# A non-smooth simulation of the dynamics of the grand piano action

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## Abstract

Two models of the grand piano key mechanism are presented: a single-degree-of-freedom model and a model based on 6 rotating bodies, 13 contact zones with nonlinear springs, 3 of them (hammer-jack, jack-escapement button, hammer-check) being also subject to Coulomb friction. The latter model introduces discontinuities on the velocities. The problems raised by the usual regular-dynamics formulation are discussed and a non-smooth dynamics approach is proposed. Based on the comparison between experimental and simulation results, it is discussed whether the simulation should be driven by the force exerted by the pianist or by the displacement of the key.

## Introduction

The piano action is made of seven rotating bodies (Fig. 1) with parallel axes and felts at contact zones. Simulating the dynamics of the key mechanism (we retain this term for clarity purposes) has several purposes: to validate a mechanical model, to run numerical experiments which account for the effect of the mechanism on the player's finger, to study how modifications are "felt" by the player.

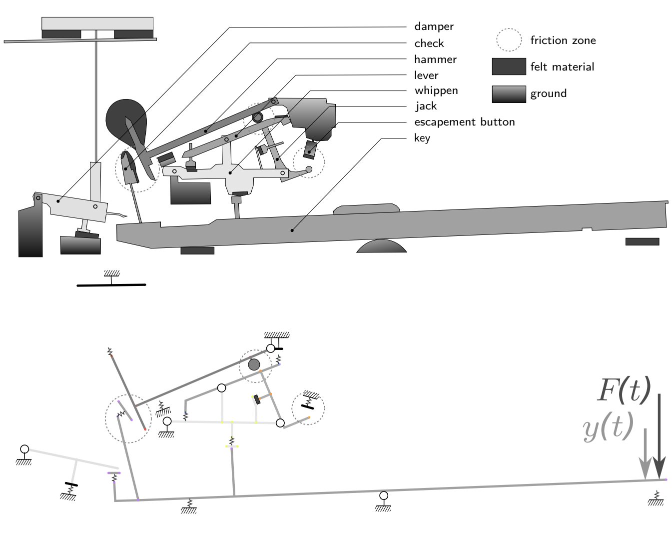


Figure 1: Top: scheme of the grand piano action. Bottom: rigid bodies model.

## 1 Model complexity and simulation input

The key motion $y(t)$ and the force $F(t)$ on the key are given by the dynamics of the mechanism and by the action imposed by the pianist (whose dynamics is also limited). If one does not describe the whole coupled system {mechanism – pianist}, which seems presently out of reach, the simulation of the mechanism only must be driven either by force data or by motion data. However, it has never been clarified whether the mechanism is better described as pseudo-impedance (force reacting to a motion imposed by the pianist) or as a pseudo-mobility (motion resulting from a force imposed by the pianist).

In order to validate a mechanical model, it is customary to compare simulation results with experimental observations. Since the dynamics of the mechanism is dominated by inertia, it appears that one can reduce the model of the whole mechanism to one single degree-of-freedom (following a dynamical equation of the form given by Eq. (1)) and yet obtain an excellent match between experimental measurements and *force-driven* simulation results. However, the corresponding motion-driven simulation results do not compare well with experiments: fine details in the time-evolution of the reacting force $F(t)$ are ignored. In other words, because of the inertia dominance, a force-driven simulation is not sufficient for accounting the details of the piano key mechanism.

## 2 Non-smooth formulation

Since an elementary model is not fully satisfactory, we used a model based on that proposed by Lozada [1]. The 7 bodies are considered as 6 rotating solids with dry and viscous friction on their axes and 13 non-linear and localized coupling springs representing the felts (Fig. 1). Any spring force is generically given by $F(g) = k\, g^r + b\, g^2\, \dot{g}$, where $g$ is the compression length of the spring (felt). The equation describing the dynamics of any rigid body in the model is of generic form:

$$J\,\ddot{\theta} + c_v\,\dot{\theta} + c_d\,\mathrm{sign}(\dot{\theta}) + F(g(\mathbf{x}))\,l + \alpha = 0 \qquad (1)$$

where $J$ is the inertia of the rigid body, $c_v$ is a viscous friction coefficient, $c_d$ is a dry friction coefficient, $\mathbf{x}$ is the vector of generalized coordinates (*i.e.* the 6 angles), $F(g(\mathbf{x}))\, l$ is the moment of the felt force (several such terms may be necessary when more than one felt act upon the considered rigid body) and $\alpha$ contains time-invariant terms such as the moment of gravity, in the small angles approximation. As usual, sign is the set-valued function defined by:

$$\text{sign}(\dot{\theta}) = \begin{cases} 1 & : \dot{\theta} > 0 \\ [-1, 1] & : \dot{\theta} = 0 \\ -1 & : \dot{\theta} < 0 \end{cases} \qquad (2)$$

so that the dry friction is described by the Coulomb model.

