.

The following model has been introduced in [1]

(1) $$\frac{d}{dt}\begin{pmatrix}\rho \\ n\end{pmatrix} = \begin{pmatrix}\frac{\alpha}{\tau}\,\rho\, n \\ \frac{\rho}{\tau}\, n\end{pmatrix}$$

Where ρ denotes reactivity, τ denote the effective neutron life time, and α the reactivity feedback coefficient *(also called power reactivity coefficient [2]).*

In [1], which was dedicated to Pressurized Water Reactors, α was assumed to be negative and constant. To take into account the RBMK behavior which is unstable at low power, we shall assume in the present paper that $\alpha = \alpha(n)$.

More precisely, we shall assume that

$$\alpha(n) = n\,\alpha_1 + (1-n)\alpha_0.$$

with $\alpha_0 > 0$ and $\alpha_1 < 0$.

Let $n_* = \frac{\alpha_0}{\alpha_0 - \alpha_1} > 0$, we see that $\alpha(n)$ is positive for $n < n_*$ and negative otherwise.

We shall prove that $n(t)$ *has a finite limit when* $t \to \infty$ and that $\rho(t) \to 0$, which means that there is no divergence. However, this does not mean that the reactor is stable as we shall see.

Remark 1 : The fuel temperature, and then the Doppler effect, depends directly on the core power. However the coolant effect depends on the water density which depends on the core power but also on the coolant flow rate.
In our model, we have implicitly assumed that the average water density was just depending on the core power.
In fact, the power coefficient of reactivity was introduced for practical use by the reactor operator.■

2. Preliminary remarks:
The curves $t \to \{\rho(t), n(t)\}$ are following a parabola in the plane $\{\rho, n\}$; in fact

$$\frac{d\rho}{dn} = \frac{d\rho/dt}{dn/dt} = \frac{\alpha\ \rho\, n/\tau}{\rho\, n/\tau} = \alpha(n) = \alpha_0 + n(\alpha_1 - \alpha_0)$$

implies that there exists a constant ρ_0 such that

(2) $\quad \rho(n) = \rho_0 + \alpha_0 n + (\alpha_1 - \alpha_0)\frac{n^2}{2}$

so that $\rho(n) \uparrow$ while $n < n_*$ and $\rho(n) \downarrow$ while $n > n_*$.

If we start from the initial point $\{n_i, \rho_i\}$ we first evaluate $\rho_0 = \rho_i - \alpha_0 n_i - (\alpha_1 - \alpha_0)\frac{{n_i}^2}{2}$.

We shall prove in §4 that $n(t) \to n_\infty$ where $\{n_\infty, 0\}$ is on the parabola and on the horizontal axis.

Remark 2 : Dividing the first equation of system (1) by ρ and the second one by n we obtain

(3) $\quad \frac{d}{dt}\begin{pmatrix} Log\ \rho \\ Log\ n \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\tau}\ \ n \\ \frac{\rho}{\tau} \end{pmatrix}$

Obviously, if we start from the quarter plane $\{\rho > 0\ n > 0\}$, we stay there.
For the quarter plane $\{\rho < 0\ n > 0\}$, the first equation is replaced by

$$\frac{d}{dt} Log\ (-\rho) = \frac{\alpha}{\tau}\ \ n,$$

and in the same way, if we start from the quarter plane $\{\rho < 0\ n > 0\}$, we stay there. ■

§3 Adjustment of α_0 and α_1.