Because of dry friction and intermittent contacts, the simulation of the model is complex. One difficulty is that Eq. (1) is not an ODE. Regularizing the sign set-valued function yields ODEs but the convergence to a physical solution when reducing the time step is not ensured. An example can be seen in the equilibrium position: null-velocities imply vanishing regularized friction forces whereas the Coulomb friction generally lets non-zero forces in the system. Another difficulty is that stick-slip transitions induce velocity discontinuities. Furthermore, the evaluation of the moment of the reaction contact forces $F(g(\mathbf{x}))$ is tedious. These difficulties can be overcome by using methods of non-smooth contact dynamics (NSCD).

Instead of writing the dynamics in the form of six coupled equations of the form (1), we use a Measure Differential Inclusion formulation [2]:

$$\begin{cases} \mathbf{M}\mathrm{d}\mathbf{v} & = \mathbf{F}^*(t)\mathrm{d}t + \mathbf{H}(\mathbf{x})\mathrm{d}\mathbf{i} \\ \mathbf{v}^+ & = (\dot{\mathbf{x}})^+ \\ (\mathbf{g}(\mathbf{x}), & \mathbf{H}^T(\mathbf{x}).\mathbf{v}^+,\ \mathrm{d}\mathbf{i}) \in K \end{cases} \qquad (3)$$

The first equation formulates the non-smooth dynamics where $\mathbf{M}$ is the mass matrix, $\mathbf{v}$ is the generalized velocity, $\mathbf{F}^*$ is the regular part of the sum of external forces, including gravity. $\mathrm{d}\mathbf{v}$ and $\mathrm{d}\mathbf{i}$ are vector-valued measures on $\mathbb{R}$ and can therefore be non-smooth. $\mathbf{H}$ relates the relative velocities to the generalized coordinates. The non-smooth laws (Coulomb and articular friction, impacts) and equality constraints are written as an inclusion in the fixed set $K$.

Eqs. (3) are discretized using a time-stepping scheme. Its solution is computed with an implicit scheme. As for smooth ODEs, it requires a root-finding algorithm (Newton's algorithm in our case). The time-discretization of the non-smooth dynamics and the non-smooth laws leads to a One-Step Non-Smooth Problem (OSNSP) [3]. This OSNSP is reformulated using a non-smooth augmented Lagrangian approach and solved using an iterative projective Gauss-Seidel-like method.

## 3 Results

We used XDE (eXtended Dynamic Engine), a software component developed at CEA, LIST. The inputs of the software are the geometrical and inertial descriptions of the pieces (here: the rigid bodies), the properties of the pivots (here: dry and viscous friction) and the contact laws (here: the coupling forces of the springs and the Coulomb friction). The software implements internally the non-smooth formulation of the dynamics and its solution, as described in Sec. 2.

An additional spring/damper association, aimed at representing the softness of the finger, has been inserted between the key mechanism and the (force- or motion-)driver of the mechanism. We measured the position of the key and the force applied by the pianist for several nuances, on one individual key. As for the results obtained with the one-degree-of-freedom model (Sec. 1), the results of a *force-driven* simulation compare correctly with the measured motion. Contrary to the results obtained with the one-degree-of-freedom model, the results of the *motion-driven* simulation compare also correctly with the measured force.

The calculation time ($\approx 20\times$ real-time) on an ordinary laptop computer could be largely improved by taking into account the particularities of the model of the key mechanism.

## References


[1] J. Lozada, *Modélisation, contrôle haptique et nouvelles réalisation de claviers musicaux*, PhD thesis, École Polytechnique, France, 2007.

[2] X. Merlhiot, *On some industrial applications of time-stepping methods for nonsmooth mechanical systems: issues, successes and challenges*, in Euromech Colloquium [516] – Nonsmooth contact and impact laws in mechanics, 2011.

[3] V. Acary and B. Brogliato, *Numerical methods for nonsmooth dynamical systems: applications in mechanics and electronics*, Springer, 2008.