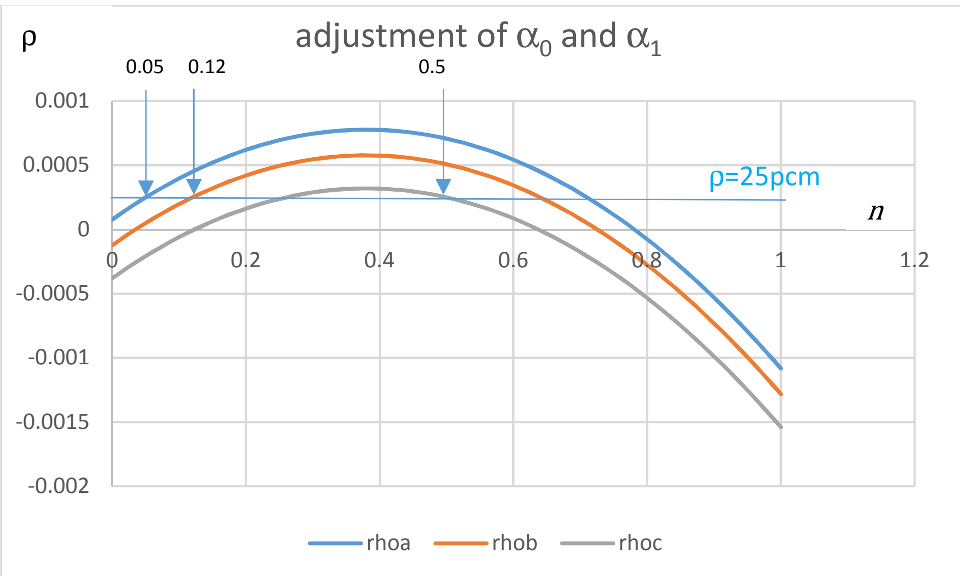

Fig 1 : parabolas starting from $n_i = 0.05$, $n_i = 0.12$, $n_i = 0.5$ and $\rho_i = 25\ pcm$

The following numbers have been communicated to us [5] for $\rho_i = 25\ pcm$:
When $n_i = 0.05$ then $n_\infty = 0.93$.
When $n_i = 0.12$ then $n_\infty = 0.68$.
When $n_i = 0.5$ then $n_\infty = 0.58$.
We have selected $\alpha_0 = 368\ pcm$ and $\alpha_1 = -600\ pcm$ to fit these results.
With such data we get $n_\infty = 0.78$, $n_\infty = 0.725$ and $n_\infty = 0.63$ which is not very far from the communicated results which were obtained for a burn-up like it was on the day of the accident.
In any case, these results show that the RBMK was not stable at low power (say below $n^* = 0.38$) that is in the domain where the curve $n \to \rho$ is increasing.
Indeed, with the same safety injection (25 pcm) the final power is larger while the initial power is smaller. However unstability does not mean divergence. ■

4. Solution of system (1)
We shall proceed by separation of variables.
We use (2) to eliminate ρ.
We get

$$\frac{dn}{dt} = \frac{1}{\tau}\, n\left[\rho_0 + \ \alpha_0 n + (\alpha_1 - \alpha_0)\frac{n^2}{2}\right]$$

We call n_1 and n_2 the solutions of

$$\rho_0 + \ \alpha_0 n + (\alpha_1 - \alpha_0)\frac{n^2}{2} = 0$$

And we let $\gamma = \frac{\alpha_1 - \alpha_0}{2}$ so that (see remark 4) :

$$\rho_0 + \ \alpha_0 n + (\alpha_1 - \alpha_0)\frac{n^2}{2} = \gamma(n - n_1)(n - n_2)$$

That is

$$\frac{dn}{n(n-n_1)(n-n_2)} = \gamma\frac{dt}{\tau}$$

We check that

$$\frac{1}{n(n-n_1)(n-n_2)} = \ \frac{a}{n} + \frac{b}{n-n_1} + \ \frac{c}{n-n_2} \qquad \text{with } \ a = \frac{1}{n_1 n_2},\ b = \frac{1}{n_1(n_1-n_2)},\ c = \frac{1}{n_2(n_2-n_1)}$$

So that we get

$$\left[\frac{a}{n} + \frac{b}{n-n_1} + \ \frac{c}{n-n_2}\right] dn = \gamma\frac{dt}{\tau}$$

So, by integrating both sides we get

$$[a\ Log\ (n) + b\ Log(n - n_1) + c\ Log(n - n_2)] = \gamma\frac{t}{\tau} + B$$

$$Log\ \left(n^a (n - n_1)^b (n - n_2)^c\right) = \gamma\frac{t}{\tau} + B$$

To get the value of B, we apply this relation at $t = 0$:

$$B = Log\ \left({n_i}^a (n_i - n_1)^b (n_i - n_2)^c\right).$$

Finally we get

$$\gamma\frac{t}{\tau} = Log\ \left(\left(\frac{n}{n_i}\right)^a \left(\frac{n-n_1}{n_i-n_1}\right)^b \left(\frac{n-n_2}{n_i-n_2}\right)^c\right)$$

(4) $$\left(\frac{n}{n_i}\right)^a \left(\frac{n-n_1}{n_i-n_1}\right)^b \left(\frac{n-n_2}{n_i-n_2}\right)^c = \exp(\gamma\frac{t}{\tau})$$

Since $\gamma < 0$, we see that $\exp(\gamma\frac{t}{\tau}) \to 0$ as $t \to \infty$.
Let us assume that $n_1 < n_2$, then n_1 may be positive or negative.

1. $n_1 < 0$ then $a < 0, b > 0, c > 0$ $n(t)$ cannot tend to zero, therefore $n(t) \to n_2$
2. $n_1 > 0$ then $a > 0, b < 0, c > 0$ $n(t)$ cannot tend to n_1, therefore either $n(t) \to 0$ or $n(t) \to n_2$
   - If $n_i > n_2$ then $\rho_i < 0$ so that $n(t)$ decreases from n_i to n_2

- If $n_1 < n_i < n_2$ then $\rho_i > 0$ so that $n(t)$ increases from n_i to n_2
- If $0 < n_i < n_1$ then $\rho_i < 0$ so that $n(t)$ decreases from n_i to 0

Remark 3 : The case where the parabola has no intersection with the horizontal axis should not be excluded so that n_1 and n_2 will not be real numbers, the above proof will not work but this is no problem since $\rho(t) < 0$ so that $n(t)$ decreases rapidly to zero.■

In conclusion we have shown that $n(t) \rightarrow n_\infty$ where n_∞ is either n_2 or 0.

5. A more accurate model

To get a more accurate model we should start from the 6 groups model (see REUSS [6]) : that is the following 7 x 7 differential system :

$$(5) \qquad \begin{cases} \frac{dn}{dt} = \frac{\rho-\beta}{\ell_0} n + \sum_i \lambda_i \, c_i \\ \frac{dc_i}{dt} = \frac{\beta_i}{\ell_0} n - \lambda_i c_i \; 1 \leq i \leq 6 \end{cases}$$

where $n = n(t)$ is the (normalized) number of neutrons, ℓ_0 neutron's life time, $c_i = c_i(t)$ the (normalized) number of precursors, λ_i the associated decreasing constant° i, and $\beta = \sum_i \beta_i$ the delayed neutrons fraction.
For uranium-based fuel, numerical values of these parameters can be found in ref [6].
To take the reactivity feedback into account we decide that ρ is the 8th variable and we complement this 7x 7 system by an 8th equation:

$$(6) \qquad \frac{d\rho}{dt} = \alpha(n) \frac{dn}{dt}$$

We also specify some initial conditions :

$$n(0) = n_0 \, ; c_i(0) = c_i^0 \, ; \rho(0) = \rho_0$$

We specify that $\frac{\beta_i}{\ell_0} n_0 - \lambda_i c_i^0$ should be equal to 0.

We can expect, like above, that $\rho(t) \rightarrow 0$ when $t \rightarrow \infty$. On the other hand from (6) we deduce that $\{n(t), \rho(t)\}$ remains on the same parabola as for the simple model.

	$T_i\ (s)$	$\beta_i\ (pcm)$
$i = 1$	78.6	24
$i = 2$	31.5	123
$i = 3$	8.6	117
$i = 4$	3.2	262
$i = 5$	0.7	108
$i = 6$	0.26	45

Tab 1 periods and proportions of the 6 precursor groups

Taking $\frac{dn}{dt} = \frac{dc_i}{dt} = 0$ in (5) we see that

$$n(t) \rightarrow n_\infty$$
$$c_i(t) \rightarrow \frac{\beta_i}{\ell_0} n_\infty$$

We obtained some numerical results with $\ell_0 = 20\ \mu s$ $\lambda_i = 1/T_i$ where T_i, β_i are given in table 1 : see Reuss [6]. We also specify $\frac{\beta_i}{\ell_0} n_0 - \lambda_i c_i^0 = 0$ which means that the precursors were in equilibrium with $\rho = 0$ before the reactivity injection ρ_i.

Here is what we find with $\rho_i = 25\ pcm$ and $\Delta t = .02\ s$ (semi-implicit scheme) :

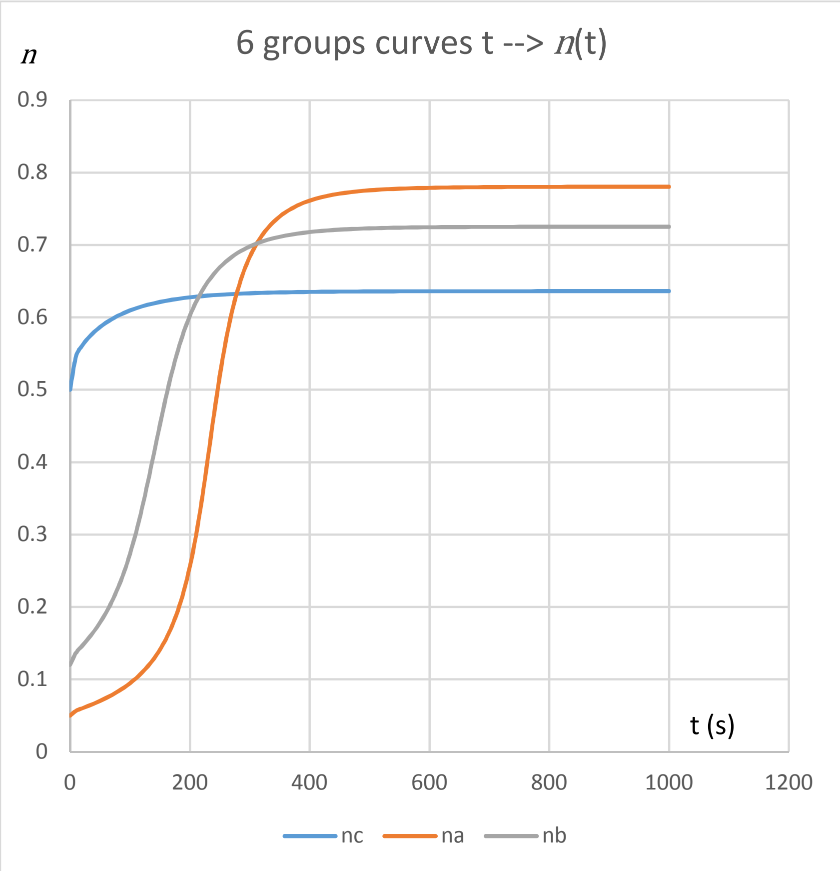

Fig 2 : Curves $t \to n(t)$ obtained with the 6 groups model for (a) $n_i = 0.05$ (b) $n_i = 0.12$ (c) $n_i = 0.5$

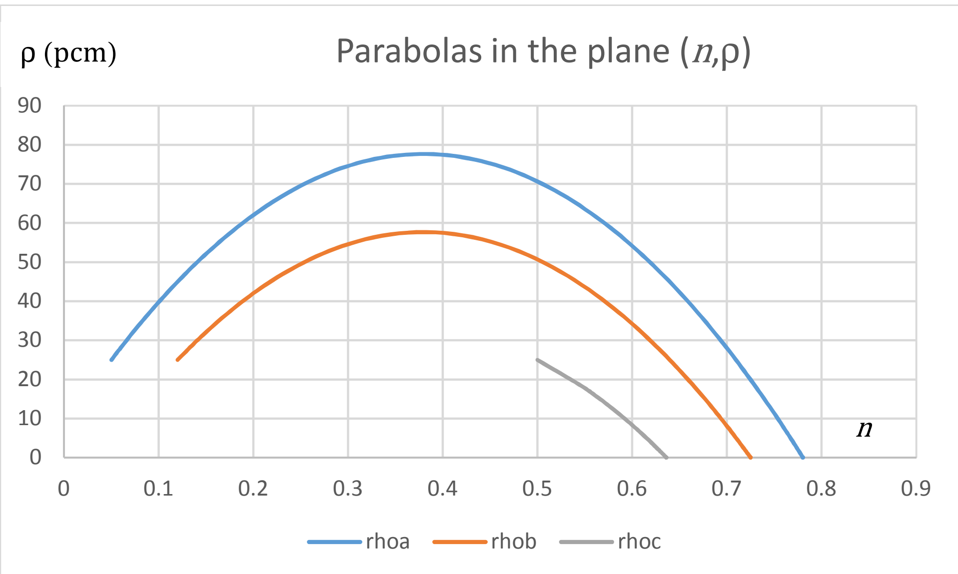

Fig 3 Curves $n \to \rho$ obtained with the 6 groups model for (a) $n_i = 0.05$ (b) $n_i = 0.12$ (c) $n_i = 0.5$

In Fig 2 we show the curves $t \to n(t)$ for 3 starting point (reminder $n = 1$ corresponds to full power).

In Fig.3 we show the 3 parabolas $n \to \rho$ obtained numerically : they coincide with those given in Fig.1.

§6 2x2 vs 8x8 model

In Fig 4 we compare the simplified 2x2 model and the more accurate 8x8 model.

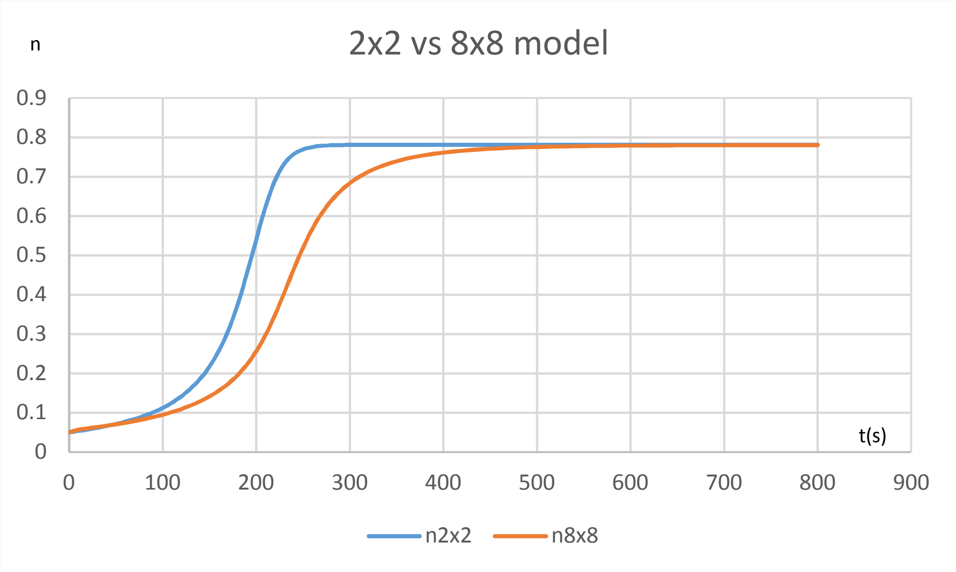

Fig 4 Comparison of the curves $\to n$: 2x2 *model with* $\tau = 0.04\ s$ *vs* 8x8 *model*

We obtain the same limit n_∞ when $\to \infty$. However the kinetics depends in a sensitive way on the choice of the time constant τ in our 2x2 model. With $\tau = 0.05\ s$ we would obtain a better kinetics.

Conclusion

Both our 2x2 and our 8 x 8 models represent qualitatively the behavior of the RBMK just before the accident, even though we can expect a better kinetics with the 8x8 model.

In fact we have shown that naturally (i-e without using the control rods), with the same reactivity injection, the lower the initial power, then the higher the final power, which is a rather unusual behaviour.

Our results explain better why Anatoly Dyatlov has written in [7] :

"The nuclear safety department of the power plant (…) measured the fast power coefficient at power levels close to nominal full power, ie in the region where it was negative. The results were used by the operators in their everyday work. The latest data prior to the accident gave a value of minus 1.7X10^{-4}*ß/MW."*

However, even if the RBMK was unstable at low power, a control law had been set up to regulate the core power, and, for the RBMK4 which exploded in Chernobyl, the problem was that below 480 MW the neutron flux was known only at the median plane of the core so that the control law was based on biased observation.

Keywords :

Reactivity feedback, Reactor kinetics, Unstability, Nonlinear differential systems, RBMK.

Competing interests :

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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